RAFFLES INSTITUTION
2025 Year 5 H2 Mathematics Promotion Examination
Questions and Solutions with comments

General Comments

- **Wrong Exam Index Numbers**
  A small number of students wrote the wrong Exam Index Number on their scripts, though fewer than during the Timed Practice.
  For example, *260227* was mistakenly written as *262027*, which belongs to another student.
  👉 **Please double-check your Index Number** before writing it down in future assessments.

- **Writing Outside the Designated Boxes**
  Quite a significant number of scripts had solutions written *outside* the boxes provided for each part of the question.
  📌 For the actual A-Level exams, your scripts will be *digitally scanned*, and any writing outside the boxes **may not be captured**.
  👉 Always keep your work **within the provided boundaries**.

- **Writing Solutions in the Wrong Question Space**
  More students than in the Timed Practice wrote solutions for one question in the space meant for another question.
  👉 **Do not do this** — different markers may be assigned to different questions, and your answer may be missed if it is in the wrong section.

- **Using Additional Pages and not using the correct Additional Page.**
  A significant number of students who used Additional Pages did not indicate this on the original allocated space. There are also a few students wrote solutions on the provided school foolscap paper and submitted together with the answer booklet
  👉 When you use an Additional Page, **always write "See Additional Page" clearly** in the allocated space, or the marker may miss your continued work.
  👉 The provided school foolscap paper is meant for **rough work**. Any work that needs to be submitted for marking should be inside the Answer Booklet or the Additional Page.

- **Multiple Solutions for the Same Question**
  Some students wrote two or three different solutions for the same question without cancelling the undesired ones.
  👉 You must **cancel any solution** that you do not want to be marked. The marker will only mark 1 solution.

- **Presentation and Legibility**
  Most scripts were well presented, with clear and logical mathematical arguments. 👍
  However, some scripts were difficult to mark because:
  - handwriting was hard to read
  - writing was too small
  - ink was too light

  ✏️ **Reminder:** Markers can only award marks for work that is legible. Please ensure your handwriting is neat, written at a reasonable size, and clearly visible.

1

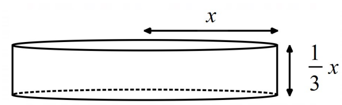


Fig. 1

A tablet is dissolving in water and is modelled as a cylinder, as shown in Fig. 1. At t seconds after being dropped into the water, the radius of the tablet is x mm, and its thickness is $\frac{1}{3}x$ mm. The circular cross-sectional area of the tablet is decreasing at a constant rate of 0.5 mm² per second. When $x = 5$, find the exact value of

(a) $\frac{\mathrm{d}x}{\mathrm{d}t}$, [2]

(b) the rate of change of the volume of the tablet. [2]

(a) **[2]**	Let A be the cross-sectional surface area at time t. Then, $A = \pi x^2$. $\frac{\mathrm{d}A}{\mathrm{d}t} = 2\pi x \frac{\mathrm{d}x}{\mathrm{d}t}$ When x= 5, $-0.5 = 2\pi(5)\frac{\mathrm{d}x}{\mathrm{d}t}$ $\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{0.05}{\pi}$ or $-\frac{1}{20\pi}$	Most students used the chain rule instead of differentiating implicitly wrt t. Both methods are accepted but it is encouraged to do implicit differentiation for this question. When chain rule was used, there were quite a few careless mistakes. There were also quite a number of bad presentation where $x = 5$ was substituted at the last step just to get the answer. Common mistakes seen: 1) When students used this chain rule: $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}A} \cdot \frac{\mathrm{d}A}{\mathrm{d}t}$, they did not take the reciprocal of $\frac{\mathrm{d}A}{\mathrm{d}x}$ when substituting for $\frac{\mathrm{d}x}{\mathrm{d}A}$. 2) Writing $-\frac{1}{20\pi}$ as $-\frac{1}{20}\pi$. 3) Did not consider "decreasing rate for A" and omitted the $-$sign, i.e used $\frac{\mathrm{d}A}{\mathrm{d}t} = 0.5$ instead of $\frac{\mathrm{d}A}{\mathrm{d}t} = -0.5$ 4) Failed to read the question carefully and did not give the "exact value" as required.

		5) Did not know the formula for the area of a circle, used $A = 2\pi x$ instead of $A = \pi x^2$
(b) **[2]**	Let V be the volume of the tablet at time t. Then, $V = \pi x^2\left(\frac{1}{3}x\right) = \frac{1}{3}\pi x^3$ $\frac{\mathrm{d}V}{\mathrm{d}t} = \pi x^2 \frac{\mathrm{d}x}{\mathrm{d}t}$ Thus, $\left.\frac{\mathrm{d}V}{\mathrm{d}t}\right\|_{x=5} = \pi\left(5^2\right)\left(-\frac{0.05}{\pi}\right) = -1.25$ The rate of change of the volume of the tablet when $x = 5$ is $-1.25\,\text{mm}^3$ per second.	This part was done better than part (a) as the chain rule was applied in a more direct manner. Students should also remember to answer the question by writing a short statement and giving the correct units. Common mistakes: 1) $V = \frac{1}{3}\pi x^3$ $\frac{\mathrm{d}V}{\mathrm{d}t} = \pi x^2$ (differentiated wrt to x but on the LHS treated it as $\frac{\mathrm{d}V}{\mathrm{d}t}$) 2) Differentiating $V = \frac{1}{3}\pi x^3$ wrongly. ($\frac{\mathrm{d}V}{\mathrm{d}x} = 3\pi x^2$)

2 **Do not use a calculator in answering this question.**

Showing your working, find the complex numbers z and w which satisfy the simultaneous equations

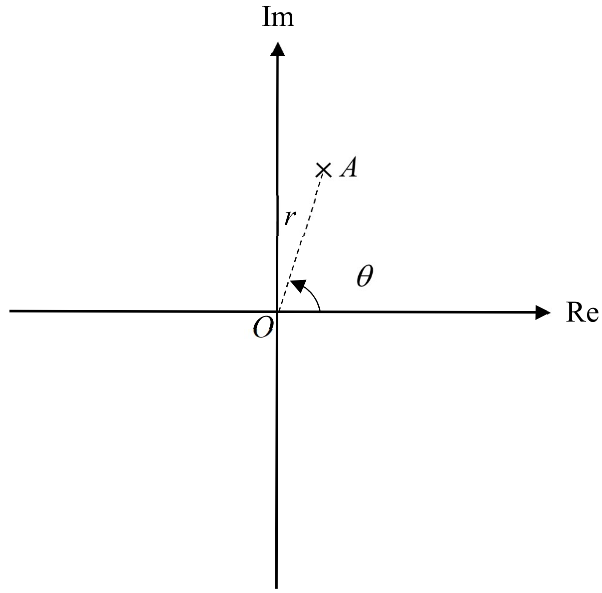
$$2\mathrm{i}z + w = 5 + 7\mathrm{i},$$
$$(4-\mathrm{i})w^* = z - 3\mathrm{i}.$$

[5]

