RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Additional Practice Questions for Chapter 2B: Vectors II – Equation of Straight Lines

1 Referred to the origin O, the position vector of the point A is $2\mathbf{i}+2\mathbf{j}-6\mathbf{k}$ and the cartesian equation of the line l is $x-1=2-y=z+6$. Find

(i) the position vector of the foot of the perpendicular from A to l, [3]

(ii) the perpendicular distance from A to l, [2]

2 The line l_1 has equation $\mathbf{r}=\begin{pmatrix}1+2\lambda\\1-2\lambda\\-3\lambda\end{pmatrix}$, where λ is a parameter.

(i) Find the exact shortest distance between the origin O and l_1. [3]

Another line l_2 has equation $\dfrac{x-1}{2}=\dfrac{3-y}{2}=-\dfrac{z}{9}$.

(ii) Write down a vector equation of l_2. [1]

(iii) Show that l_1 and l_2 are skew lines. [3]

(iv) Find a vector that is perpendicular to both l_1 and l_2. [1]

The points P and Q lie on l_1 and l_2 respectively, such that the line PQ is perpendicular to both l_1 and l_2.

(v) Find a vector equation of the line PQ. [3]

3 The line l_1 has equation $\dfrac{x-5}{3}=\dfrac{z-5}{m}$, $y=2$ and the line l_2 has equation $\dfrac{x}{-5}=y$, $z=0$, where m is a constant. It is given that l_1 and l_2 intersect at point A.

(i) Find the value of m, and the coordinates of A. [4]

(ii) Find the position vector of the point P on l_1 such that OP is perpendicular to l_1, where O is the origin. [3]

(iii) Find a vector equation of the line which is a reflection of l_2 in l_1. [3]

4 The points A and B have position vectors $\begin{pmatrix} 8 \\ 3 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix}$ respectively.

(i) Show that $AB = 2\sqrt{26}$. [1]

(ii) Find the cartesian equation for the line AB. [2]

(iii) The line l has equation $\mathbf{r} = \begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix} + t\begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix}$.

Find the length of the projection of AB onto l. [2]

(iv) Calculate the acute angle between AB and l, giving your answer correct to the nearest degree. [2]

(v) Find the position vector of the foot N of the perpendicular from A to l. Hence find the position vector of the image of A in the line l. [4]

5 With reference to the origin O, the points A and B have position vectors $\mathbf{a} = -\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 2\mathbf{j} + 5\mathbf{k}$ respectively.

(i) Find a vector equation of the line l_1 that passes through point A and is parallel to the vector **a**. [1]

(ii) Find the exact length of projection of **b** on l_1. Hence find d, the exact perpendicular distance from the point B to l_1. [4]

(iii) Using the value of d found in part **(ii)**, find the position vector of the point C, the foot of perpendicular from the point B to l_1. [3]

(iv) The line l_2 passes through point B and is parallel to vector **b**. Find a cartesian equation of l_3 which is the reflection of l_2 in l_1. [3]

6 The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} -6 \\ -3 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + \mu\begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} \text{ respectively, where } \lambda,\ \mu \in \mathbb{R}.$$

(i) Find the acute angle between l_1 and l_2. [2]

(ii) The points P and R lie on l_1 and l_2 respectively such that P is the reflection of the point R in the line l. The 3 lines intersect at the point $Q(0,3,-6)$.

Find a possible pair of vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$ such that $\angle PQR$ is acute and $\left|\overrightarrow{QP}\right| = 5$.

