



HWA CHONG INSTITUTION
2026 JC2 Preliminary Examination
Higher 2

FURTHER MATHEMATICS

Question Paper

9649/02
22 September 2026
3 hours
Total marks: 100

Candidate Name	
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CT Group	25
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READ THESE INSTRUCTIONS FIRST

Do not write anything on the List of Formulae (MF27).

Answer **all** the questions.

Write your answer to each question in the space provided in the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [] at the end of each question or part of question.

Remarks for Candidates:

- a) **INSTR:** Did not follow **instructions** stated in the question (e.g. correct significant figures, exact values, coordinates, required form, etc.).
- b) **NOT:** Incorrect or inappropriate use of mathematical **notation**.
- c) **ACC:** **Inaccurate** answer (e.g. due to premature rounding, omission of "+C" in indefinite integrals, etc.).

This document consists of 7 printed pages and 1 blank page.

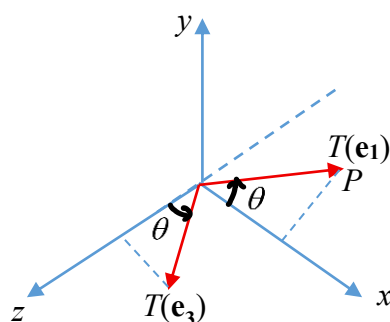
Section A: Pure Mathematics [50 marks]

- 1** The sequence $\{a_n\}$ is given by $a_{n+2} + a_{n+1} + a_n = 0$, where $n \geq 1$, $a_1 = 1$ and $a_2 = 2$.
- (a) Find a_n in terms of n , giving your answer in the form $P \cos(n\alpha) + Q \sin(n\alpha)$ where P , Q and α are constants to be determined. [4]
- (b) Determine the set of all possible values that a_n can take as n varies. [1]
- (c) Deduce the value of $\sum_{n=1}^{2026} a_n$. [2]
- 2** (a) Consider the gradient function $\frac{dy}{dx} = f(x)$. By applying the trapezium rule to approximate the integral of $f(x)$ over the interval $x \in [kh, (k+1)h]$, derive the improved Euler formula $y_{k+1} \approx y_k + \frac{h}{2} [f(kh) + f((k+1)h)]$ where y_k denotes the value of y when $x = kh$. [3]
- (b) A function y satisfies $\frac{dy}{dx} = \frac{1}{x} + \frac{1}{y}$, where $y = 2$ when $x = 3$. Use two steps of the improved Euler method to calculate an approximation of the value of y , when $x = 3.2$, giving your answers to three decimal places. [4]
- 3** (a) (i) If z is a complex number and z^* is its conjugate, show that $zz^* = |z|^2$. [1]
- (ii) Let z and w be complex numbers which do not lie on the unit circle. Given that $\left| \frac{z - 2w}{2 - z^*w} \right| = 1$, find the value of $|z|$. [4]
- (b) The complex number $z = x + iy$ where x, y are real, satisfies the locus L , which is defined by $|z - k| = |z - 2ki|$ where k is a positive real constant.
- (i) Show that the cartesian equation of the locus L can be expressed as $4y - 2x = 3k$. [2]
- (ii) A circle C is defined by the equation $|z + 3 - 6i| = \sqrt{20}$. It is given that the point on L which is nearest to the origin lies on the circumference of circle C . Find the possible value(s) of k . [4]

- 4 Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear transformation representing the anticlockwise rotation about the y axis through an angle θ and let $\mathbf{e}_1, \mathbf{e}_2$ and $\mathbf{e}_3$ be the standard basis vectors of $\mathbb{R}^3$, where

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ and } \mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

The diagram below illustrates the transformations of $\mathbf{e}_1$ and $\mathbf{e}_3$ to $T(\mathbf{e}_1)$ and $T(\mathbf{e}_3)$ respectively.



