Class	Register Number	Name

南洋女子中学校

NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2019
Secondary 3

INTEGRATED MATHEMATICS 1 — **2 hours**

Thursday **10 October 2019** **0845 – 1045**

READ THESE INSTRUCTIONS FIRST

1. Write your name, register number and class on all the work you hand in.
2. Answer **all** questions.
3. Write your answers and working on the separate writing paper provided, unless otherwise stated.
4. Write in dark blue or black ink on both sides of the paper.
5. You may use an HB pencil for any diagrams or graphs.
6. Do not use staples, paper clips, glue or correction tape/fluid.
7. Omission of essential working will result in loss of marks.
8. The use of an electronic calculator is expected, where appropriate.
9. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degree to one decimal place.

   For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
10. At the end of the examination, fasten all your work securely together.
11. The number of marks is given in brackets [ ] at the end of each question or part question.
12. The total number of marks for this paper is 80.

Mathematical Formulae

1. TRIGONOMETRY

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

2. MENSURATION

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2} r^2 \theta$, where θ is in radians

1 (a) Given $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & -5 \\ 3 & 1 \end{pmatrix}$, find k and m if $\mathbf{A}^2 - k\mathbf{B} = \begin{pmatrix} 13 & -10 \\ m & 3 \end{pmatrix}$. [4]

(b) Ginger Tea, Jasmine Tea and Milk Tea are drinks sold by three Bubble Tea outlets. During Christmas, all proceeds from the three outlets are donated to charity. The number of cups of each type of drink sold during this year's Christmas is shown in the table below.

		Number of cups sold		
		Shop *A*	Shop *B*	Shop *C*
Types of Drink	Ginger Tea	70	120	30
	Jasmine Tea	130	90	160
	Milk Tea	50	160	50

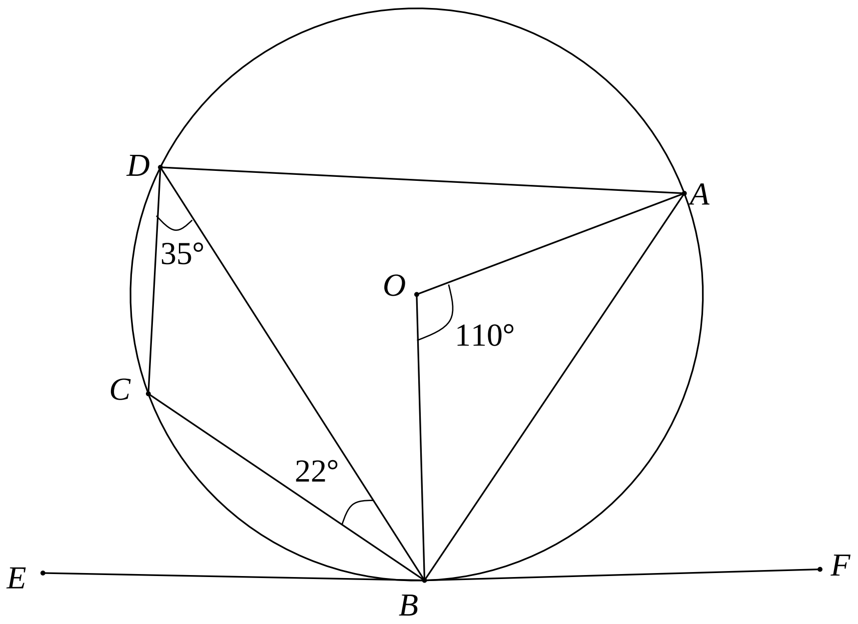
(i) Calculate $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 70 & 120 & 30 \\ 130 & 90 & 160 \\ 50 & 160 & 50 \end{pmatrix}$. [1]

(ii) Explain what your answer represents. [1]

For the Christmas charity drive, each outlet charges a fixed price for all types of drink. Shop *A* sells its drinks at \$3.50 per cup, Shop *B* at \$4.50 per cup while Shop *C* charges \$4 per cup.

(iii) By multiplying your answer from **(i)** with another matrix, find the total amount of money that will be collected from all the three Bubble Tea outlets. [2]

2 The diagram below shows a circle with centre O, and A, B, C and D are points on the circumference of the circle. EF is a tangent to the circle at point B. Angle $AOB = 110°$, angle $CBD = 22°$ and angle $BDC = 35°$.



