SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY FOUR
O-LEVEL PROGRAMME

CANDIDATE NAME	**Solution**								
CLASS	4				REGISTER NUMBER				
CENTRE NUMBER					INDEX NUMBER				

ADDITIONAL MATHEMATICS **4049/01**

Paper 1

Wednesday **30 August 2022** **2 hours 15 minutes**

Candidates answer on the Question Paper.
No Additional Materials are required.

1. A quadratic curve is given by the equation $\mathrm{f}(x) = 6x - 13 - 3x^2$.

By expressing f (x) in the form $a(x-h)^2 + k$, explain why the curve has no real roots. [4]

	$\mathrm{f}(x) = 6x - 13 - 3x^2$ $= -3\left(x^2 - 2x + \frac{13}{3}\right)$ $= -3\left[(x-1)^2 - 1 + \frac{13}{3}\right]$ $= -3\left[(x-1)^2 + \frac{10}{3}\right]$ $= -3(x-1)^2 - 10$ Since the curve has a **maximum point** of (1, –10) which is **below the *x*-axis**, the curve **doesn't cut the *x*-axis** and thus it has no real roots. **<u>Alternative Method</u>** Since $(x-1)^2 \geq 10$, $-3(x-1)^2 \leq 0$ $-3(x-1)^2 - 10 \leq -10$ $\therefore \mathrm{f}(x) \leq -10$, $\mathrm{f}(x) \neq 0$ for all values of x Hence, the curve has no real roots.		

2. **(a)** **Without the use of a calculator**, find the value of 6^n, given that $2^{n+3} - 2^n = \frac{3^{n+1}}{9^n}$. [3]

(b) The value of an antique is estimated to increase by k % per year. Given that its value at the beginning of 2017 was \$20 000. The value V, after t years, can be modelled by $V = A(1.11)^t$.

(i) State the value of A and of k. [2]

(ii) Calculate the year when its estimated value will first reach \$190 000. [2]

(a)	$2^{n+3} - 2^n = \frac{3^{n+1}}{9^n}$ $2^n\left(2^3 - 1\right) = \frac{3}{3^n}$ $6^n = \frac{3}{7}$		
(b) (i) **(ii)**	$A = 20\ 000$, $k = 11$ $190\,000 = 20\,000(1.11)^t$ $1.11^t = 9.5$ $t = \frac{\lg 9.5}{\lg 1.11}$ $t = 21.6$ The year 2038		

3. The term containing the highest power of x in the polynomial $\mathrm{f}(x)$ is $3x^4$. Two of the roots of the equation $\mathrm{f}(x) = 0$ are $x = -1$ and $x = k$, where k in an integer.

Given that $x^2 - 2x + 6$ is a quadratic factor of $\mathrm{f}(x)$ and $\mathrm{f}(x)$ leaves a remainder of – 36 when divided by x,

(a) show that $k = 2$. [3]

Hence,

(b) determine the number of real roots of the equation $\mathrm{f}(x) = 0$. [3]

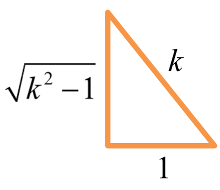
(a) **(b)**	$\mathrm{f}(x) = 3(x^2 - 2x + 6)(x+1)(x-k)$ $\mathrm{f}(0) = -36$ $3(6)(-1)(k) = -36$ $k = 2$ (shown) $x^2 - 2x + 6 = 0$ Discriminant = $(-2)^2 - 4(6) < 0$ $\therefore$ no real roots When $\mathrm{f}(x) = 0$, there are 2 real roots.		

4. Given that $\cos 20^\circ = \frac{1}{k}$, where $k > 0$, find an expression, in terms of k, for the following trigonometric ratios.

(a) $\sec(-20^\circ)$, [1]

(b) $\sin 160^\circ$, [2]

(c) $\tan 110^\circ$. [1]

(a) **(b)** **(c)**	$\sec(-20^\circ) = k$ $\sin 160^\circ = \sin 20^\circ$ $= \frac{\sqrt{k^2-1}}{k}$ $\tan 110^\circ = -\tan 70^\circ$ $= -\frac{1}{\sqrt{k^2-1}}$  		

5. The concentration of a drug, C(t), in a patient's bloodstream, t hours after an injection of the drug, can be modelled by $C(t)=-\frac{t^3}{75}+\frac{16t}{5}$.