- (a) Explain why $T(\mathbf{e}_1) = \begin{pmatrix} \cos \theta \\ 0 \\ -\sin \theta \end{pmatrix}$, $T(\mathbf{e}_3) = \begin{pmatrix} \sin \theta \\ 0 \\ \cos \theta \end{pmatrix}$ and find $T(\mathbf{e}_2)$. [3]

$S: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is a linear transformation that scales a vector by non-zero scale factors of α and β along the x and z axes respectively. Let $\mathbf{A}$ and $\mathbf{B}$ be the matrices representing T and S respectively.

- (b) Give a geometrical interpretation of $\mathbf{A}^{-1}$ and hence find $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$. [3]

A curve C_1 in the xz -plane has an equation $2x^2 + z^2 = 36$. Another curve C_2 is obtained by rotating C_1 30° anticlockwise about the y -axis followed by scaling parallel to the x -axis and z -axis by scale factors of 2 and 3 respectively.

- (c) Hence find a cartesian equation of C_2 , leaving your answer in a simplified form. [4]

- 5 (a) Solve $x \frac{du}{dx} - 2u = x^3 e^x$, leaving your answer for u in terms of x . [3]

Sketch, on a single diagram, three distinct members of the family of solution curves. [2]

- (b) Use the substitution $y = xz$ to show that $x^2 \frac{d^2 z}{dx^2} - 2z = x g(x)$ can be written as

$$x \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} = x g(x). \quad [2]$$

- (c) Solve the differential equations

(i) $x^2 \frac{d^2 z}{dx^2} - 2z = x^3 e^x$, [4]

(ii) $x^2 \frac{d^2 z}{dx^2} - 2z = 1$. [4]

giving each answer for z in the form $z = f(x)$.

Section B: Probability and Statistics [50 marks]

- 6 A publisher is interested in testing whether the number of printing errors per 100 pages follows a Poisson distribution. The observed frequencies from examining 400 randomly selected sets of 100 pages are as follows:

Number of errors	0	1	2	3	4	5	6	7	8
Frequency	80	100	86	67	33	25	6	2	1

- (a) Test at the 5% level of significance whether the Poisson distribution provides a good fit for the number of printing errors per 100 pages. [5]
- (b) Referring to the two largest contributions to the test statistic, discuss what the test indicates about the printing errors. [1]

- 7 The probability density function (p.d.f) of a normal distribution is given by

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad x \in \mathbb{R}$$

where μ and σ^2 are the mean and variance of the normal distribution respectively.

- (a) Using the given p.d.f, show that $\int_0^\infty e^{-2x^2} dx = \sqrt{\frac{\pi}{8}}$. [2]

A continuous random variable X has its probability distribution given by

$$g(x) = \begin{cases} ke^{-2x^2}, & x \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

- (b) Sketch the graph of $g(x)$. [1]
- (c) Using the result found in part (a), find the value of k . [1]
- (d) With the value of k found in part (c), find $\text{Var}(X)$, leaving your answer in exact form. [3]
- (e) Find the median value of X . [2]

- 8 A biased blue die is rolled. The probability that “1” is obtained is 0.4. Let X be the number of attempts to obtain the first “1” on the blue die.

- (a) Find the probability that at least 3 attempts are required to obtain a “1”. [1]
 (b) Given that no “1” was obtained in the first 6 attempts, find the probability that the first “1” is obtained on the 10th attempt. [2]

Another biased red die with the same probability of obtaining “1” is rolled independently.

Let Y be the number of attempts to obtain the first “1” on the red die. Let W be the minimum of X and Y .

- (c) Find $P(W = w)$, in terms of w . Hence or otherwise, find $P(W = 2)$. [4]
 (d) Let $T = X + Y$. Find $P(T = 4)$. [2]
 (e) Using your result from parts (c) and (d), show that T and W are not independent. [2]

- 9 A sugar company produces bags of sugar with a target weight of 1 kg per bag. The mass of the packaging is negligible. A quality control team randomly selects 40 bags and records the mass of the sugar in each bag in grams. The data are summarized as follows.

$$\sum x = 39889.4 \quad \sum x^2 = 39779879.2$$

- (a) Construct a 95% confidence interval for the mean mass of the sugar based on the data, leaving your answer correct to two decimal places. [4]
 (b) State, with a reason, what conclusion the control team should reach. [2]

After receiving feedback from this initial assessment, the company makes changes to the production line.