[5]	$2\mathrm{i}z + w = 5 + 7\mathrm{i}$ ----(1) $(4-\mathrm{i})w^* = z - 3\mathrm{i}$ ----(2) From (2), $(4-\mathrm{i})w^* = z - 3\mathrm{i}$ $z = (4-\mathrm{i})w^* + 3\mathrm{i}$ ----(3) Sub (3) into (1), $2\mathrm{i}\left[(4-\mathrm{i})w^* + 3\mathrm{i}\right] + w = 5 + 7\mathrm{i}$ $(2+8\mathrm{i})w^* - 6 + w = 11 + 7\mathrm{i}$ $(2+8\mathrm{i})w^* + w = 11 + 7\mathrm{i}$ Let $w = x + \mathrm{i}y$. $\therefore (2+8\mathrm{i})(x-\mathrm{i}y) + (x+\mathrm{i}y) = 11 + 7\mathrm{i}$ $2x + 8\mathrm{i}x - 2\mathrm{i}y + 8y + x + \mathrm{i}y = 11 + 7\mathrm{i}$ $(3x+8y) + (8x-y)\mathrm{i} = 11 + 7\mathrm{i}$ Comparing Re and Im parts, $\begin{cases} 3x+8y=11 \\ 8x-y=7 \end{cases} \Rightarrow \begin{cases} 8(8x-7) = 11-3x \Rightarrow 67x = 67 \\ y = 8x-7 \end{cases}$ $\therefore x = 1,\ y = 1$ Sub $w = 1 + \mathrm{i}$ into (3), $z = (4-\mathrm{i})(1-\mathrm{i}) + 3\mathrm{i}$ $= 4 - 5\mathrm{i} - 1 + 3\mathrm{i}$ $= 3 - 2\mathrm{i}$ $\therefore w = 1 + \mathrm{i},\ z = 3 - 2\mathrm{i}$	There were a few methods to find the complex numbers. Some students let $w = a + \mathrm{i}b$ and $z = c + \mathrm{i}d$, substituted them into the given equations, compared real and imaginary components to get 4 equations to solve for a, b, c, d. This was accepted. There were quite a number of careless mistakes made when they were solving the equations simultaneously. As the use of calculator is not allowed for this question, students must show clear working when they are solving simultaneous equations. Common mistakes: **1)** Make w the subject first and write: $w = 5 + (7-2z)\mathrm{i}$ $w^* = 5 - (7-2z)\mathrm{i}$ Take note that z is also a complex number with real and imaginary components. And so, $\mathrm{Im}(w) \neq 7 - 2z$. It should be $w^* = 5 - (7-2z^*)\mathrm{i}$ **2)** Using GC to solve for unknowns even when the question does not allow the use of calculator. Similar Qns: • Promo Topical Revision Exercise Page 10 Qn 5, 6(a)(i) • RI 2024 Promo Qn 7(b)

3 The point A on the Argand diagram below represents the complex number w_1 with modulus r and argument θ. It is given that the complex numbers w_2 and w_3 satisfy the equations $w_2 = -w_1$ and $w_3 = \mathrm{i}w_2$.

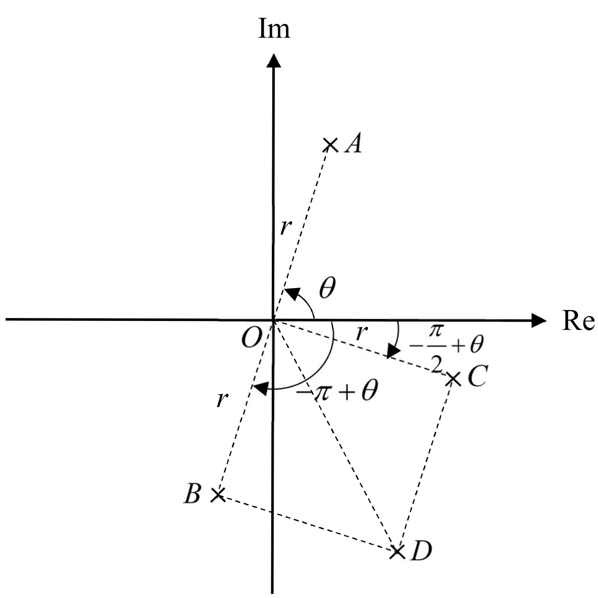
Let B, C and D represent the complex numbers w_2, w_3 and $w_2 + w_3$ respectively.



(a) On the copy of the Argand diagram in the Printed Answer Book, plot the points B, C and D, indicating clearly the modulus and argument of w_2 and w_3. [3]

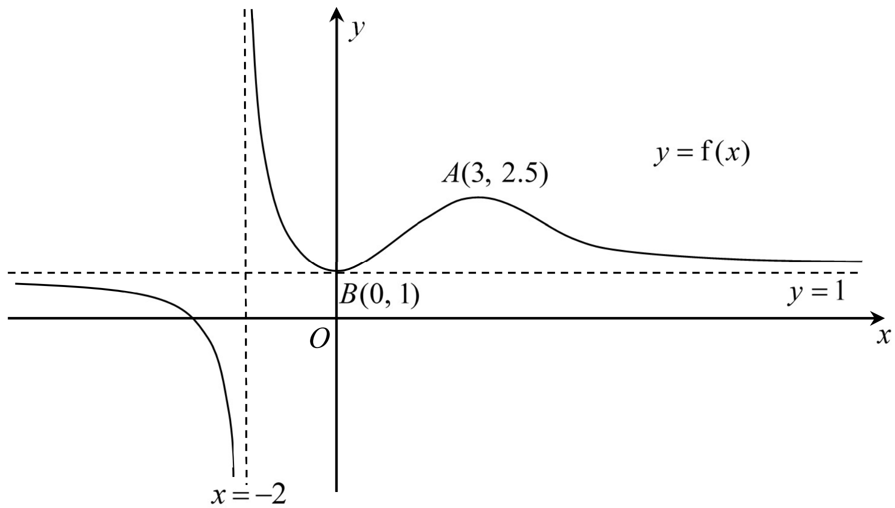
(b) State, in radians, the angle BOC. [1]

(c) By considering the quadrilateral $OBDC$, find $|w_2 + w_3|$ in terms of r and $\arg(w_2 + w_3)$ in terms of θ. [2]

(a) **[3]**		Most students were able to locate the points in the correct quadrant, but were not able to find the argument. Students need to know that arguments are measured from the positive Re axis, and points above the real axis has positive argument and points below the real axis has negative argument. While the question did not give a specific point, students could arbitrary let point A be (say 1+i) and calculate out the argument for B and C and put back the argument in terms of θ at the end.

(b) **[1]**	$\measuredangle BOC = \dfrac{\pi}{2}$	
(c) **[2]**	By Pythagora's Theorem, $\lvert w_2 + w_3 \rvert = \sqrt{(r)^2 + (r)^2} = \sqrt{2}r$ Since OD bisects $\measuredangle BOC$, $\measuredangle COD = \dfrac{1}{2}\left(\dfrac{\pi}{2}\right) = \dfrac{\pi}{4}$ $\therefore \arg(w_2 + w_3) = -\dfrac{\pi}{2} + \theta - \dfrac{\pi}{4} = -\dfrac{3\pi}{4} + \theta$	The modulus part was generally well done. For the argument part, students who were not able to see the bisection generally could not get the correct argument.

4 **(a)** The diagram shows the curve $y = \mathrm{f}(x)$, with turning points at $A(3,\ 2.5)$ and $B(0,\ 1)$. The lines $y = 1$ and $x = -2$ are asymptotes of the curve.



On separate diagrams, sketch the following graphs, clearly stating the equations of the asymptotes and the coordinates of the points corresponding to A and B where appropriate.

