DUNMAN SECONDARY SCHOOL

CANDIDATE NAME			
CLASS		INDEX NUMBER	

PRELIMINARY EXAMINATION 2026
SECONDARY 4 EXPRESS/ 5 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS **4049/01**

Paper 1

2 hours 15 minutes

Marking Scheme

Question	Answer	Marks	Partial Marks	Guidance
1	$\dfrac{2x^2-3x-8}{(x+2)^2(x-1)}=\dfrac{A}{x+2}+\dfrac{B}{(x+2)^2}+\dfrac{C}{x-1}$		M1	Correct partial fractions, accept $\dfrac{Ax+B}{(x+2)^2}$
	$2x^2-3x-8=A(x+2)(x-1)+B(x-1)+C(x+2)^2$		M1	Comparing numerator
	Let $x=1$, $9C=-9$ $C=-1$		B1, B1	For any two correct A, B or C
	Let $x=-2$, $6=-3B$ $B=-2$			
	Let $x=0$, $-6=-2A$ $A=3$			
	$\dfrac{2x^2-3x-8}{(x+2)^2(x-1)}=\dfrac{3}{x+2}-\dfrac{2}{(x+2)^2}-\dfrac{1}{x-1}$	5	A1	All correct with partial fractions given

2	$\log_3 x - 2\log_9 3 = \log_9(x+4)$ $\log_3 x - \log_9 3^2 = \log_9(x+4)$ $\log_3 x - 1 = \log_9(x+4)$ $\log_3 x - 1 = \dfrac{\log_3(x+4)}{\log_3 9}$ $\log_3 x - 1 = \dfrac{\log_3(x+4)}{2}$ $2\log_3 x - 2 = \log_3(x+4)$ $\log_3 x^2 - \log_3(x+4) = 2$ $\log_3 \dfrac{x^2}{(x+4)} = 2$ $\dfrac{x^2}{(x+4)} = 9$ $x^2 = 9x + 36$ $x^2 - 9x - 36 = 0$ $(x-12)(x+3) = 0$ $x = 12$ or $x = -3$ (rej.)	5	M1 M1 M1 M1 A1	Realises $2\log_9 3 = 1$ or combine with log term Change of base formula into a common base Getting rid of log Solving quadratic equation

3	$\dfrac{1-\sin 2x}{\cos^2 x}=\left(\tan x-1\right)^2$ $\text{LHS}=\dfrac{1-2\sin x\cos x}{\cos^2 x}$ $=\dfrac{\cos^2 x+\sin^2 x-2\sin x\cos x}{\cos^2 x}$ $=\tan^2 x-2\tan x+1$ $=\left(\tan x-1\right)^2$ $=\text{RHS}$		M1 M1 M1 AG1	Double angle $\sin 2x$ Identity $\cos^2 x+\sin^2 x=1$ Divide to get in terms of $\tan x$
	Alternative: $\text{LHS}=\dfrac{1-2\sin x\cos x}{\cos^2 x}$ $=\dfrac{1}{\cos^2 x}-2\tan x$ $=\sec^2 x-2\tan x$ $=\tan^2 x-2\tan x+1$ $=\left(\tan x-1\right)^2$ $=\text{RHS}$	4	M1 M1 M1 AG1	Double angle $\sin 2x$ Get $\sec^2 x$ Identity $\sec^2 x=1+\tan^2 x$

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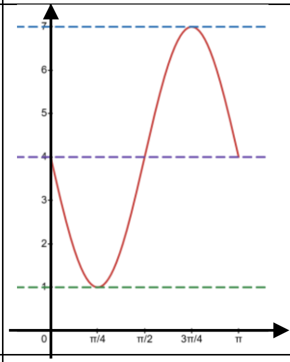
4	$60(1.1)^{n-1}=150$ $(1.1)^{n-1}=2.5$		M1	Of the form $60(1.1)^{n-1}$ or $60(1.1)^{n}$
	$(n-1)\lg 1.1=\lg 2.5$ $n-1=\dfrac{\lg 2.5}{\lg 1.1}$		M1	Taking log or changing to log form $\log_{1.1} 2.5$
	$n=10.61$		M1	Making n the subject and obtaining the value (either n = 9.61 or 10.61 is fine at this stage)
	Therefore, **Week 11.**	4	A1	

Question	Answer	Marks	Partial Marks	Guidance
5	$y = x - 3$ subst into $x^2 - xy + y^2 = 19$, $x^2 - x(x-3) + (x-3)^2 = 19$ $x^2 - x^2 + 3x + x^2 - 6x + 9 = 19$ $x^2 - 3x - 10 = 0$ $(x-5)(x+2) = 0$ $x = 5$ or $x = -2$	4	M1 M1 M1 A1	Subst. to get equation in one variable Simplify into general form Solving quadratic equation