- (c) The quality control team calculated the 95% confidence interval for the mass of the sugar after the change in production line. The confidence interval is (998.01, 1000.59). By comparing it with the confidence interval found in part (a), state if it can be concluded that there is a change in the mean mass of sugar. [1]
 (d) Subsequently, another 40 bags of sugar are randomly sampled, and their masses are recorded in grams. The sample mean mass is 999.3 grams while the sample variance is 16.799 grams². Using this sample and the data from part (a), test at 5% level of significance whether there is evidence that the mean mass of sugar differs before and after the production line change. [4]

- 10** AceDrink and BrightDrink are two popular drinks. A company carried out a survey to determine which drink is preferred by consumers. Ten consumers who took part in a taste test were asked to rate the two drinks on a scale of 1 to 10, 10 being the best. The following scores were recorded.

AceDrink	9	4	7	8	3	6	9	10	10	5
BrightDrink	3	6	4	4	8	5	2	2	1	5

- (a) Give one reason why a paired-sample t -test may be inappropriate in testing whether consumers prefer one drink to the other. [1]
- (b) Use the Wilcoxon signed rank test to determine whether consumers prefer AceDrink to BrightDrink at the 5% significance level. [4]
- (c) Explain why it is generally better to use a Wilcoxon matched pair signed rank test than a sign test. Give an instance where a sign test has to be used instead of a Wilcoxon signed rank test. [2]
- (d) Suppose that the company carried out a similar survey on n participants, and 20 of the participants gave a higher score for AceDrink as compared to BrightDrink. The company used a one-tailed sign test at the 5% level of significance and was able to conclude that consumers prefer AceDrink. Assuming no ties, find the largest possible value of n . [3]
- (e) In a separate survey, 20 participants were asked to indicate which of the two beverages they prefer. 13 of them indicated a preference for AceDrink, and 5 indicated a preference for BrightDrink, and 2 of them have no preference. Use a sign test to determine at the 5% significance level if this shows a significant difference in the preference for either drink. [3]

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Mark Scheme

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This document consists of **22** printed pages.

Section A: Pure Mathematics [50 marks]

- 1 The sequence $\{a_n\}$ is given by $a_{n+2} + a_{n+1} + a_n = 0$, where $n \geq 1$, $a_1 = 1$ and $a_2 = 2$.
- (a) Find a_n in terms of n , giving your answer in the form $P \cos(n\alpha) + Q \sin(n\alpha)$ where P , Q and α are constants to be determined. [4]
- (b) Determine the set of all possible values that a_n can take as n varies. [1]
- (c) Deduce the value of $\sum_{n=1}^{2026} a_n$. [2]

	Suggested Solution
(a)	The characteristic equation $x^2 + x + 1 = 0$ Roots are $x_1 = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \text{ and } x_2 = \cos \frac{2\pi}{3} - i \sin \frac{2\pi}{3}.$ $r = 1$ $\theta = \frac{2\pi}{3}$ Therefore $a_n = r^n [P \cos(n\theta) + Q \sin(n\theta)]$ $= P \cos\left(\frac{2n\pi}{3}\right) + Q \sin\left(\frac{2n\pi}{3}\right)$ When $n = 1$, $a_n = 1$: $P\left(-\frac{1}{2}\right) + Q\left(\frac{\sqrt{3}}{2}\right) = 1$ When $n = 2$, $a_n = 2$: $P\left(-\frac{1}{2}\right) + Q\left(-\frac{\sqrt{3}}{2}\right) = 2$ Solving: $P = -3$, $Q = -\frac{\sqrt{3}}{3}$. $\therefore a_n = -3 \cos \frac{2n\pi}{3} - \frac{\sqrt{3}}{3} \sin \frac{2n\pi}{3}$ Alternative: $a_n = A_1 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)^n + A_2 \left(\cos \frac{2\pi}{3} - i \sin \frac{2\pi}{3} \right)^n$