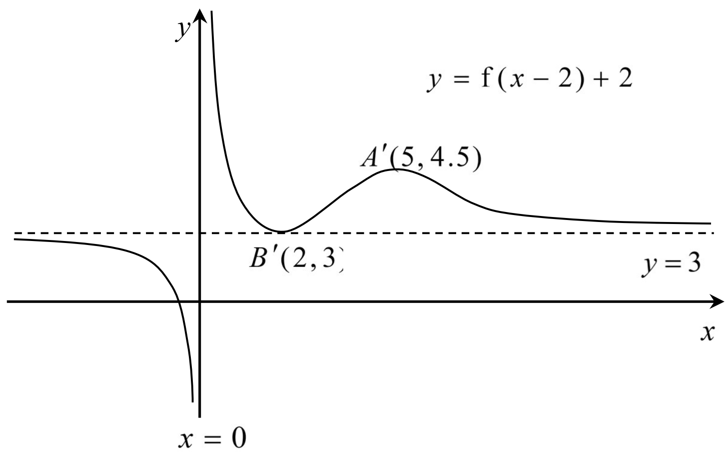
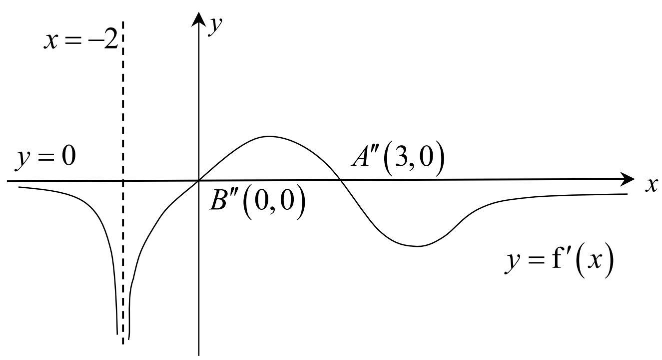
(i) $y = \mathrm{f}(x-2)+2$ [2]

(ii) $y = \mathrm{f}'(x)$ [3]

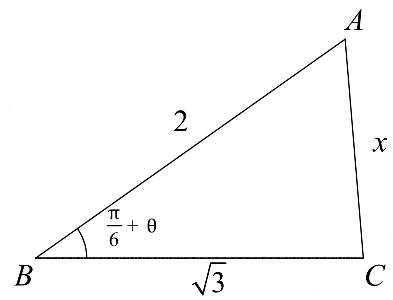
(b) The parametric equations of a curve are given by

$$x = t^2 - 2t,\ y = t^3 - 3t,\ t > 0.$$

The curve is reflected in the y-axis. Find the exact coordinates of the point(s) where the reflected curve intersects the x-axis. [2]

(i) **[2]**	We can translate the graph of $y = \mathrm{f}(x)$ 2 unit in the positive x-axis direction first, then followed by a translation of 2 units in the positive y-axis direction. Under the transformation, $A(3, 2.5) \rightarrow A(5,2.5) \rightarrow A(5,4.5)$ $B(0,1) \rightarrow B(2,1) \rightarrow B(2,3)$ 	Most students handled this part well. Just a general reminder to ensure that your graph is drawn to scale — the relative positioning of key points (such as intercepts, turning points, and asymptotes) should accurately reflect their calculated values. A correct shape with incorrect scale may cost marks in future assessments
(ii) **[3]**		Most students were able to draw the derivative graph well.
(b) **[2]**	When the curve intersect the x-axis, $y = 0$ $t^3 - 3t = 0$ $t = \sqrt{3}$, since t is positive When $t = \sqrt{3}$, $x = 3 - 2\sqrt{3}$. When curve is reflected about y-axis, replace x by $-x$. Hence required coordinate is $\left(2\sqrt{3} - 3, 0\right)$.	Students were generally able to begin correctly by solving $y = 0$. However, many did not discard the negative values of t and $t = 0$ indicating a lack of careful reading of the question. This led to unnecessary loss of marks

5



In the triangle ABC, $AB = 2$, $BC = \sqrt{3}$, $AC = x$ and angle $ABC = \dfrac{\pi}{6} + \theta$ radians (see diagram).

(a) Show that $x^2 = 7 - 6\cos\theta + 2\sqrt{3}\sin\theta$. [2]

(b) Show that $x\dfrac{\mathrm{d}^2x}{\mathrm{d}\theta^2} + \left(\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 = 3\cos\theta - \sqrt{3}\sin\theta$. [2]

(c) Use the result from part **(b)** to find the Maclaurin expansion for x, up to and including the term in θ^3. Give the coefficients as exact values in their simplest form. [3]

(d) Using the result from part **(c)**, deduce the approximate value of x when angle ABC is 35°, giving your answer correct to 6 decimal places. [2]

(a) **[2]**	By cosine rule, $x^2 = 2^2 + \left(\sqrt{3}\right)^2 - 2(2)\left(\sqrt{3}\right)\cos\left(\dfrac{\pi}{6}+\theta\right)$ $= 7 - 4\sqrt{3}\left(\dfrac{\sqrt{3}}{2}\cos\theta - \dfrac{1}{2}\sin\theta\right)$ $= 7 - 6\cos\theta + 2\sqrt{3}\sin\theta$	There are various ways to do this question. The direct method is to use the cosine rule since the lengths of the 3 sides of the triangle are stated with an included angle. Note that this is a "show" question, so it is important **not** to skip steps in your working. **Draw a diagram** in your solution if you need to introduce new points or lengths. Similar Qns: • Tut 7C Sect B Qn 8(a), 10(i)
(b) **[2]**	$x^2 = 7 - 6\cos\theta + 2\sqrt{3}\sin\theta$ Differentiate with respect to θ, $2x\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 6\sin\theta + 2\sqrt{3}\cos\theta \quad ----(1)$ Differentiate with respect to θ, $2x\dfrac{\mathrm{d}^2x}{\mathrm{d}\theta^2} + 2\left(\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 = 6\cos\theta - 2\sqrt{3}\sin\theta$ $x\dfrac{\mathrm{d}^2x}{\mathrm{d}\theta^2} + \left(\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 = 3\cos\theta - \sqrt{3}\sin\theta$	Notice that we want an equation involving $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}$ and $\dfrac{\mathrm{d}^2x}{\mathrm{d}\theta^2}$. So, the method is to differentiate the equation found in part **(a)** **implicitly** with respect to θ. Quite a few scripts wrote to "differentiate with respect to x" and unable to make much progress. There are also quite a few scripts that make x the subject from **(a)** and then differentiate twice, giving very "ugly" expressions. Only a few are able to get desired answer from there.

(c) **[3]**	When $\theta = 0$, $x^2 = 7 - 6 + 0 \Rightarrow x = \pm 1$ But $x > 0$, so $x = 1$. Then, from (1), when $\theta = 0$, $$2(1)\frac{\mathrm{d}x}{\mathrm{d}\theta} = 6\sin 0 + 2\sqrt{3}\cos 0$$ $$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \frac{6(0)+2\sqrt{3}(1)}{2} = \sqrt{3}$$ From the result in part **(b)**, when $\theta = 0$, $$(1)\frac{\mathrm{d}^2x}{\mathrm{d}\theta^2} + \left(\sqrt{3}\right)^2 = 3\cos 0 - \sqrt{3}\sin 0$$ $$\frac{\mathrm{d}^2x}{\mathrm{d}\theta^2} = 3 - \sqrt{3}(0) - 3 = 0$$ Differentiate the result from part **(b)** with respect to θ, $$x\frac{\mathrm{d}^3x}{\mathrm{d}\theta^3} + \frac{\mathrm{d}x}{\mathrm{d}\theta}\frac{\mathrm{d}^2x}{\mathrm{d}\theta^2} + 2\frac{\mathrm{d}x}{\mathrm{d}\theta}\frac{\mathrm{d}^2x}{\mathrm{d}\theta^2} = -3\sin\theta - \sqrt{3}\cos\theta$$ $$x\frac{\mathrm{d}^3x}{\mathrm{d}\theta^3} + 3\frac{\mathrm{d}x}{\mathrm{d}\theta}\frac{\mathrm{d}^2x}{\mathrm{d}\theta^2} = -3\sin\theta - \sqrt{3}\cos\theta$$ So, when $\theta = 0$, $$(1)\frac{\mathrm{d}^3x}{\mathrm{d}\theta^3} + 3(\sqrt{3})(0) = -3\sin 0 - \sqrt{3}\cos 0$$ $$\frac{\mathrm{d}^3x}{\mathrm{d}\theta^3} = -\sqrt{3}$$ So, $x \approx 1 + \sqrt{3}\theta - \frac{\sqrt{3}}{3!}\theta^3 = 1 + \sqrt{3}\theta - \frac{\sqrt{3}}{6}\theta^3$	Most are able to get the answers. Only a minority got the signs wrong (careless mistake) or forgot to divide by 3! for the coefficient of θ^3.
(d) **[2]**	$\measuredangle ABC = 35^\circ \Rightarrow \theta = 5^\circ = \frac{5}{180}\pi$ So, from part **(c)**, $$x \approx 1 + \sqrt{3}\left(\frac{5}{180}\pi\right) - \frac{\sqrt{3}}{6}\left(\frac{5}{180}\pi\right)^3$$ $= 1.150958$ (6 d.p.)	Not many are able to get the correct answer. Common mistakes: • Use $\theta = 35^\circ$ or $\theta = \frac{35}{180}\pi$ (Actually, $\measuredangle ABC = 35^\circ$) • Use $\theta = 5^\circ$ (It is given in the question that θ is in radians, so need to convert) • $x \approx 1.15096$ (the question asks for 6 d.p. answer) • Some use part **(a)** equation to find the value of x to get 1.150977 (the question states to use the result from part **(c)**)