Question	Answer	Marks	Partial Marks	Guidance
6(a)	Let x be the side of the square. $x^2+x^2=\left(2+\sqrt{8}\right)^2$ $2x^2=4+8+4\sqrt{8}$ $x^2=6+2\sqrt{8}$ $x^2=6+4\sqrt{2}$	3	M1 M1 A1	Pythagoras' theorem expansion
6(b)	$V=\frac{1}{3}x^2h$ $10+6\sqrt{2}=\frac{1}{3}\left(6+4\sqrt{2}\right)h$ $h=\frac{30+18\sqrt{2}}{6+4\sqrt{2}}\times\frac{6-4\sqrt{2}}{6-4\sqrt{2}}$ $=\frac{180-120\sqrt{2}+108\sqrt{2}-144}{4}$ $=\frac{36-12\sqrt{2}}{4}$ $=9-3\sqrt{2}$	4	M1 M1 M1 A1	Volume formula Rationalise denominator Expansion

Question	Answer	Marks	Partial Marks	Guidance
7(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(xe^{3x}\right)=3xe^{3x}+e^{3x}$	2	B1, B1	Differentiating e^{3x} correctly, All correct
7(b)	$\int 3xe^{3x}+e^{3x}\ \mathrm{d}x=xe^{3x}+c$ $\int 3xe^{3x}\ \mathrm{d}x=xe^{3x}-\int e^{3x}\ \mathrm{d}x+c$ $\int 3xe^{3x}\ \mathrm{d}x=xe^{3x}-\frac{e^{3x}}{3}+c+c_1$ $\int xe^{nx}\ \mathrm{d}x=\frac{xe^{3x}}{3}-\frac{e^{3x}}{9}+c_2$	4	M1 M1 M1 A1	Using reverse of differention Making $\int 3xe^{3x}\ \mathrm{d}x$ the subject Integrate e^{3x}

Question	Answer	Marks	Partial Marks	Guidance
8(a)	$x^2+4xh=432$ $4xh=432-x^2$ $h=\dfrac{432-x^2}{4x}$ $V=x^2h$ $=x^2\left(\dfrac{432-x^2}{4x}\right)$ $=\dfrac{432x-x^3}{4}$	3	B1 M1 AG1	Surface area h in terms of x
8(b)	$\dfrac{\mathrm{d}V}{\mathrm{d}x}=\dfrac{432-3x^2}{4}$ $\dfrac{432-3x^2}{4}=0$ $3x^2=432$ $x^2=144$ $x=12$ (rej. -ve) $\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2}=-\dfrac{3x}{2}=-18<0$ Therefore, stationary value is **maximum**.	4	B1 B1 M1 A1	Correct differentiation x = 12 and derivative = 0 valid test (or 1st derivative test) Conclusion

Question	Answer	Marks	Partial Marks	Guidance
9(a)	Amplitude = 3 Period = π or $180°$	 2	B1 B1	
9(b)		3	B1 B1 B1	Max and min values, and shape 1 cycle All critical angles shown
9(c)	$3\pi \sin 2x = -4x$ $3\sin 2x = -\frac{4x}{\pi}$ $-3\sin 2x = \frac{4x}{\pi}$ $4 - 3\sin 2x = 4 + \frac{4x}{\pi}$ $\therefore y = \frac{4x}{\pi} + 4$ Draw line $y = \frac{4x}{\pi} + 4$ Since the line lies above the curve for $x > \frac{3\pi}{4}$, there are no intersections. Hence the equation has no solution for $x > \frac{3\pi}{4}$.	 3	M1 M1 A1	Attempt to make $4 - 3\sin 2x$ the subject Based on student's graph (must be linear, and consist of π in the gradient) Only award for correct graph and explanation

Question	Answer	Marks	Partial Marks	Guidance
10(a)	$(2-kx)^8 = 2^8 + \binom{8}{1}2^7(-kx) + \binom{8}{2}2^6(-kx)^2 + \binom{8}{3}2^5(-kx)^3 + \ldots$ $= 256 - 1024kx + 1792k^2x^2 - 1792k^3x^3 + \ldots$	3	B3,2,1,0	
10(b)	$\left(5+\frac{3}{kx}\right)(2-kx)^8 = \left(5+\frac{3}{kx}\right)\left(256-1024kx+1792k^2x^2-1792k^3x^3+\ldots\right)$ coefficient of $x^2 = 5\left(1792k^2\right) + \frac{3}{k}\left(-1792k^3\right)$ $8960k^2 - 5376k^2 = 896$ $3584k^2 = 896$ $k^2 = \frac{1}{4}$ $k = \frac{1}{2}$ $(\because k > 0)$	4	M1 M1 M1 A1	Identifying $5 \times x^2$-term Identifying $\frac{3}{k} \times x^3$ -term Simplifying quadratic equation