	$a_n = A_1 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)^n + A_2 \left(\cos \frac{2\pi}{3} - i \sin \frac{2\pi}{3} \right)^n$ $= A_1 \left(\cos \frac{2n\pi}{3} + i \sin \frac{2n\pi}{3} \right) + A_2 \left(\cos \frac{2n\pi}{3} - i \sin \frac{2n\pi}{3} \right)$ Substitute $a_1 = 1$ and $a_2 = 2$: $A_1 \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i \right) + A_2 \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i \right) = 1 \quad \text{-----(1)}$ $A_1 \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i \right) + A_2 \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i \right) = 2 \quad \text{-----(2)}$ $(1) \times \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i \right) - (2) \times \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i \right)$ $\Rightarrow A_2 = \frac{-9 - \sqrt{3} i}{6}$ Then $A_1 = \frac{-9 + \sqrt{3} i}{6}$, Therefore $a_n = \frac{-9 + \sqrt{3} i}{6} \left(\cos \frac{2n\pi}{3} + i \sin \frac{2n\pi}{3} \right) + \frac{-9 - \sqrt{3} i}{6} \left(\cos \frac{2n\pi}{3} - i \sin \frac{2n\pi}{3} \right)$ $= -3 \cos \frac{2n\pi}{3} - \frac{\sqrt{3}}{3} \sin \frac{2n\pi}{3}$
(b)	When $n=1$, $a_1 = 1$ $n=2$, $a_2 = 2$ $n=3$, $a_3 = -3$ $n=4$, $a_4 = 1$ $n=5$, $a_5 = 2$ $n=6$, $a_6 = -3$ So possible set of values are $\{1, 2, -3\}$
(c)	$\sum_{n=1}^{2026} a_n = \sum_{n=1}^{2025} a_n + a_{2026}$ $= a_{2026}$ $= a_1$ $= 1$

- 2 (a) Consider the gradient function $\frac{dy}{dx} = f(x)$. By applying the trapezium rule to approximate the integral of $f(x)$ over the interval $x \in [kh, (k+1)h]$, derive the improved Euler formula $y_{k+1} \approx y_k + \frac{h}{2} [f(kh) + f((k+1)h)]$ where y_k denotes the value of y when $x = kh$. [3]
- (b) A function y satisfies $\frac{dy}{dx} = \frac{1}{x} + \frac{1}{y}$, where $y = 2$ when $x = 3$. Use two steps of the improved Euler method to calculate an approximation of the value of y , when $x = 3.2$, giving your answers to three decimal places. [4]

	Suggested Solution
(a)	$\because \frac{dy}{dx} = f(x)$ $\therefore y = \int f(x) dx$ $y_{k+1} - y_k = \int_{kh}^{(k+1)h} f(x) dx$ $\therefore y_{k+1} = y_k + \int_{kh}^{(k+1)h} f(x) dx$ By the trapezium rule, $\int_{kh}^{(k+1)h} f(x) dx = \frac{h}{2} [f(kh) + f((k+1)h)]$ Hence, $y_{k+1} \approx y_k + \frac{h}{2} [f(kh) + f((k+1)h)]$, which is the improved Euler formula since in this case f is a function of x only.
(b)	$\frac{dy}{dx} = \frac{1}{x} + \frac{1}{y}$ $f(x, y) = \frac{1}{x} + \frac{1}{y}$ $x_0 = 3, y_0 = 2, h = 0.1$ $u_1 = y_0 + h \left(\frac{1}{x_0} + \frac{1}{y_0} \right) = 2 + 0.1 \left(\frac{1}{3} + \frac{1}{2} \right) = \frac{25}{12}$ $y_1 = 2 + \frac{0.1}{2} \left[\left(\frac{1}{3} + \frac{1}{2} \right) + \left(\frac{1}{3.1} + \frac{1}{25} \right) \right] = 2.081796 = 2.082$ $u_2 = y_1 + h \left(\frac{1}{x_1} + \frac{1}{y_1} \right)$ $= 2.081796 + 0.1 \left(\frac{1}{3.1} + \frac{1}{2.081796} \right)$ $= 2.162090$