6 **(a)** The non-zero vectors **a**, **b**, and **c** are such that $3\mathbf{a}\times\mathbf{b}=2\mathbf{b}\times\mathbf{c}$, where $3\mathbf{a}\neq-2\mathbf{c}$. Find a linear relationship between **a**, **b**, and **c**. [3]

(b) The variable position vector **v** satisfies the equation

$$\mathbf{v}\times\left(-\mathbf{i}+2\mathbf{j}+\mathbf{k}\right)=a\mathbf{i}+\mathbf{j}+\mathbf{k},$$

where a is a non-zero constant.

(i) Show that $a=3$. [1]

(ii) Hence, by finding the set of position vectors **v**, describe geometrically the set of points represented by the position vectors **v**. [3]

(a) **[3]**	$3\mathbf{a}\times\mathbf{b}=2\mathbf{b}\times\mathbf{c}$ $3\mathbf{a}\times\mathbf{b}-2\mathbf{b}\times\mathbf{c}=\mathbf{0}$ $3\mathbf{a}\times\mathbf{b}+2\mathbf{c}\times\mathbf{b}=\mathbf{0}$ $\left(3\mathbf{a}+2\mathbf{c}\right)\times\mathbf{b}=\mathbf{0}$ Then, $3\mathbf{a}+2\mathbf{c}=\mathbf{0}$ or $\mathbf{b}=\mathbf{0}$ or $3\mathbf{a}+2\mathbf{c}$ is parallel to **b**. Since we are given that $3\mathbf{a}\neq-2\mathbf{c}$ and **b** is a non-zero vector, then $3\mathbf{a}+2\mathbf{c}$ is parallel to **b**. $\therefore\mathbf{b}=k\left(3\mathbf{a}+2\mathbf{c}\right),k\in\mathbb{R}\setminus\{0\}$	Common Mistakes: • $\mathbf{b}\times\mathbf{c}=\mathbf{c}\times\mathbf{b}$ (it should be $\mathbf{b}\times\mathbf{c}=-\mathbf{c}\times\mathbf{b}$) • $\left(3\mathbf{a}+2\mathbf{c}\right)\times\mathbf{b}=\mathbf{0}\Rightarrow 3\mathbf{a}+2\mathbf{c}\parallel\mathbf{b}$ (there is also possibility that 3**a**+2**c** or **b** being a zero vector. See Tut 2A Sect B Qn 3(i).) Similar Qns: • Chap 2A notes Example 13 • 2023 RI Promo Qn 3(b) The reason for excluding 0 for the value of k is that if $k=0$, then $\mathbf{b}=\mathbf{0}$, but it is given that **b** is a non-zero vector.
(b)(i) **[1]**	**Method 1** Since $\begin{pmatrix}a\\1\\1\end{pmatrix}$ is perpendicular to $\begin{pmatrix}-1\\2\\1\end{pmatrix}$, to find a, $\begin{pmatrix}-1\\2\\1\end{pmatrix}.\begin{pmatrix}a\\1\\1\end{pmatrix}=0$ $-a+2+1=0$ $\therefore a=3$ **Method 2** Let $\mathbf{v}=\begin{pmatrix}x\\y\\z\end{pmatrix}$.	Recall from the lecture notes Chap 2A Page 15 that "$\mathbf{a}\times\mathbf{b}$ is a vector perpendicular to both **a** and **b**". And so, $\begin{pmatrix}a\\1\\1\end{pmatrix}$ is perpendicular to $\begin{pmatrix}-1\\2\\1\end{pmatrix}$. Not many are able to recall this important property of the cross product. Quite a few scripts uses Method 2.

	Then, $\mathbf{v}\times(-\mathbf{i}+2\mathbf{j}+\mathbf{k})=a\mathbf{i}+\mathbf{j}+\mathbf{k}$ $\begin{pmatrix}x\\y\\z\end{pmatrix}\times\begin{pmatrix}-1\\2\\1\end{pmatrix}=\begin{pmatrix}a\\1\\1\end{pmatrix}$ $\begin{pmatrix}y-2z\\-(x+z)\\2x+y\end{pmatrix}=\begin{pmatrix}a\\1\\1\end{pmatrix}$ $\begin{cases}y-2z=a----(1)\\-x-z=1----(2)\\2x+y=1----(3)\end{cases}$ (3)+2(2), $2x+y+2(-x-z)=1+2$ $y-2z=3$ Comparing with (1), $a=3$.	
(ii) **[3]**	Let $\mathbf{v}=\begin{pmatrix}x\\y\\z\end{pmatrix}$. Then, $\mathbf{v}\times(-\mathbf{i}+2\mathbf{j}+\mathbf{k})=a\mathbf{i}+\mathbf{j}+\mathbf{k}$ $\begin{pmatrix}x\\y\\z\end{pmatrix}\times\begin{pmatrix}-1\\2\\1\end{pmatrix}=\begin{pmatrix}3\\1\\1\end{pmatrix}$ $\begin{pmatrix}y-2z\\-(x+z)\\2x+y\end{pmatrix}=\begin{pmatrix}3\\1\\1\end{pmatrix}$ $\begin{cases}y-2z=3\\-x-z=1\\2x+y=1\end{cases}$ From GC, $x=-1-z$ and $y=3+2z$. We get $\mathbf{v}=\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-1-z\\3+2z\\z\end{pmatrix}$ So, $\left\{\mathbf{v}\in\mathbb{R}^3:\mathbf{v}=\begin{pmatrix}-1\\3\\0\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\1\end{pmatrix},\lambda\in\mathbb{R}\right\}$ The set of $\mathbf{v}$ forms a straight line passing through $(-1,3,0)$ and parallel to $\begin{pmatrix}-1\\2\\1\end{pmatrix}$.	The question states "describe geometrically" so some geometrical description of the answer is expected. In this case, students are required to describe the straight line, giving the coordinates of a point on a line and the direction vector of the line, and not just stating the equation of the line. Quite a few scripts state that "the line starts from $(-1,3,0)$". Note that the line passes through $(-1,3,0)$, but need not start from there. Note that with $\lambda\in\mathbb{R}$, the line does **not** have end-points. So, it is not right to say that the line starts from $(-1,3,0)$.

7 The curve C_1 has equation $y=\dfrac{ax^2+bx+3}{x+2}$, where a and b are constants. It is given that C_1 has an asymptote $y=x-3$.

(a) State the value of a and show that $b=-1$. [3]

(b) Using an **algebraic method**, find the set of values that y cannot take. [3]

The locus of a point is defined as the path traced out by that point as it moves.