(a) Find, giving reasons for each answer,

(i) angle DAB, [2]

(ii) angle DAO, [2]

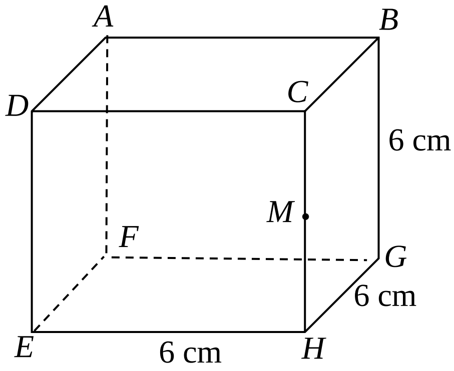
(iii) angle EBC, [1]

(iv) angle ADB. [1]

(b) A point P is such that angle BPC is 30°. Is P inside, outside or on the circumference of the circle? Justify your answer with a brief reason. [2]

(c) Explain whether points A, O and C are collinear. [2]

3 The diagram shows a cube *ABCDEFGH* of sides 6 cm.

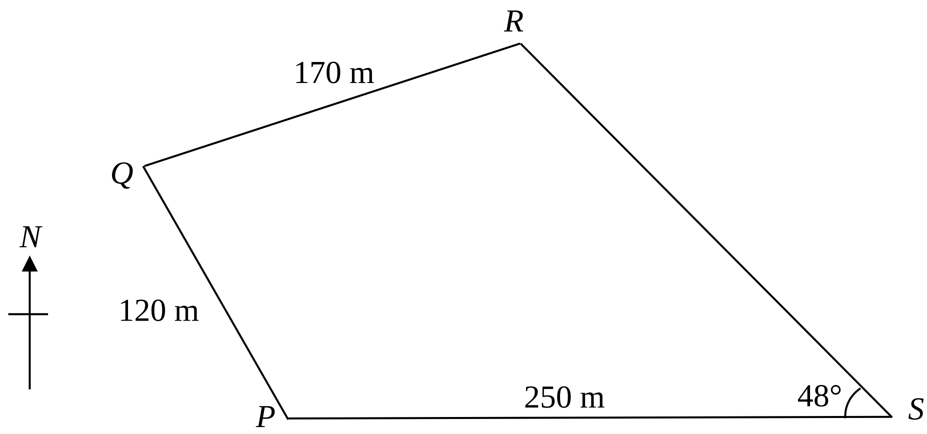


Find

(a) the length of *AM*, where *M* is the midpoint of *CH*, [2]

(b) angle *AMF*. [2]

4 In the diagram, *P*, *Q*, *R* and *S* are four points on level ground. The bearing of *Q* from *P* is 320° and the bearing of *R* from *Q* is 055°. *PQ* = 120 m, *PS* = 250 m, *QR* = 170 m and angle *PSR* = 48°.



(a) **(i)** Find angle *PQR*. [2]

(ii) Hence, show that the length of *PR* is 199 m. [2]

(b) Calculate, using the value of *PR* given in **(a)(ii)**,

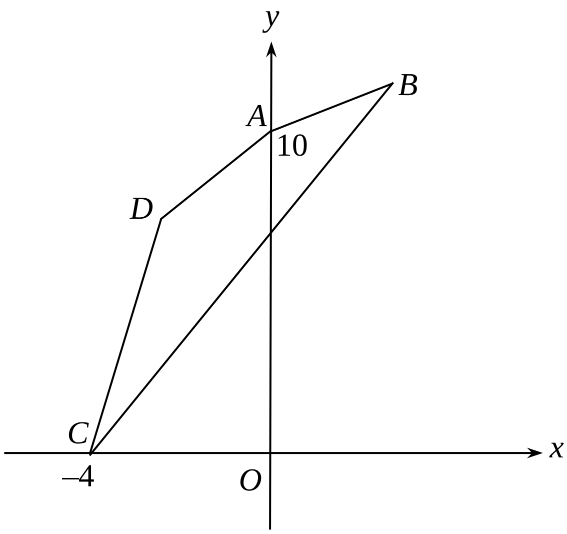
(i) the acute angle *PRS*, [2]

(ii) the area of triangle *PRS*. [2]

(c) A tree stands at *S*. An eagle sits on top of the tree, looking at a mouse as the mouse runs along *PR*. Given that the greatest angle of depression from the eagle to the mouse is 3.6°, find the height of the tree. [3]

5 **Solutions to this question by accurate drawing will not be accepted.**

The diagram below (not drawn to scale) shows a quadrilateral *ABCD*. *AB* meets the y-axis at 10, and *BC* meets the x-axis at –4. Points *A* and *C* lie on the y-axis and x-axis respectively.