	$y_2 = 2.081796 + \frac{0.1}{2} \left[\left(\frac{1}{3.1} + \frac{1}{2.081796} \right) + \left(\frac{1}{3.2} + \frac{1}{2.162090} \right) \right]$ $= 2.160694$ $= 2.161$
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3 (a) (i) If z is a complex number and z^* is its conjugate, show that $zz^* = |z|^2$. [1]

(ii) Let z and w be complex numbers which do not lie on the unit circle. Given that

$$\left| \frac{z-2w}{2-z^*w} \right| = 1, \text{ find the value of } |z|. \quad [4]$$

(b) The complex number $z = x + iy$ where x, y are real, satisfies the locus L , which is defined by $|z-k| = |z-2ki|$ where k is a positive real constant.

(i) Show that the cartesian equation of the locus L can be expressed as $4y - 2x = 3k$. [2]

(ii) A circle C is defined by the equation $|z+3-6i| = \sqrt{20}$. It is given that the point on L which is nearest to the origin lies on the circumference of circle C . Find the possible value(s) of k . [4]

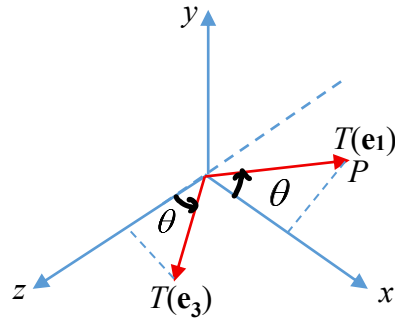
	Suggested Solution
(a)(i)	Let $z = x + iy$ $zz^* = (x + iy)(x - iy)$ $= x^2 + y^2$ $= z ^2$
(a)(ii)	$\left \frac{z-2w}{2-z^*w} \right = 1$ $ z-2w ^2 = 2-z^*w ^2$ $(z-2w)(z-2w)^* = (2-z^*w)(2-z^*w)^*$ $(z-2w)(z^*-2w^*) = (2-z^*w)(2-zw^*)$ $zz^* - 2zw^* - 2wz^* + 4ww^* = 4 - 2zw^* - 2z^*w + zz^*ww^*$ $z^*z + 4ww^* = 4 + zz^*ww^*$ $ z ^2 + 4 w ^2 = 4 + z ^2 w ^2$ $ z ^2 + 4 w ^2 - 4 - z ^2 w ^2 = 0$ $(z ^2 - 4)(w ^2 - 1) = 0$ $\therefore z = 2 \quad (\because w \neq 1)$

(b)(i)	$ z - k = z - 2ki $ Let $z = x + iy$ $ x + iy - k = x + iy - 2ki $ $(x - k)^2 + y^2 = x^2 + (y - 2k)^2$ $x^2 - 2kx + k^2 + y^2 = x^2 + y^2 - 4ky + 4k^2$ $-2kx + k^2 + 4ky - 4k^2 = 0$ $\therefore k > 0$ $4y - 2x = 3k \quad (\because k > 0)$
(b)(ii)	$4y - 2x = 3k$ $y = \frac{1}{2}x + \frac{3}{4}k$ $\therefore$ the point nearest to the origin lies on the line $y = -2x$. Substitute $y = -2x$ into $4y - 2x = 3k$, $4(-2x) - 2x = 3k$ $x = -0.3k$ The coordinates of the point nearest the origin is $(-0.3k, 0.6k)$ Since this point lies on the circle C , $(-0.3k + 3)^2 + (0.6k - 6)^2 = 20$ $0.09(k - 10)^2 + 0.36(k - 10)^2 = 20$ $(k - 10)^2 = \frac{20}{0.45}$ $k = \frac{10}{3}$ or $\frac{50}{3}$

- 4 Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear transformation representing the anticlockwise rotation about the y axis through an angle θ and let $\mathbf{e}_1, \mathbf{e}_2$ and $\mathbf{e}_3$ be the standard basis vectors of $\mathbb{R}^3$, where

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ and } \mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

The diagram below illustrates the transformation of $\mathbf{e}_1$ to $T(\mathbf{e}_1)$.