(c) Let $P(x,y_1)$ be a point on the curve C_1 and $Q(x,y_2)$ be a point on the curve C_2 with equation $y=\dfrac{3x-x^2}{x+2}$. Find the cartesian equation of the locus of the midpoint of P and Q as x varies. [2]

(a) **[3]**	$a=1$ By long division, $y=x+(b-2)+\dfrac{7-2b}{x+2}$. $b-2=-3\Rightarrow b=-1$ (shown) **Alternatively,** $y=\dfrac{x^2+bx+3}{x+2}$ $=x-3+\dfrac{A}{x+2}$ $=\dfrac{(x-3)(x+2)+A}{x+2}$ Comparing the numerators, Coefficient of x: $b=2-3=-1$	This part was very well done. Students use either long division or form the related equation using the given asymptote and the remainder quotient to answer the question. Students who did not show their method clearly will not receive credit for showing $b=-1$. Care should be taken to avoid mistakes so as not to affect the subsequent parts. Similar Qns: • Tut 8A Qn 4(i) • Add Prac 8A Qn 6(i), 9(i) • 2024 RI Promo Qn 8(b)(i)
(b) **[3]**	If y can take the value k, the graphs of $y=\dfrac{x^2-x+3}{x+2}$ and $y=k$ must intersect. Suppose they intersect, then at the point of intersection, $\dfrac{x^2-x+3}{x+2}=k$ $x^2-x+3=kx+2k$ $x^2-(k+1)x+(3-2k)=0$ For the quadratic equation **not** to have real roots, discriminant < 0 $(k+1)^2-4(3-2k)<0$ $k^2+10k-11<0$ $(k+11)(k-1)<0$ $-11<k<1$ Hence set of values that y cannot take is $(-11,1)$.	There were two methods used in this part, although the use of differentiation is discouraged as it is not an algebraic method. Furthermore, detailed analysis of the nature of the two stationary points and the asymptotic behavior of the graph is required to get full credit. In both methods, careless algebraic manipulations were seen and this resulted in students spending more time solving for y. Some students worked on a wrong expression that they had obtained from part **(a)** and hence could not get the correct answer. Some who managed to get the correct answer, but did not express it in the appropriate set or interval notation. Sometimes the wrong bracket or interval was provided e.g. [-11,1], {-11,1} or (1,-11). Similar Qns: • Tut 8A Qn 3(i) • Add Prac 8A Qn 6(ii)

(c) **[2]**	The midpoint of P and Q has coordinates $\left(x, \dfrac{y_1+y_2}{2}\right)$. For a fixed value of x, $\dfrac{y_1+y_2}{2}=\dfrac{x^2-x+3+-x^2+3x}{2(x+2)}=\dfrac{2x+3}{2(x+2)}$ Hence the locus of the midpoint of P and Q lies on the curve traced out by the equation $y=\dfrac{2x+3}{2x+4},\ x\neq -2$.	This part was mostly left blank. Common mistakes: • using the wrong mid-point formula $\left(x, \dfrac{y_1-y_2}{2}\right)$. • finding two mid-points and fitting a straight line through these two points. • equating C_1 and C_2. • seeing the two graphs as reflections of each other in some horizontal line (the locus). For those who were successful with this part, $x\neq -2$ was usually left out from the cartesian equation of the locus.

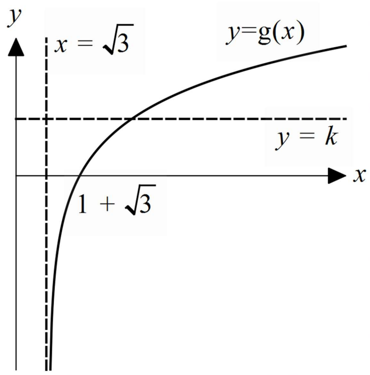
8 The function g is defined by $g : x \mapsto \ln\left(x-\sqrt{3}\right)$, for $x \in \mathbb{R},\ x > \sqrt{3}$.

(a) Explain how you know g^{-1} exists. [2]

(b) Find $g^{-1}(x)$ and write down its domain. [3]

(c) Solve $g(x) = 0$ exactly. [1]

(d) Hence, without using a calculator, solve the inequality $\dfrac{g(x)}{(x-3)(x+1)} \geq 0$. [3]

(a) **[2]**	Since every horizontal line $y = k$, where $k \in \mathbb{R}$, cuts the graph of g at one and only one point, g is a 1-1 function and hence g^{-1} exists.  The underlined phrases are expected in your answer.	About half the cohort did not put in the vertical asymptote correctly. Those who use the fact that all strictly increasing functions are one-one need to establish why $g'(x) > 0$ on $x > \sqrt{3}$ instead of merely quoting the result.
(b) **[3]**	$y = \ln(x-\sqrt{3})$ $e^y = x-\sqrt{3}$ $x = e^y + \sqrt{3}$ $\therefore g^{-1}(x) = e^x + \sqrt{3}$ Note the way to give the rule $g^{-1}(x)$. $D_{g^{-1}} = R_g = \mathbb{R}$	Those who wrote R_g correctly scored full marks here.
(c) **[1]**	$\ln\left(x-\sqrt{3}\right) = 0$ $x-\sqrt{3} = e^0 = 1$ $x = 1+\sqrt{3}$	Question requested for the exact answer.

(d) **[3]**	For $\dfrac{\mathrm{g}(x)}{(x-3)(x+1)} \geq 0$, Note that $x \neq -1, 3$ **Case 1** : $\mathrm{g}(x) \geq 0$ **<u>and</u>** $(x-3)(x+1) > 0$: $\mathrm{g}(x) \geq 0 \quad \Rightarrow \quad x \geq 1+\sqrt{3}$ $(x-3)(x+1) > 0 \quad \Rightarrow \quad x < -1$ or $x > 3$ Taking intersection, $x > 3$. g is defined for $x > \sqrt{3}$ **Case 2** : $\mathrm{g}(x) \leq 0$ **<u>and</u>** $(x-3)(x+1) < 0$: $\mathrm{g}(x) \leq 0 \quad \Rightarrow \quad \sqrt{3} < x \leq 1+\sqrt{3}$ $(x-3)(x+1) < 0 \quad \Rightarrow \quad -1 < x < 3$ Taking intersection, $\sqrt{3} < x \leq 1+\sqrt{3}$ Hence $\dfrac{\mathrm{g}(x)}{(x-3)(x+1)} \geq 0 \Rightarrow \sqrt{3} < x \leq 1+\sqrt{3}$ **or** $x > 3$	The phrase "without using a calculator" means <u>no complicated graph allowed</u> (ie answers derived from the graphs such as $y=\dfrac{\mathrm{g}(x)}{(x-3)(x+1)}$, $y=\mathrm{g}(x)(x-3)(x+1)$, or even $y=\mathrm{g}(x)(x-3)$ are <u>not</u> allowed). This means you need to explain <u>how the resultant sign is positive for each case</u>.

9 **(a)** It is given that $\displaystyle\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}$.

(i) State the sum to infinity of the series. [1]

(ii) Find the sum of the first n terms of the series

$$\frac{2}{17\times18\times19} + \frac{2}{18\times19\times20} + \frac{2}{19\times20\times21} + \cdots.$$ [2]

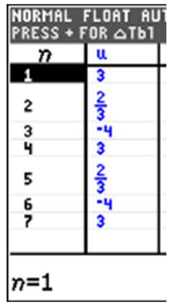
(iii) Find $\displaystyle\sum_{r=2}^{n} \frac{1}{r(r^2-1)}$. [3]

(b) The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 3$, $u_{n+1} = 2 - \dfrac{4}{u_n}$, $n \geq 1$.