Find the length of time in which the drug concentration in the bloodstream is increasing after an injection [4]

	$C'(t)=-\frac{t^2}{25}+\frac{16}{5}$ $C'(t)>0 \Rightarrow \quad -\frac{t^2}{25}+\frac{16}{5}>0$ $80-t^2>0$ $t^2-80<0$ $(t+\sqrt{80})(t-\sqrt{80})<0$ $-\sqrt{80}<t<\sqrt{80}$ Since $t\geq 0$, $\quad 0\leq t<\sqrt{80}$ or $0\leq t<4\sqrt{5}$ Duration $=4\sqrt{5}$ h or 8.94 h		

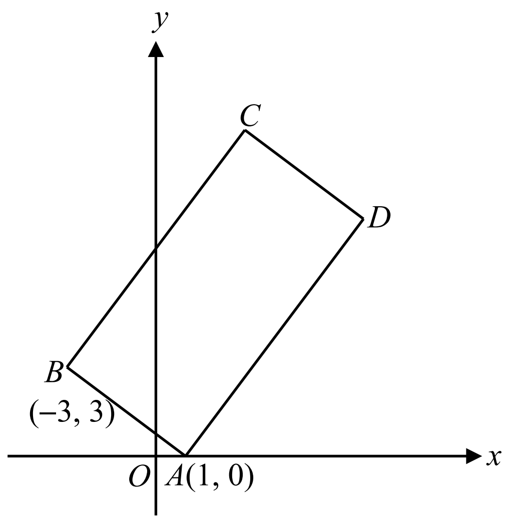
6. **(a)** Express $\frac{1-3x-2x^2}{x(1+2x)^2}$ in partial fractions. [5]

(b) Find the value of $\int_1^2 \frac{1-3x-2x^2}{x(1+2x)^2}\,\mathrm{d}x$. [4]

(a)	Let $\frac{1-3x-2x^2}{x(1+2x)^2}=\frac{A}{x}+\frac{B}{1+2x}+\frac{C}{(1+2x)^2}$ $\Rightarrow 1-3x-2x^2=A(1+2x)^2+B(x)(1+2x)+C(x)$ When $x=-\frac{1}{2}$, $\quad 1+\frac{3}{2}-\frac{1}{2}=C\left(-\frac{1}{2}\right)$ $\therefore C=-4$ When $x=0$, $\quad 1=A(1)$ $\therefore A=1$ When $x=1$, $\quad 1-3-2=1(9)+B(1)(3)-4(1)$ $\therefore B=-3$ Or Comparing coefficient of x^2, $\quad 4A+2B=-2$ $\therefore B=-3$ $\therefore \frac{1-3x-2x^2}{x(1+2x)^2}=\frac{1}{x}-\frac{3}{1+2x}-\frac{4}{(1+2x)^2}$		

(b)	$\int_1^2 \frac{1-3x-2x^2}{x(1+2x)^2}\,\mathrm{d}x$ $=\int_1^2\left(\frac{1}{x}-\frac{3}{1+2x}-\frac{4}{(1+2x)^2}\right)\mathrm{d}x$ $=\left[\ln x-\frac{3}{2}\ln(1+2x)-\frac{4}{(-1)(2)(1+2x)}\right]_1^2$ $=\left[\ln x-\frac{3}{2}\ln(1+2x)+\frac{2}{1+2x}\right]_1^2$ $=\left[\ln 2-\frac{3}{2}\ln 5+\frac{2}{5}\right]-\left[\ln 1-\frac{3}{2}\ln 3+\frac{2}{3}\right]$ $=-1.321009+0.98125$ $=-0.340$		

7.



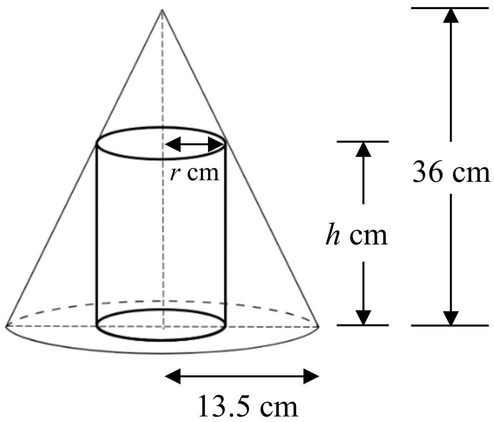
The diagram shows a rectangle *ABCD*. The vertices of the rectangle are $A(1, 0)$, $B(-3\ , 3)$, *C* and *D*. The area of the rectangle is 50 square units.

