NANYANG JUNIOR COLLEGE

JC2 Timed Practice

Higher 2

FURTHER MATHEMATICS 9649/01

Paper 1

3rd July 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks is 100.

Section A: Pure Mathematics [66 marks]

1 A rectangular box is placed in the region $R=\{(x,y,z):x,y,z\geq 0\}$ with one corner at the origin, three faces in the coordinate planes and the corner P diagonally opposite the origin on the plane $x+y+z=3$.

Use partial differentiation to find the maximum possible surface area for such a box, showing your working clearly and justifying your conclusion. [5]

2 The curve C has polar equation $r=2\sin\theta(1-\cos\theta)$ for $0\leq\theta\leq\pi$.

(a) Find $\dfrac{\mathrm{d}r}{\mathrm{d}\theta}$ and hence find the polar coordinates of the point P on C that is furthest from the pole. [3]

(b) Sketch C, indicating the polar coordinates of the point P. [2]

(c) Find the exact area of the region bounded by C and the two lines $\theta=0$ and $\theta=\dfrac{\pi}{4}$. [4]

3 The sequence $\{u_n\}$ is given by the recurrence relation $u_{n+2}=\frac{1}{3}(2u_{n+1}+u_n-24)$ for $n\geq 1$, together with initial terms $u_1=p$ and $u_2=q$, where p and q are positive integers. The sequence $\{v_n\}$ is given by $v_n=u_n+6n$ for $n\geq 0$.

(a) Find an expression for v_n in terms of n, p and q. [5]

(b) Find the possible values of p and q for v_n to converge to 13. [2]

(c) Hence, find u_n in terms of n, p and q. State what happens to u_n for large values of n. [2]

4 The function $\mathrm{f}(x)=\ln x-x+2$ roots $x=\alpha$ and $x=\beta$, where $\alpha<\beta$.

(a) It is given that β lies in the interval $(k,k+1)$, where k is an integer. State the value of k. Justify that there is indeed a root in the interval $(k,k+1)$. [2]

(b) Show that one application of linear interpolation using the interval found in **(a)** will give a first approximation for β as 3.138439, correct to 6 decimal places. [2]

(c) Hence, use the Newton-Rhapson method to approximate β, correct to 4 decimal places. Justify that the approximation is correct up to 4 decimal places. [3]

(d) It is known that the Newton-Rhapson method, when applied to $\mathrm{f}(x)$ can approximate the root α, as long as the first approximation lies in the interval $(0,c)$. Find the largest value of c, leaving your answer in exact form. [3]

5 The country of Ganyan is looking to export timbre to boost its economy. In 2025, the government of Ganyan acquired a timbre plantation and decided to let the plantation grow.

After t number of years, the amount of timbre, T thousand tonnes, can be modelled as

$$\frac{\mathrm{d}T}{\mathrm{d}t} = kT\left(1-\frac{T}{a}\right),$$

where a and k are positive constants.

(a) State the significance of a, in the context of the question. [1]

For the rest of the question, let $a = 1000$. It is also given that when $t = 0$, $T = 200$ and $\frac{\mathrm{d}T}{\mathrm{d}t} = 16$.

(b) Find the time taken for the timbre to double its initial amount. [4]

Upon reaching $T = 800$, the government of Ganyan starts harvesting a fixed amount of timbre, h thousand tonnes per year.

(c) Given that $k = 0.1$, find the range of values of h for the harvest to be sustainable in the long run. [2]

(d) For $h = 15$, find the equilibrium amount of timbre in the long run. [3]

6 The prestigious Nanyang D' Art Gallery was preparing for its grand reopening. The gallery's owner, Frankie, had commissioned a local artist, Stein, to create a stunning installation. Stein decided to use a linear transformation to transform a set of 4D vectors into a new set of vectors that would form the basis of this installation. The linear transformation was represented by a 4×4 matrix,

$$\mathbf{A} = \begin{pmatrix} 2\alpha & 1 & 0 & 1 \\ 1 & 2 & \alpha & 1 \\ 0 & 1 & 2 & 2 \\ 1 & 0 & 1 & 2 \end{pmatrix}.$$

(a) It is given that $\alpha = 1$.