- (a) Explain why $T(\mathbf{e}_1) = \begin{pmatrix} \cos \theta \\ 0 \\ -\sin \theta \end{pmatrix}$, $T(\mathbf{e}_3) = \begin{pmatrix} \sin \theta \\ 0 \\ \cos \theta \end{pmatrix}$ and find $T(\mathbf{e}_2)$. [3]

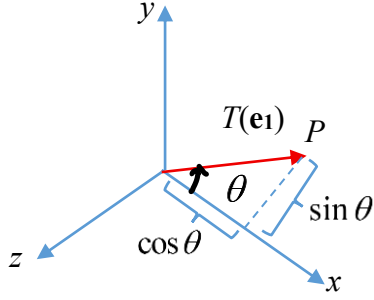
$S: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is a linear transformation that scales a vector by non-zero scale factors of α and β along the x and z axes respectively. Let $\mathbf{A}$ and $\mathbf{B}$ be the matrices representing T and S respectively.

- (b) Give a geometrical interpretation of $\mathbf{A}^{-1}$ and hence find $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$. [3]

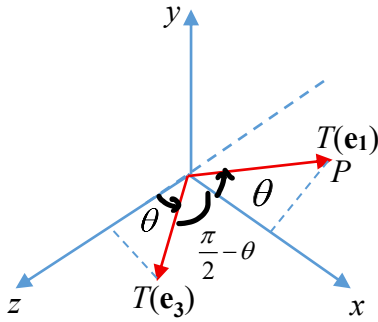
A curve C_1 in the xz -plane has an equation $2x^2 + z^2 = 36$. Another curve C_2 is obtained by rotating C_1 30° anticlockwise about the y -axis followed by scaling parallel to the x -axis and z -axis by scale factors of 2 and 3 respectively.

- (c) Hence find a cartesian equation of C_2 , leaving your answer in a simplified form. [4]

Suggested Solution	
(a)	Let P be the coordinates of the point by rotating $(1,0,0)$ anticlockwise about y axis through an angle θ . By referring to the following right-angle triangle,



$$P(\cos \theta, 0, -\sin \theta) \Rightarrow T(\mathbf{e}_1) = \begin{pmatrix} \cos \theta \\ 0 \\ -\sin \theta \end{pmatrix}$$



$$\text{Similarly, } T(\mathbf{e}_3) = \begin{pmatrix} \cos\left(\frac{\pi}{2} - \theta\right) \\ 0 \\ \cos \theta \end{pmatrix} = \begin{pmatrix} \sin \theta \\ 0 \\ \cos \theta \end{pmatrix}$$

$$\text{Since } \mathbf{e}_2 \text{ is invariant by } T, T(\mathbf{e}_2) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

(b)

The geometrical interpretation of $\mathbf{A}^{-1}$ is the anticlockwise rotation about the y axis through an angle $-\theta$ (i.e. clockwise rotation about the y axis through an angle θ).

$$\mathbf{A} = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \beta \end{pmatrix}$$

$$\mathbf{A}^{-1} = \begin{pmatrix} \cos(-\theta) & 0 & \sin(-\theta) \\ 0 & 1 & 0 \\ -\sin(-\theta) & 0 & \cos(-\theta) \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix}$$