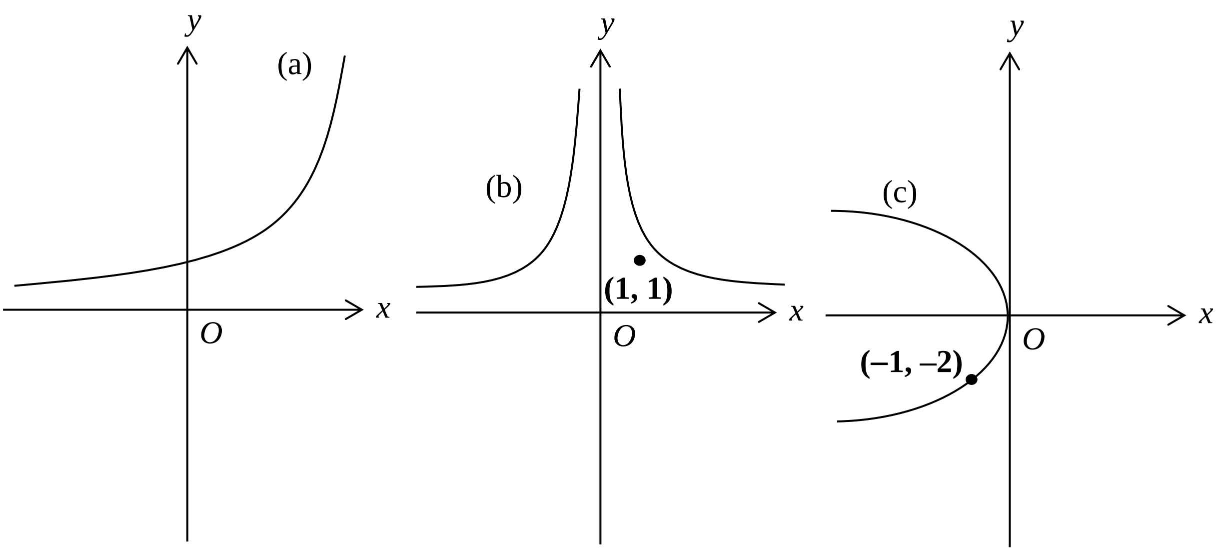
(a) *BC* is parallel to the line $y - 2x + 9 = 0$. Find the equation of *BC*. [2]

(b) The length of *AB* is $2\sqrt{2}$ units. Show that the coordinates of *B* are (2, 12). [3]

(c) The point *D* $(-3, p)$ is equidistant from points *B* and *C*. Find the value of p. [4]