(i) Find the values of u_2, u_3, u_4 and u_{100}. [2]

(ii) Hence find $\displaystyle\sum_{r=1}^{3n-2} u_r$ in terms of n, simplifying your answer. [2]

(a)(i) **[1]**	The sum to infinity is $\frac{1}{2}$	Most students were able to state the sum to infinity of this series correctly. A handful of students incorrectly assumed that the given series must be arithmetic or geometric, and hence had trouble with the whole of part **(a)**.
(ii) **[2]**	$\displaystyle\sum_{r=17}^{n+16} \frac{2}{r(r+1)(r+2)}$ $\displaystyle= \sum_{r=1}^{n+16} \frac{2}{r(r+1)(r+2)} - \sum_{r=1}^{16} \frac{2}{r(r+1)(r+2)}$ $\displaystyle= \frac{1}{2} - \frac{1}{(n+17)(n+18)} - \left(\frac{1}{2} - \frac{1}{17\times18}\right)$ $\displaystyle= \frac{1}{306} - \frac{1}{(n+17)(n+18)}$	This part proves challenging for most students. Common mistakes include: • Misread this as a sum to infinity. • Used ‘n’ instead of ‘n+16’ for the ending value of the index of the summation. Similar Qns: • Promo Topical Revision Exercise Page 15 Qn 2(i)
(iii) **[3]**	$\displaystyle\sum_{r=2}^{n} \frac{1}{r(r^2-1)}$ $\displaystyle= \frac{1}{2}\sum_{r=2}^{n} \frac{2}{(r-1)r(r+1)}$ $\displaystyle= \frac{1}{2}\left(\frac{2}{1\times2\times3} + \cdots + \frac{2}{(n-1)n(n+1)}\right)$ $\displaystyle= \frac{1}{4} - \frac{1}{2n(n+1)}$	Responses for this question were mixed. Most students attempted to re-index the summation so that they could use the given formula, but had difficulties figuring out the starting and ending values of r after re-indexing. Even those who managed to get the re-indexing correct often missed out the factor of $\frac{1}{2}$. Similar Qns: • Promo Topical Revision Exercise Page 15 Qn 1(b) • T4W1 In-Class Revision on Seq and Series Qn 2(b).

2(b) **[2]**	$u_2 = 2 - \frac{4}{3} = \frac{2}{3}$ $u_3 = 2 - \frac{4}{\frac{2}{3}} = -4$ $u_4 = 2 - \frac{4}{-4} = 3$ $u_{100} = u_{97} = u_{94} = \cdots = u_1 = 3$  Or use GC to obtain the values.	Most students were able to state the required values correctly.
(ii) **[2]**	$\sum_{r=1}^{3n-2} u_r = n\left(3 + \frac{2}{3} - 4\right) - \frac{2}{3} + 4 = \frac{10-n}{3}$	Most students were able to recognize that the sum occurs in groups of $3, \frac{2}{3}$ and -4. However not many were able to recognize that because there are $3n-2$ terms, so the final group had only the term 3, without $\frac{2}{3}$ and -4.

10 With reference to the origin O, the point A has position vector $-\mathbf{i}-3\mathbf{j}+3\mathbf{k}$. The plane π and the line l have equations

$$\mathbf{r}=\begin{pmatrix}4\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}-1\\3\\2\end{pmatrix}+\mu\begin{pmatrix}2\\3\\-4\end{pmatrix} \text{ and } \mathbf{r}=\begin{pmatrix}2\\1\\1\end{pmatrix}+\beta\begin{pmatrix}3\\t\\-2\end{pmatrix},$$

respectively, where λ, μ and β are parameters and t is a real value.

(a) It is given that l and π intersect at a point.

(i) Find the range of values of t. [2]

(ii) Find, in terms of t, the coordinates of the point of intersection, B, of l and π. [2]

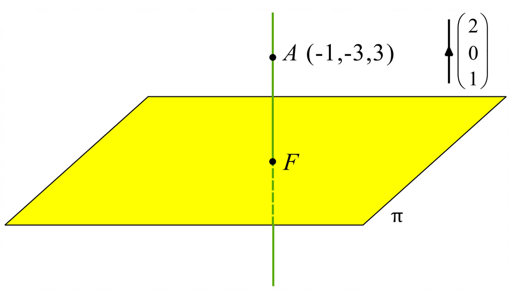
(b) It is given that A lies on l.

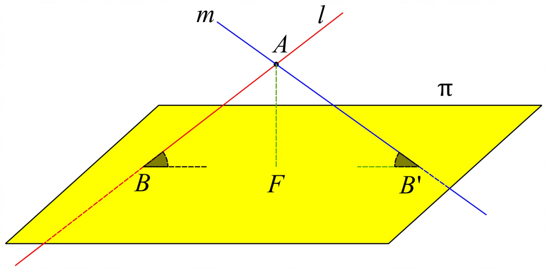
(i) Show that $t=4$. [1]

(ii) Find the position vector of the foot of perpendicular, F, of A onto π. [3]

(iii) Find the cartesian equation of another line m through A, that lies on the plane containing A, B and F, and m makes the same angle with π as l makes with π. [4]

(a)(i) **[2]**	A normal to plane π: $\begin{pmatrix}-1\\3\\2\end{pmatrix}\times\begin{pmatrix}2\\3\\-4\end{pmatrix}=\begin{pmatrix}-18\\0\\-9\end{pmatrix}=-9\begin{pmatrix}2\\0\\1\end{pmatrix}$ Since l and π intersect at a point, then l is not parallel to π. $\begin{pmatrix}3\\t\\-2\end{pmatrix}.\begin{pmatrix}2\\0\\1\end{pmatrix}\neq 0 \Rightarrow 4+0t\neq 0 \quad \cdots(1)$ The range of values of t that satisfy (1) is $\mathbb{R}$	It's better to check that the cross product is correct as the normal vector will be used in subsequent parts as well: If $\mathbf{a}\times\mathbf{b}=\mathbf{n}$, check that $\mathbf{a}\cdot\mathbf{n}=0$ and $\mathbf{b}\cdot\mathbf{n}=0$
(a)(ii) **[2]**	Method 1: $\pi:\ \mathbf{r}\cdot\begin{pmatrix}2\\0\\1\end{pmatrix}=\begin{pmatrix}4\\1\\1\end{pmatrix}\cdot\begin{pmatrix}2\\0\\1\end{pmatrix}=9$ At B, $\left[\begin{pmatrix}2\\1\\1\end{pmatrix}+\beta\begin{pmatrix}3\\t\\-2\end{pmatrix}\right].\begin{pmatrix}2\\0\\1\end{pmatrix}=9$ $(4+1)+\beta(6-2)=9$ $\beta=1$ So, $\overrightarrow{OB}=\begin{pmatrix}2\\1\\1\end{pmatrix}+1\begin{pmatrix}3\\t\\-2\end{pmatrix}=\begin{pmatrix}5\\1+t\\-1\end{pmatrix}$ B has coordinates $(5,1+t,-1)$	To find the intersection between line and plane, it's usually easier to substitute the equation of line into the equation of the plane in scalar product form.