(i) Find a basis for the range space of matrix **A**, that gives the set of transformed vectors. [2]

(ii) This stunning installation must pass through the point $(1,2,1,0)$. By finding a basis for the null space, determine the set of vectors that satisfies this condition. [3]

(b) Using row operations, determine if it is possible to find another value of α such that the dimension of the range space of matrix **A** is the same as that in **(a)(i)**. [5]

7 **(a)** Empirical studies suggest that the surface area of a human body is related to its weight and height. If the weight is w kg and the height is h cm, then the surface area S, in square metres, is approximately given by

$$S = 0.0072w^{0.425}h^{0.725}.$$

(i) Find expressions in terms of w and h for $\dfrac{\partial S}{\partial w}$ and $\dfrac{\partial S}{\partial h}$. [2]

(ii) A boy weighs 17 kg and stands at 100 cm on his birthday. Given that he is growing at a rate of 2 kg per year and 9 cm per year, find the rate at which his surface area is increasing on his birthday. [2]

(b) Given that $\mathrm{f}(x, y) = \mathrm{e}^{xy} + y\sin x$ where $x, y \in \mathbb{R}$. Let P be the point $(0, -2)$ on the xy-plane.

(i) Find the directional derivative of $\mathrm{f}(x, y)$ at P in the direction of the vector $\mathbf{i} + \sqrt{3}\mathbf{j}$. [3]

(ii) Find the greatest possible rate of change of $\mathrm{f}(x, y)$ at P. [1]

(iii) Find a unit vector along which $\mathrm{f}(x, y)$ is stationary at P. [3]

(iv) Find the equation of the tangent plane to the surface $z = \mathrm{f}(x, y)$ at P. [2]

Section B: Probability and Statistics [34 marks]

8 A factory produces microchips 24 hours a day in two non-overlapping shifts. In shift A (8am to 8pm), the average number of defective microchips per hour is 2.5.

(a) State the conditions in which the number of defective microchips per hour produced in shift A can be well modelled by a Poisson distribution. [1]

You should assume that these conditions hold.

(b) Find the probability that, in a randomly chosen 6 hour period in shift A, the total number of defective microchips is more than 12. [2]

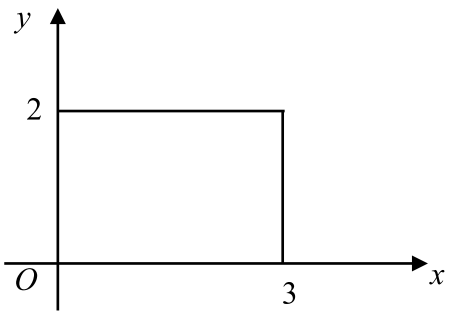
In shift B (8pm to 8am), the number of defective microchips per hour follows a Poisson distribution with mean 4.5.

(c) Find the probability that, in a randomly chosen day, the total number of defective microchips is less than 80. [2]

(d) Find the probability that, in a randomly chosen day, the total number of defective microchips is less than 80, given that there were 23 defective microchips in shift A. [2]

9 Two independent random variables $X \sim \mathrm{U}(0,3)$ and $Y \sim \mathrm{U}(0,2)$ are the length and width of a rectangle respectively. The area of this rectangle is denoted by A.

(a) The diagram below shows the region satisfying the inequalities $0 \le x \le 3$ and $0 \le y \le 2$. By considering a suitable curve in the diagram, find the probability that the area of the rectangle, A, is less than a, where $0 \le a \le 6$. [4]



(b) Hence find the probability density function of A. [2]

(c) Find the mean and variance of A. [2]

10 A member of the King's Air Rifle Club is practising target shooting. The member fires at a target until he hits the bullseye, then moves on to the next target. The probability of hitting the bullseye with any given shot is p, which remains constant for each target. $X(n)$ represents the number of shots required to hit the bullseye for the nth target.

(a) Write down the distribution of $X(n)$ and an assumption under this context. Hence find $\mathrm{P}\left(X\left(n\right)\leq n+1\right)$. [4]

The total number of shots required to hit n bullseyes is denoted by $S\left(n\right)$.

(b) Find the probability that the nth bullseye is hit with only one shot given that a total of at most $n+1$ shots are required for the first n bullseyes. [4]

(c) It is given that the member has 60 bullets and $p=0.8$. Find the probability that the first 50 bullseyes are hit before the member runs out of bullets. [3]

11 A study by the Nanyang Research Academy investigated the impact of alcohol on reaction time. Fifty randomly selected male participants, aged 35, consumed 178ml of wine. Thirty minutes later, their reaction times were measured using a standardized driving simulation. The participants' reaction times (in milliseconds) were recorded. The results are as follows:

$$\sum x=k \text{ and } \sum x^2=10\ 989\ 354.$$

(a) The typical reaction time of a 35-year-old male driver who did not consume alcohol is 400 milliseconds. If $k=22\ 243$, is there sufficient evidence, at 5% significance level, to suggest that the consumption of 178ml of wine increase the reaction time of a 35-year-old male driver? [5]

(b) The Tippler Winery Association aims to challenge the findings of the study in **(a)**. Given that the population variance is 20 514, find the range of values of k that would suggest that wine consumption does not significantly impact reaction time. [3]

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