

FM Recurrence Relations with Complex Roots

1. 2023/VJC/FM Promo/Q5 (modified) [8marks, 14mins]

The sequence $\{v_n\}$ is given by $v_1 = 2$, $v_2 = 0$ and $v_{n+2} - 2v_{n+1} + 2v_n = 4$ for $n \geq 1$. The sequence $\{w_n\}$ satisfies a second order homogeneous recurrence relation and $v_n + w_n = k$, where k is a constant.

- (a) Write down a second order homogeneous recurrence relation for $\{w_n\}$ and hence find the value of k . [2]
- (b) Obtain an expression for v_n in the form of a trigonometric function. [4]
- (c) Hence show that v_{4n+2} can be expressed as $k[1 + f(n)]$, where the two possible forms of $f(n)$ are to be determined. [2]

H2 Math Sequences and Series

2. 2022/VJC/H2 Math Promo/Q11 [12marks, 22mins]

In a laboratory experiment, a tank is transported to and fro between station A and station B.

On the 1st visit to station A, 200 ml of water is added into the tank and on subsequent visits, 200 ml more water than the previous visit to station A will be added. That is, 400 ml and 600 ml of water will be added into the tank on the 2nd and 3rd visit to station A respectively. On each visit to station B, b ml of water is removed from the tank.

It is given that the tank is empty at the start of the experiment and it starts from station A.

- (a) On the m^{th} visit to station A, 3200 ml of water is added into the tank. Find m . [2]
- (b) It is given that immediately after the 16th visit to station B, there is more than 25 litres of water in the tank. Find the set of values that b can take. [3]

In another laboratory experiment, a tank is transported to and fro between station A and station C. 10% of the water in the tank is removed on each visit to station C. It is given that the tank is empty at the start of the experiment, and it starts from station A.

- (c) Show that the amount of water in the tank immediately after the 3rd visit to station C is 1009.8 ml. [2]
- (d) Show that the amount of water in the tank immediately after the n^{th} visit to station C is $1800n - 16200(1 - 0.9^n)$. [4]
- (e) The experiment stops when the amount of water in the tank immediately after it leaves station C exceeds 10 litres. After the k^{th} visit to station C, the experiment stopped. Find k . [1]