(a) Show that *x*-coordinate of *C* satisfies the equation $x^2 + 6x - 27 = 0$. [6]
(b) Find the coordinates of *C* and *D*. [3]

(a)	$m_{AB} = \dfrac{3-0}{-3-1} = -\dfrac{3}{4}$ $m_{BC} = \dfrac{4}{3}$ Equation of *BC*, $y - 3 = \dfrac{4}{3}(x+3)$ $y = \dfrac{4x}{3} + 7$ $AB = \sqrt{4^2 + 3^2}$ $= 5$ units $\therefore\ BC = 10$ units $BC^2 = 100$ unit^2 $(x+3)^2 + \left(\dfrac{4x}{3} + 7 - 3\right)^2 = 100$ $x^2 + 6x + 9 + \dfrac{(4x+12)^2}{9} = 100$ $9x^2 + 54x + 81 + 16x^2 + 96x + 144 = 900$ $25x^2 + 150x - 675 = 0$ $x^2 + 6x - 27 = 0$		
(b)	$(x-3)(x+9) = 0$ $x = 3,\ \ -9$ (NA) $y = \dfrac{4(3)}{3} + 7 = 11$ $\therefore C(3, 11)$		

	Let $D(x, y)$ Midpoint of AC = Midpoint of BD $\frac{x+(-3)}{2}=\frac{1+3}{2} \Rightarrow x=7$ $\frac{y+3}{2}=\frac{11+0}{2} \Rightarrow y=8$ $\therefore D(7, 8)$		

8. The diagram shows the vertical cross-section of a piece of solid conical wood. The solid cone has base radius 13.5 cm and height 36 cm. A solid cylinder of radius r cm and height h cm was to be carved out of the solid cone.



(a) Show that the total surface area of the solid cylinder, A cm^2, is given by

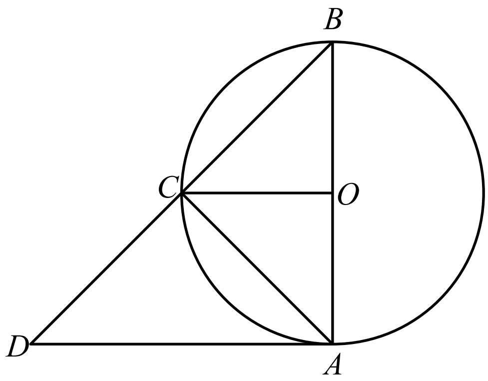
$$A = 72\pi r - \frac{10}{3}\pi r^2 .$$ [4]

(b) Given that r can vary, find the stationary value of A and determine its nature. [5]

(a)	By similar triangles, $\frac{r}{36-h}=\frac{13.5}{36}$ $8r = 108-3h$ $h=\frac{108-8r}{3}$ or $36-\frac{8r}{3}$ $A = 2\pi r^2 + 2\pi rh$ $= 2\pi r^2 + 2\pi r\left(\frac{108-8r}{3}\right)$ $= 2\pi r^2 + \frac{216\pi r}{3} - \frac{16\pi r^2}{3}$ $= 72\pi r - \frac{10}{3}\pi r^2$ (Shown)		

(b)	$\frac{\mathrm{d}A}{\mathrm{d}r} = 72\pi - \frac{20}{3}\pi r$ When $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$, $72\pi - \frac{20}{3}\pi r = 0$ $r = 10.8$ $\frac{\mathrm{d}^2 A}{\mathrm{d}r^2} = -\frac{20}{3}\pi < 0 \Rightarrow A$ is maximum $A = 72\pi(10.8) - \frac{10}{3}\pi(10.8)^2$ $= 1221.45$ $\therefore A = 1220$ or $\frac{1944\pi}{5}$ cm^2		

9. The diagram shows a circle, centre O, with diameter AB. The point C lies on the circle. The tangent to the circle at A meets BC extended at D and CO is perpendicular to AB.