Hence, or otherwise, find the vector equation of the line l. [4]

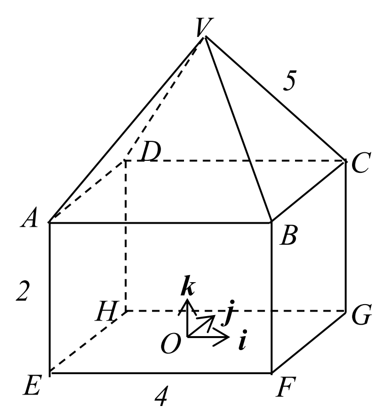
7 Relative to the origin O, the points A, B and C have position vectors $\mathbf{i}+2\mathbf{j}+5\mathbf{k}$, $\mathbf{i}-\mathbf{k}$ and $2\mathbf{i}+\mathbf{j}$ respectively.
The line l_1 passes through A and is parallel to the vector $3\mathbf{i}+2\mathbf{j}$. The line l_2 passes through the points B and C.

(i) Find a vector equation of the line l_1. [1]

(ii) Show that the lines l_1 and l_2 intersect and that the position vector $\overrightarrow{OX}$ of the point of intersection is $7\mathbf{i}+6\mathbf{j}+5\mathbf{k}$. [3]

The vector $a\mathbf{i}+b\mathbf{j}+\mathbf{k}$ is perpendicular to both l_1 and l_2. Find the values of a and b.
Hence give an equation of the line l_3 that is concurrent with l_1 and l_2 and is perpendicular to both l_1 and l_2.
If V is a point on l_3 such that $VABX$ is a tetrahedron with base ABX and height $5\sqrt{14}$ units, find the coordinates of V. [7]

8 The figure shows a right pyramid *VABCD* with a square base *ABCD*, standing horizontally on a cuboid *ABCDEFGH*. It is given that *VA* = *VB* = *VC* = *VD* = 5 cm, *EF* = *FG* = 4 cm and *AE* = 2 cm, as shown in the diagram. *O* is the centre of the square base *EFGH*. Perpendicular unit vectors **i**, **j**, **k** are parallel to *EF*, *FG*, *EA* respectively.



(i) Show that the height of the figure, *OV* is $2+\sqrt{17}$. [2]

(ii) State the geometrical meaning of $\dfrac{\left|\overrightarrow{VA}\times\overrightarrow{OV}\right|}{2+\sqrt{17}}$. [1]

(iii) Find the equation of the line passing through *B* and *V*. [2]

9 Relative to an origin O, points A and B have position vectors $\begin{pmatrix}3\\4\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\2\\0\end{pmatrix}$ respectively.

The line l has vector equation $\mathbf{r}=\begin{pmatrix}6\\a\\0\end{pmatrix}+t\begin{pmatrix}1\\3\\a\end{pmatrix}$, where t is a real parameter and a is a constant. The line m passes through the point A and is parallel to the line OB.

(i) Find the position vector of the point P on m such that OP is perpendicular to m. [4]
(ii) Show that the two lines l and m have no common point. [3]
(ii) If the acute angle between the line l and the $z-$axis is $60°$, find the exact values of the constant a. [3]

10 Referred to an origin O, the position vectors of two points A and B are $\mathbf{a}$ and $\mathbf{b}$ respectively. A line l has vector equation given by $\mathbf{r} = \frac{1}{3}\mathbf{a}+\lambda\left(2\mathbf{b}-\mathbf{a}\right)$, where $\lambda\in\mathbb{R}$.

The point N is the foot of perpendicular from A to l. It is given that $|\mathbf{a}|=2$, $|\mathbf{b}|=1$ and $\mathbf{a}$ is perpendicular to $\mathbf{b}$.

Find the position vector of N in terms of $\mathbf{a}$ and $\mathbf{b}$. [5]

11 The four points A, B, C, D have position vectors $p\mathbf{i}$, $\mathbf{j}+q\mathbf{k}$, $\mathbf{k}$ and $\mathbf{i}+\mathbf{j}$ respectively, where p and q are positive real numbers. Find vector equations for the lines AB and CD.

(i) Given that the lines AB and CD intersect at a point R, express q in terms of p, and show that the two lines cannot be perpendicular. Show also that $\dfrac{AR}{AB}=\dfrac{CR}{CD}$.
(ii) Given that AB and CD are perpendicular, express q in terms of p, and show that the angle between AC and BD is equal to the angle between AD and BC.