Question	Answer	Marks	Partial Marks	Guidance
11(a)	$y = px^3 + qx^2 + 2x - 2$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 3px^2 + 2qx + 2$ $3px^2 + 2qx + 2 > 0$ for all real values of x $(2q)^2 - 4(3p)(2) < 0$ $4q^2 - 24p < 0$ $q^2 < 6p$	3	B1 M1 AG1	derivative Discriminant Discriminant < 0 and correct simplification
11(b)	$y = 6x^3 + x^2 + 2x - 2$ When $x = \frac{1}{2}$, $y = 6\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 2$ $= \frac{3}{4} + \frac{1}{4} + 1 - 2$ $= 0$ Hence, $x = \frac{1}{2}$ is an x-intercept.	1	B1	Must show $y = 0$
11(c)	Since $1^2 < 6(6) = 36$, $q^2 < 6p$, the curve is increasing. Therefore, it only cuts the x-axis once at $x = \frac{1}{2}$.	2	B1 B1	Calculation Showing increasing function, cutting x-axis only once

Question	Answer	Marks	Partial Marks	Guidance
12(a)	$\angle QRT = \angle QPT = x$ (tangent-chord theorem) $\angle QOP = 2x$ ($\angle$ at centre = $2\times\angle$ at circum.) $\angle OPU = 90°$ (radius $\perp$ tangent) $\angle OUP = 180° - 90° - 2x$ ($\angle$ sum of triangle) $= 90° - 2x$ $= 90° - 2\angle QPT$ (shown)	4	B1 M1 M1 AG1	Tangent-chord theorem/alt. segment
12(b)	Prove: triangle *RPT* is similar to triangle *RQP* $\angle PRT = \angle QRP$ (common angle) $\angle RQP = 90°$ (right angle in semicircle) $\angle RQP = \angle RPT = 90°$ Therefore triangle *RPT* is similar to triangle *RQP*. $\frac{RP}{RQ} = \frac{RT}{RP}$ (ratios of corresponding sides are equal) $RP^2 = RT \times RQ$ (shown)	4	B1 B1 M1 AG1	Prove similar triangles

Question	Answer	Marks	Partial Marks	Guidance
13(a)	$y = x + 3$ $q = p + 3$ $AD = \sqrt{(14-6)^2 + (0-15)^2} = 17$ units Since perimeter = 60 units, $2(AB + AD) = 60$ $AB = 13$ units $\sqrt{(p-14)^2 + q^2} = 13$ $(p-14)^2 + q^2 = 169$ $(p-14)^2 + (p+3)^2 = 169$ $p^2 - 28p + 196 + p^2 + 6p + 9 = 169$ $2p^2 - 22p + 36 = 0$ $p^2 - 11p + 18 = 0$ $p^2 = 11p - 18$ (shown)	6	B1 M1 B1 M1 M1 AG1	 Find AD AB = 13 units Equating distance Subst. to get equation in terms of p
13(b)	$p^2 - 11p + 18 = 0$ $(p-2)(p-9) = 0$ $p = 2$ or $p = 9$ (rej.) $q = 5$ $\therefore B(2,5)$. Diagram is necessary to determine that B lies to the left of D and hence smaller x-coordinate than that of D.	3	M1 A1 B1	Solving Correct coordinates Sound explanation

Question	Answer	Marks	Partial Marks	Guidance
14(a)	$\frac{dy}{dx}=\frac{2}{2x+1}$ At P, $\frac{dy}{dx}=\frac{2}{5}$ Gradient of normal = $-\frac{5}{2}$ $y-\ln 5=-\frac{5}{2}(x-2)$ $y=-\frac{5}{2}x+5+\ln 5$ $y=-\frac{5}{2}x+5\ln e+\ln 5$ $y=-\frac{5}{2}x+\ln e^5+\ln 5$ $y=-\frac{5}{2}x+\ln 5e^5$, where $m=-\frac{5}{2}$, $k=5e^5$	4	B1 M1 M1 A1	Derivative of curve Gradient of normal Equation of normal
14(b)	$y=\ln(2x+1)$ $e^y=2x+1$ $x=\frac{e^y-1}{2}$	1	B1	Or equivalent

14(c)	Area bounded by curve and y-axis: $\int_0^{\ln 5} \frac{e^y - 1}{2}\ dy$ $= \left[\frac{e^y - y}{2}\right]_0^{\ln 5}$ $= \left(\frac{e^{\ln 5} - \ln 5}{2}\right) - \left(\frac{e^0 - 0}{2}\right)$ $= 2 - \frac{1}{2}\ln 5$ y-intercept of normal $= \ln 5e^5$ Area of triangle $= \frac{1}{2} \times 2 \times 5 = 5$ units^2 shaded area $= 7 - \frac{1}{2}\ln 5$ units^2 $= 6.20$ units^2 (3 s.f.)		B1 M1 M1 M1 A1	Correct limits Integral – from (b) Subst. of limits Area of triangle