(a) Prove that C is the midpoint of BD. **[4]**

(b) Prove that triangle ADC is isosceles. **[3]**

(a)	$\angle BAD = 90°$ (tan $\perp$ radius) $= \angle BOC$ ($CO \perp AB$) $\angle CBO = \angle DBA$ (common angle) ΔBOC and ΔBAD are similar. (AA) $\therefore \frac{BO}{BA} = \frac{BC}{BD} = \frac{1}{2}$ Hence, C is the midpoint of BD. **Alternative Method** $\angle BAD = 90°$ (tan $\perp$ radius) $= \angle BOC$ ($CO \perp AB$) $\therefore CO$ is parallel to DA. $\angle CBA = \angle DBA$ (common angle) $\angle BOC = \angle BAD$ (alternate angles) ΔBOC and ΔBAD are similar. (AA) $\therefore \frac{BO}{BA} = \frac{BC}{BD} = \frac{1}{2}$ Hence, C is the midpoint of BD.	
(b)	$\angle DAC = \angle CBA$ (alternate segment theorem) $\angle BCO = \angle CBA$ (base angles of an isosceles Δ) $\angle BCO = \angle CDA$ (corresponding angles of similar Δ) $\therefore \angle CDA = \angle DAC$ Hence, triangle ADC is isosceles.	

10. **(a)** Show that $\dfrac{1}{\sec x+1}+\dfrac{1}{\sec x-1}=\dfrac{2\cos x}{\sin^2 x}$. [3]

(b) Hence find, for $-\dfrac{\pi}{2}\le x\le\dfrac{\pi}{2}$, the exact solutions of the equation

$\dfrac{1}{\sec x+1}+\dfrac{1}{\sec x-1}=4\cos x$ [4]

(a)	$\dfrac{1}{\sec x+1}+\dfrac{1}{\sec x-1}$ $=\dfrac{2\sec x}{\sec^2 x-1}$ $=\dfrac{2\sec x}{\tan^2 x}$ $=\dfrac{2}{\cos x}\left(\dfrac{\cos^2 x}{\sin^2 x}\right)$ $=\dfrac{2\cos x}{\sin^2 x}$ (shown)	
(b)	$\dfrac{2\cos x}{\sin^2 x}=4\cos x$ $2\cos x=4\sin^2 x\cos x$ $2\cos x\left(1-2\sin^2 x\right)=0$ $2\cos x=0$ or $1-2\sin^2 x=0$ $x=\pm\dfrac{\pi}{2}$ $\quad$ $\sin x=\pm\dfrac{1}{\sqrt{2}}$ $x=\pm\dfrac{\pi}{4}$	

11. Given the equation $\dfrac{6}{\sin^2\theta} = 7 + \cot\theta$ where $0^\circ < \theta < 360^\circ$, find the values of θ. [6]

	$\dfrac{6}{\sin^2\theta} = 7 + \cot\theta$ $6\operatorname{cosec}^2\theta = 7 + \cot\theta$ $6\left(1+\cot^2\theta\right) = 7 + \cot\theta$ $6\cot^2\theta - \cot\theta - 1 = 0$ $\left(3\cot\theta + 1\right)\left(2\cot\theta - 1\right) = 0$ $\cot\theta = -\dfrac{1}{3}$ or $\cot\theta = \dfrac{1}{2}$ $\tan\theta = -3$ or $\tan\theta = 2$ Basic $\angle =$ 71.565° Basic $\angle =$ 63.434° $\theta = 108.4^\circ,\ 288.4^\circ$ $\theta = 63.4^\circ,\ 243.4^\circ$ OR $\dfrac{6}{\sin^2\theta} = 7 + \dfrac{\cos\theta}{\sin\theta}$ $6 = 7\sin^2\theta + \cos\theta\sin\theta$ $6\left(\sin^2\theta + \cos^2\theta\right) = 7\sin^2\theta + \cos\theta\sin\theta$ $\sin^2\theta + \sin\theta\cos\theta - 6\cos^2\theta = 0$ $\left(\sin\theta + 3\cos\theta\right)\left(\sin\theta - 2\cos\theta\right) = 0$ $\sin\theta = -3\cos\theta$ or $\sin\theta = 2\cos\theta$ $\tan\theta = -3$ or $\tan\theta = 2$		

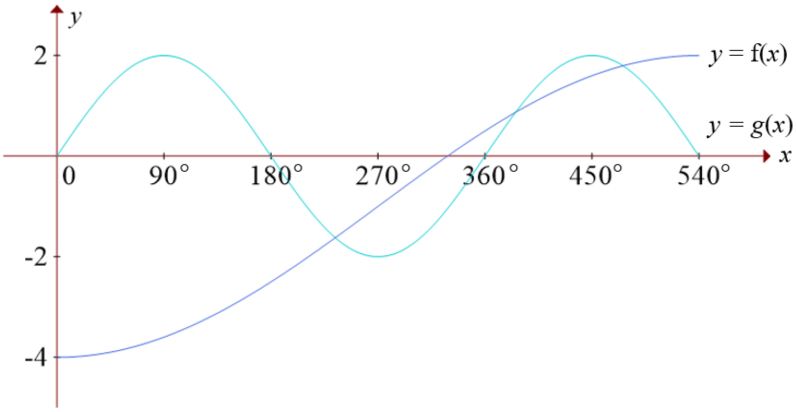
12. It is given that $f(x) = c - 3\cos\left(\frac{x}{b}\right)$ and $g(x) = 2\sin x$ for $0^\circ \le x \le 540^\circ$.