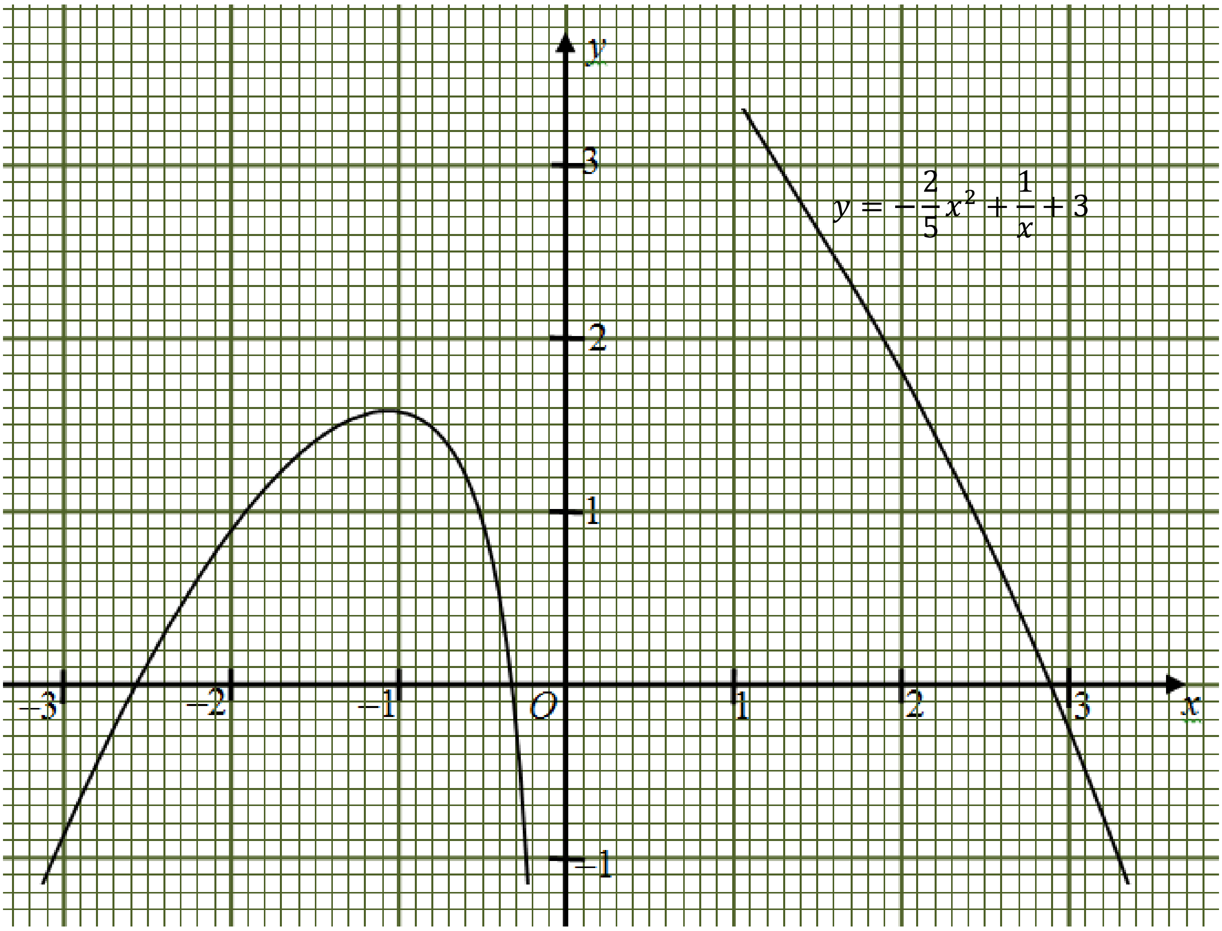
(d) Calculate the area of quadrilateral *ABCD*. [2]

6 Write down a possible equation for each of the following graphs. [3]



7 **Answer the whole of this question on the insert provided.**

The diagram below shows part of the graph of $y = -\frac{2}{5}x^2 + \frac{1}{x} + 3$.



(a) By drawing a tangent, find the gradient of the graph at the point where $x = -2$. [2]

(b) Use your graph to solve the inequality $-\frac{4}{5}x^2 + \frac{2}{x} + 2 \geq -5$ in the range of $-3 \leq x \leq 3.5$. [3]

(c) State the range of values of k for which the equation $-\frac{2}{5}x^2 + \frac{1}{x} + 3 = k$ has less than two solutions. [1]

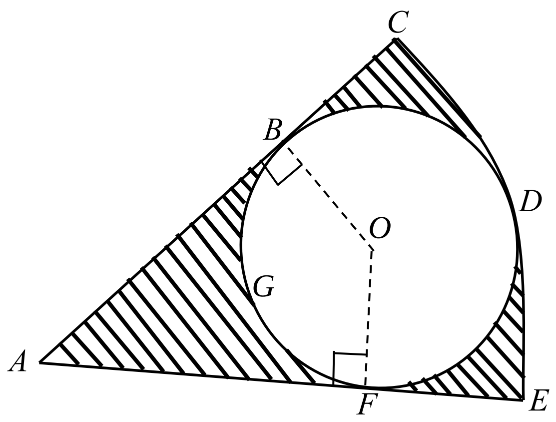
(d) **(i)** On the same axes, draw the graph of $2y + x = 2$. [1]

(ii) Write down the x-coordinates of the points at which the two graphs intersect. [2]

(iii) Find the equation, in the form $4x^3 + ax^2 + bx + c = 0$, which is satisfied by the values of x found in **d(ii)**, given that a, b, and c are integers. [2]

[Turn over]

8 The diagram shows a sector *ABCDEF* with an inscribed circle *BDFG*, centre *O*. *ABC* and *AFE* are tangents to the circle at points *B* and *F* respectively. The arc *CDE* touches the circle at *D*. The radius of the circle *BDFG* is 7 cm.