	Method 2: $\begin{pmatrix}4\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}-1\\3\\2\end{pmatrix}+\mu\begin{pmatrix}2\\3\\-4\end{pmatrix}=\begin{pmatrix}2\\1\\1\end{pmatrix}+\beta\begin{pmatrix}3\\t\\-2\end{pmatrix}$ $-\lambda+2\mu-3\beta=-2 \quad \cdots(1)$ $3\lambda+3\mu-t\beta=0 \quad \cdots(2)$ $2\lambda-4\mu+2\beta=0 \quad \cdots(3)$ Solving (1) and (3) gives $\beta=1$ $\overrightarrow{OB}=\begin{pmatrix}2\\1\\1\end{pmatrix}+1\begin{pmatrix}3\\t\\-2\end{pmatrix}=\begin{pmatrix}5\\1+t\\-1\end{pmatrix}$ B has coordinates $(5,1+t,-1)$	The solution can also be obtained by equating the given equations of the plane and line, then use GC to solve the equations.
(b)(i) **[1]**	Since A lies on l, $\begin{pmatrix}-1\\-3\\3\end{pmatrix}=\begin{pmatrix}2\\1\\1\end{pmatrix}+\beta\begin{pmatrix}3\\t\\-2\end{pmatrix}$ for $\beta\in\mathbb{R}$ $-1=2+3\beta \quad\Rightarrow\quad \beta=-1$ $-3=1-t \quad\Rightarrow\quad t=4$	To show clearly, find β, then t
(ii) **[3]**	 line AF: $\mathbf{r}=\begin{pmatrix}-1\\-3\\3\end{pmatrix}+\alpha\begin{pmatrix}2\\0\\1\end{pmatrix},\ \alpha\in\mathbb{R}$ At F, $\left[\begin{pmatrix}-1\\-3\\3\end{pmatrix}+\alpha\begin{pmatrix}2\\0\\1\end{pmatrix}\right].\begin{pmatrix}2\\0\\1\end{pmatrix}=9$ $(-2+3)+\alpha(4+1)=9$ $\alpha=\frac{8}{5}$ $\overrightarrow{OF}=\begin{pmatrix}-1\\-3\\3\end{pmatrix}+\frac{8}{5}\begin{pmatrix}2\\0\\1\end{pmatrix}=\begin{pmatrix}\frac{11}{5}\\-3\\\frac{23}{5}\end{pmatrix}$	Note that $\mathbf{r}$ in the line equation is the position vector of any point on the line, so $\mathbf{r}\neq\overrightarrow{AF}$ The foot of perpendicular F from point A to plane is the point of intersection between line AF and the plane. It's incorrect to use $\overrightarrow{AF}.\begin{pmatrix}2\\0\\1\end{pmatrix}=9$, $\overrightarrow{AF}\cdot\mathbf{n}=0$ or $\overrightarrow{OF}\cdot\mathbf{n}=0$

(iii) **[4]**	 The line m is the reflection of l in the line AF. Let B' be the mirror image of B in the line AF. Then, $\dfrac{\overrightarrow{AB'}+\overrightarrow{AB}}{2}=\overrightarrow{AF}$ $\overrightarrow{AB'}=2\overrightarrow{AF}-\overrightarrow{AB}$ $=2\times\dfrac{8}{5}\begin{pmatrix}2\\0\\1\end{pmatrix}-\left(\begin{pmatrix}5\\5\\-1\end{pmatrix}-\begin{pmatrix}-1\\-3\\3\end{pmatrix}\right)$ $=\begin{pmatrix}\frac{2}{5}\\-8\\\frac{36}{5}\end{pmatrix}$ $=\dfrac{2}{5}\begin{pmatrix}1\\-20\\18\end{pmatrix}$ $m:\quad \mathbf{r}=\begin{pmatrix}-1\\-3\\3\end{pmatrix}+s\begin{pmatrix}1\\-20\\18\end{pmatrix},\ s\in\mathbb{R}$ $x+1=\dfrac{y+3}{-20}=\dfrac{z-3}{18}$	For vectors, a sketch usually helps. Here, use a sketch to see that line m is the reflection of l in the line AF. Use ratio theorem or other methods to find direction vector of line m.

11 An arithmetic series has positive first term a and common difference d. The first, fifth and second terms of the arithmetic series are the first three consecutive terms of a geometric series respectively. It is also given that the terms of the arithmetic series are decreasing.

(a) Show that $d=-\frac{7}{16}a$. [3]

(b) Give a reason why the geometric series converges. [2]

It is given that the sum to infinity of the geometric series is $\frac{32}{7}$.

(c) Find the sum of the first twenty terms of the arithmetic series. [2]

(d) Given that the sum of all the terms after the nth term of the geometric series is greater than 2% of the sum of the first n terms of the geometric series, find the possible values of n. [4]

(a) **[3]**	$r=\frac{a+4d}{a}=\frac{a+d}{a+4d}$ $(a+4d)^2=a(a+d)$ $a^2+8ad+16d^2=a^2+ad$ $d(7a+16d)=0$ $d=0$ or $7a+16d=0$ $d\neq 0$ since the series is decreasing So, $7a+16d=0$ $d=-\frac{7}{16}a$	Most students were able to relate the three consecutive terms of GP and apply the concept of common ratio. A handful of students mixed up the order of $a+d$ and $a+4d$ under GP or careless mistake in algebraic manipulation. Note that $d\neq 0$ is crucial in proving the final statement. Similar Qns • Tut 7B Sect B Qn 2 • T4W1 In-Class Revision on Seq and Series Qn 6(b).
(b) **[2]**	$r=\frac{a+4d}{a}=\frac{a+4\left(-\frac{7}{16}a\right)}{a}=-\frac{3}{4}$ Since $\lvert r\rvert=\frac{3}{4}<1$, the sum to infinity exists	$r<1$ is different from $\lvert r\rvert<1$. The sum to infinity exists when $\lvert r\rvert<1$. It is **incorrect** to assume that $u_r=a\left(-\frac{3}{4}\right)^{r-1}$ approaches 0 when r approaches infinity then the sum to infinity exists. In other words, $u_r\to 0$ does not imply the sum to infinity exists.
(c) **[2]**	$\frac{a}{1-\left(-\frac{3}{4}\right)}=\frac{32}{7} \quad \Rightarrow \quad a=8$ The required sum is $\frac{20}{2}\left(2a+19\left(-\frac{7}{16}a\right)\right)=-505$	Read the question carefully and interpret the sum correctly.

(d) **[4]**	$S_\infty - S_n > \frac{1}{50} S_n$ $\frac{a}{1-r} > \frac{51}{50} \cdot \frac{a\left(1-r^n\right)}{1-r}$ $\frac{50}{51} > 1-r^n$ since $\frac{a}{1-r} = \frac{32}{7} > 0$ $\left(-\frac{3}{4}\right)^n > \frac{1}{51}$ $\left(\frac{3}{4}\right)^n > \frac{1}{51}$ and n must be even $n \ln\left(\frac{3}{4}\right) > \ln\frac{1}{51}$ $n < \frac{\ln\frac{1}{51}}{\ln\left(\frac{3}{4}\right)} = 13.7$ Possible values of n are 2, 4, 6, 8, 10, 12	Quite a number of students were careless and unable to simplify into $\left(-\frac{3}{4}\right)^n > \frac{1}{51}$ before finding the final answer. For $\left(-\frac{3}{4}\right)^n > \frac{1}{51}$, it is important to know that only even powers work for the inequality. As $\ln\left(\frac{3}{4}\right) < 0$, when solving $n \ln\left(\frac{3}{4}\right) > \ln\frac{1}{51}$, remember to change the sign of inequality when crossing over $\ln\left(\frac{3}{4}\right)$.

12

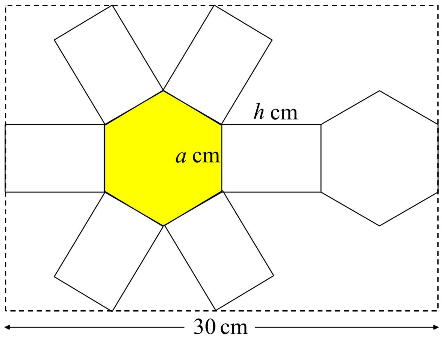


Fig. 1

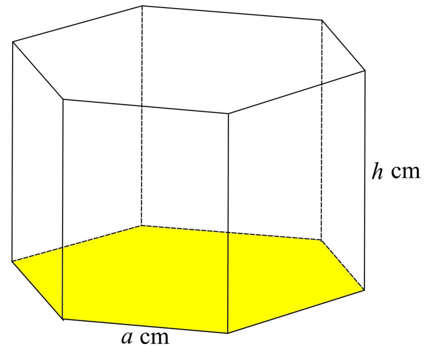


Fig. 2

Fig. 1 shows the net of a right regular hexagon-based prism cut from a rectangular cardboard of one side length 30 cm. The net consists of two regular hexagons of side length a cm and six rectangles, each with sides of length a cm and h cm. The net is folded to form a prism which has a hexagonal base of side length a cm and vertical height h cm, as shown in Fig. 2.