$f(x)$ has a period of 1080° and a minimum value of -4.

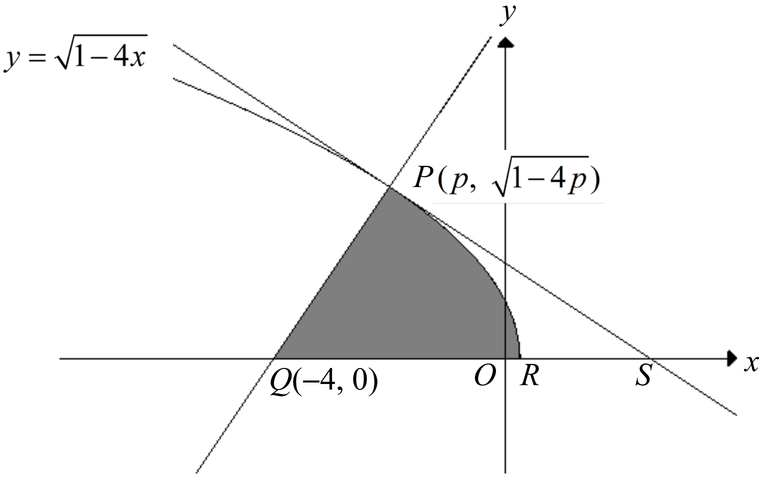
(a) State the value of b and of c. [2]

(b) Sketch, on the same axes, the graphs of $y = f(x)$ and $y = g(x)$. [4]

(c) Write down the smallest integer value of k for which $f(x) = g(x) + k$ has exactly two solutions. [1]

(a)	$b = 3$ $c = -1$	
(b)		
(c)	$k = -4$	

13. The diagram shows part of the curve $y = \sqrt{1-4x}$ passing though the point $P(p, \sqrt{1-4p})$, where p is a negative constant. The curve meets the x-axis at the point R. The normal and tangent to the curve meet the x-axis at the points $Q(-4, 0)$ and S.



(a) Show that $p = -2$. [5]
(b) Find the equation of the tangent at P. [2]
(c) Find the area of the shaded region PQR. [4]

(a)	$\frac{dy}{dx} = \left(\frac{1}{2}\right)(1-4x)^{-\frac{1}{2}}(-4)$ $= -\frac{2}{\sqrt{1-4x}}$ When $x = p$, gradient of tangent $= \frac{dy}{dx}$ $= -\frac{2}{\sqrt{1-4p}}$ Gradient of normal $= \frac{\sqrt{1-4p}}{p+4}$ Gradient of normal × Gradient of tangent = −1 $-\frac{2}{\sqrt{1-4p}} \times \frac{\sqrt{1-4p}}{p+4} = -1$ $\frac{2}{p+4} = 1$ $p+4 = 2$ $p = -2$ **<u>Alternative Method</u>** Gradient of normal, $\frac{\sqrt{1-4p}}{p+4} = \frac{\sqrt{1-4p}}{2}$ $p+4 = 2$ $p = -2$	

[Turn over

(b)	Equation of tangent, $y-3=-\frac{2}{3}(x+2)$ $y=-\frac{2}{3}x+\frac{5}{3}$	
(c)	Area of shaded region $=\frac{1}{2}\times(2)(3)+\int_{-2}^{\frac{1}{4}}(1-4x)^{\frac{1}{2}}\mathrm{d}x$ $=3+\left[\frac{(1-4x)^{\frac{3}{2}}}{\frac{3}{2}\times(-4)}\right]_{-2}^{\frac{1}{4}}$ $=3+\left[-\frac{1}{6}(1-4x)^{\frac{3}{2}}\right]_{-2}^{\frac{1}{4}}$ $=3+\left[0+\frac{1}{6}[1-4(2)]^{\frac{3}{2}}\right]$ $=\frac{15}{2}$ or $7\frac{1}{2}$ or 7.5 unit^2	

End of Paper 1