(a) The arc length of the arc *BGF* is $\frac{14\pi}{3}$ cm. Show that angle *BAF* = $\frac{\pi}{3}$ radians. [3]

(b) Calculate the length of *OA*. [2]

(c) Find the area of the shaded region. Leave your answer in terms of π. [3]

9 The variables *x* and *y* are related by the equation $xy = \frac{x+py}{q}$. When the graph of $\frac{x}{y}$ against *x* is drawn, a straight line is obtained which has a gradient of -5 and passes through the point $(4, -12)$.

(a) Find the value of *p* and of *q*. [4]

(b) Find the value of *x* when $3y = x$. [2]

(c) The equation $2x + axy - 4by = 0$ is plotted on the same axes of $\frac{x}{y}$ against *x*. It has an infinite number of solutions when solved simultaneously with $xy = \frac{x+py}{q}$. Find the value of *a* and of *b*. [3]

10 A circle C_1 of equation $x^2 + y^2 + 16x - 30y = 0$ meets a second circle C_2 at points (0, 0) and (7, 7). Given that the radii of C_1 and C_2 are equal, find

(a) the radius of C_1, [2]

(b) the equation of C_2. [3]

END OF PAPER

Answers to IM1

1(a)	k = –2, m = –2
(b)(i)	$\begin{pmatrix} 250 & 370 & 240 \end{pmatrix}$
(ii)	250 represents the total number of drinks sold by *A*, 370 represents the total number of drinks sold by *B*, and 240 represents the total number of drinks sold by *C*.
(iii)	$3500
2(a)(i)	$57°$
(ii)	$22°$
(iii)	$35°$
(iv)	$55°$
(b)	*P* is **outside** the circumference of the circle. P cannot be *on* the circumference. According to the *property "angles in the same segment"*, if *P* is on the circumference of the circle, angle *BPC* will be equal to angle BDC, which is $35°$. P cannot be *inside* the circumference, because BPC is less than 35°. Since angle *BPC* is less than $35°$, *P* must be *outside* the circle.
(c)	$\angle CDA$ $= \angle CDB + \angle BDA$ $= 35° + 55°$ $= 90°$ $\angle CDA$ satisfies the property *"right angle in semicircle"*. Hence, *triangle CDA* is a right angle triangle in the semicircle *AOCD*. Hence *AOC* is the diameter of the circle. Therefore, *A*, *O* and *C* are collinear points (proven)
3(a)	9 cm
(b)	$38.9°$
4(a)(i)	$85°$
(b)(i)	$69.0°$
(b)(ii)	22200 m^2
(c)	$14.0m$
5(a)	$y = 2x + 8$
(c)	7
(d)	15 square units
6(a)	$y = (k)^x$ where $k > 1$

(b)	$y = \frac{k}{x^2}\ or\ \frac{k}{\lvert x \rvert}\ , where\ k > 1$
(c)	$y^2 = -4x$
7(a)	Gradient = 1.35 [acceptable range: 1.25 – 1.45]
(b)	$-2.8 \leq x \leq -0.3$ [acceptable range: $-2.85\ \ to - 2.75$ & $-\ 0.35\ to - 0.25$] $0 < x \leq 3.1$ or $1.05 < x \leq 3.1$ [acceptable range: 3.05 to 3.15] [acceptable range: 1.00 to 1.10]
(c)	$k > 1.6$
(d)(ii)	$x = -0.7 \pm 0.05$, or -1.2 ± 0.05, or 3.1 ± 0.05.
(iii)	$4x^3 - 5x^2 - 20x - 10 = 0$
8(b)	OA = 14 cm
(c)	$\frac{49}{2}\pi$ cm^2
9(a)	$p = -8$, $q = -5$
(b)	$x = 1$
(c)	$a = 10$, $b = 4$
10(a)	17 units
(b)	$(x - 15)^2 + (y + 8)^2 = 289$