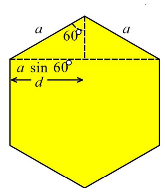
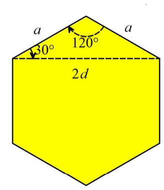
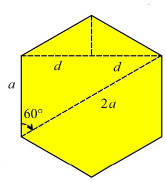
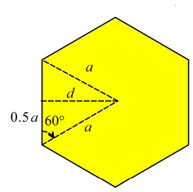
(a) Show that $h+a\sqrt{3}=15$. [2]

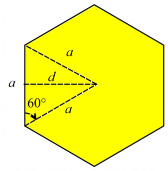
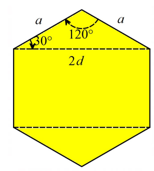
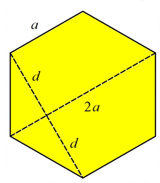
(b) Show that the area of the hexagonal base is $\frac{3\sqrt{3}}{2}a^2$ cm^2. [1]

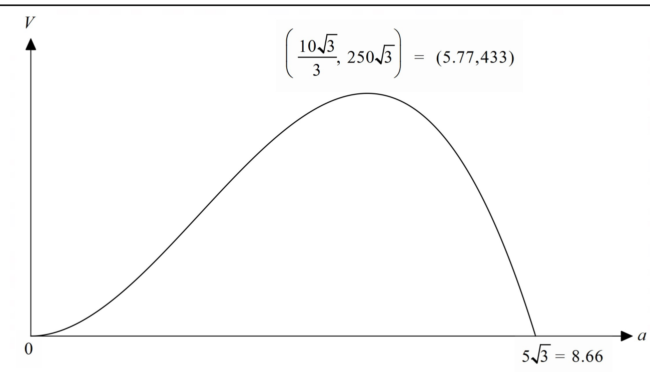
(c) Use differentiation to find the exact value of the maximum possible volume of the prism and prove that this value is a maximum. [6]

(d) Find the ratio of h : a that gives this maximum volume. [1]

(e) Sketch the graph showing the volume of the prism as the value of a varies. [2]

(a)	The interior angle of a regular hexagon is 120° Then, $h+2a\sin 60^\circ+h+2a\sin 60^\circ=30$ $2h+2a\sqrt{3}=30$ $h+a\sqrt{3}=15$  **Or** using sine rule, $\frac{2d}{\sin 120°}=\frac{a}{\sin 30°}$  **Or** using cosine rule, $(2d)^2=a^2+a^2-2a^2\cos 120°$ **Or** $\tan 60°=\frac{2d}{a}$  **Or** $\tan 60°=\frac{d}{0.5a}$ 	It is a "show" question. Students need to be explicit in their workings. A diagram labelling the sides, angles and lengths of the hexagon will be very helpful. Students should avoid terms such as "height of hexagon", "length of hexagon".

(b)	Area of the hexagonal base $= 6\left(\frac{1}{2}a^2\sin 60^\circ\right)$  $= \frac{3\sqrt{3}}{2}a^2 \text{ cm}^2$ **Or** Area of the hexagonal base $= 6\left(\frac{1}{2}ad\right)$ $= \frac{3\sqrt{3}}{2}a^2 \text{ cm}^2$ **Or** Area of the hexagonal base $= 2\left(\frac{1}{2}a^2\sin 120^\circ\right) + a\left(2a\sin 60^\circ\right)$ $= \frac{\sqrt{3}}{2}a^2 + \sqrt{3}a^2 = \frac{3\sqrt{3}}{2}a^2 \text{ cm}^2$  **OR** Area of the hexagonal base $= 2 \times$ Area of trapezium $= 2 \times \frac{1}{2}(a+2a)d$  	Similarly to **(a)**, this is another "show" question. Students must explain their workings sufficiently. Eg, using the sum of 6 identical (equilateral) triangles. Area of triangle $= \frac{1}{2}ab\sin C$. **OR** Total area = area of 2 identical triangles (top and bottom) + Area of rectangle (middle)
(c)	Volume of the prism, $V = \frac{3\sqrt{3}}{2}a^2h$ $= \frac{3\sqrt{3}}{2}a^2\left(15 - a\sqrt{3}\right)$ $= \frac{45\sqrt{3}}{2}a^2 - \frac{9}{2}a^3 \text{ cm}^3$ $\frac{\mathrm{d}V}{\mathrm{d}a} = 45\sqrt{3}a - \frac{27}{2}a^2$ For stationary values of V, $\frac{\mathrm{d}V}{\mathrm{d}a} = 0$ $45\sqrt{3}a - \frac{27}{2}a^2 = 0$ $a\left(45\sqrt{3} - \frac{27}{2}a\right) = 0$ $\frac{27}{2}a = 45\sqrt{3}$ (since $a \neq 0$) $a = \frac{10\sqrt{3}}{3}$	Most students were able to substitute $h = 15 - a\sqrt{3}$ into the expression for V. However, several students made algebraic mistakes in the expansion. Some students made mistakes in the differentiation. Factorizing would be the easiest method to solve for a. Several students used the general formula, which complicated the algebra involved and made mistakes as a result. Students are reminded to leave the answer for a in exact form.

	$\frac{\mathrm{d}^2V}{\mathrm{d}a^2} = 45\sqrt{3} - 27a$ $\left.\frac{\mathrm{d}^2V}{\mathrm{d}a^2}\right\|_{a=\frac{10\sqrt{3}}{3}} = 45\sqrt{3} - 27\left(\frac{10\sqrt{3}}{3}\right) = -45\sqrt{3} < 0$ Then, $a = \frac{10\sqrt{3}}{3}$ gives the maximum value of V The maximum volume $= \frac{3\sqrt{3}}{2}\left(\frac{10\sqrt{3}}{3}\right)^2\left(15 - \left(\frac{10\sqrt{3}}{3}\right)\sqrt{3}\right)$ $= \frac{3\sqrt{3}}{2}\left(\frac{100}{3}\right)5 = 250\sqrt{3}\ \text{cm}^3$	Most students used the 2nd Derivative test to show that the volume obtained is a maximum which is the easier approach for this question. Those who used the 1st derivative test should be cautioned on the proper presentation of the test to be credited marks. As stated in the question, the exact value of V is expected of students in this part.
(d)	$h = 15 - a\sqrt{3}$ $= 15 - \left(\frac{10}{\sqrt{3}}\right)\sqrt{3}$ $= 5$ $\frac{h}{a} = 5 \div \frac{10}{\sqrt{3}} = \frac{\sqrt{3}}{2}$ So, $h : a = \sqrt{3} : 2$	Students who did not get this question correct were mainly due to the incorrect value of a from the previous part or careless mistakes made, eg switching the positions of h and a in the ratio. Students are reminded that ratios cannot be squared on both sides (in this case) to get rid of square roots.
(e)		This is a real-life context question where $a \geq 0$ and $V \geq 0$. When $a = 0$, it will yield a minimum V. Thus, note the behaviour of the stationary point at the origin. Students are reminded to label the graph, especially the stationary points and x-intercepts.