	$\mathbf{B}^{-1} = \begin{pmatrix} \frac{1}{\alpha} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\beta} \end{pmatrix}$
(c)	Let $ST: \begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{A}^{-1} \mathbf{B}^{-1} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ $= \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix} \begin{pmatrix} \frac{1}{\alpha} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\beta} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ $= \begin{pmatrix} \frac{\cos \theta}{\alpha} & 0 & -\frac{\sin \theta}{\beta} \\ 0 & 1 & 0 \\ \frac{\sin \theta}{\alpha} & 0 & \frac{\cos \theta}{\beta} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ $x = \frac{\cos \theta}{\alpha} X - \frac{\sin \theta}{\beta} Z$ $y = Y$ $z = \frac{\sin \theta}{\alpha} X + \frac{\cos \theta}{\beta} Z$ Since the curve is in the xz-plane, $y = 0$, $Y = 0$. $2x^2 + z^2 = 36$ $\Rightarrow 2 \left(\frac{\cos \theta}{\alpha} X - \frac{\sin \theta}{\beta} Z \right)^2 + \left(\frac{\sin \theta}{\alpha} X + \frac{\cos \theta}{\beta} Z \right)^2 = 36$ When $\theta = 30^\circ$, $\alpha = 2$, $\beta = 3$, $2 \left(\frac{\sqrt{3}}{4} X - \frac{1}{6} Z \right)^2 + \left(\frac{1}{4} X + \frac{\sqrt{3}}{6} Z \right)^2 = 36$ $2 \left(\frac{3}{16} X^2 - \frac{\sqrt{3}}{12} XZ + \frac{1}{36} Z^2 \right) + \left(\frac{1}{16} X^2 + \frac{\sqrt{3}}{12} XZ + \frac{3}{36} Z^2 \right) = 36$

	$\frac{7}{16}X^2 - \frac{\sqrt{3}}{12}XZ + \frac{5}{36}Z^2 = 36$ Hence the equation of C_2 is $\frac{7}{16}x^2 - \frac{\sqrt{3}}{12}xz + \frac{5}{36}z^2 = 36$
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- 5 (a) Solve $x \frac{du}{dx} - 2u = x^3 e^x$, leaving your answer for u in terms of x . [3]

Sketch, on a single diagram, three distinct members of the family of solution curves. [2]

- (b) Use the substitution $y = xz$ to show that $x^2 \frac{d^2 z}{dx^2} - 2z = x g(x)$ can be written as

$$x \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} = x g(x). \quad [2]$$

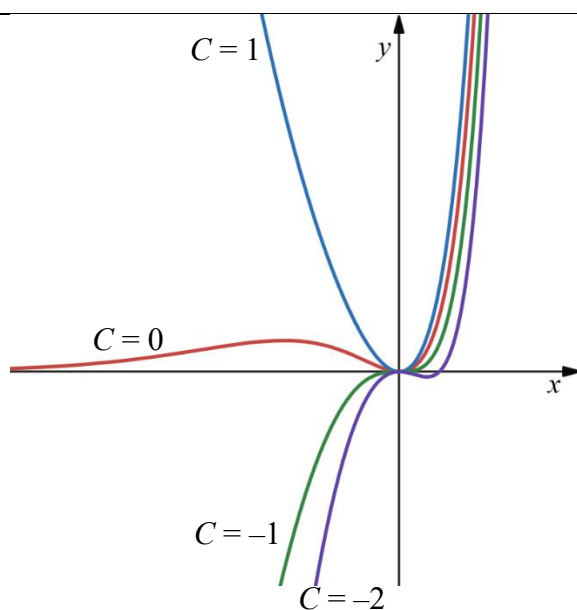
- (c) Solve the differential equations

(i) $x^2 \frac{d^2 z}{dx^2} - 2z = x^3 e^x$, [4]

(ii) $x^2 \frac{d^2 z}{dx^2} - 2z = 1$. [4]

giving each answer for z in the form $z = f(x)$.

	Suggested Solution
(a)	$x \frac{du}{dx} - 2u = x^3 e^x$ $\frac{du}{dx} - \frac{2u}{x} = x^2 e^x$ Integrating Factor: $e^{\int -\frac{2}{x} dx} = e^{-2 \ln x } = x ^{-2} = \frac{1}{x^2}$ $\frac{1}{x^2} \frac{du}{dx} - \frac{2u}{x^3} = e^x$ $\frac{d}{dx} \left(\frac{u}{x^2} \right) = e^x$ $\frac{u}{x^2} = e^x + C$ $u = x^2 e^x + Cx^2$



(b)	$y = xz$ Differentiating w.r.t x : $\frac{dy}{dx} = x \frac{dz}{dx} + z$ $\frac{d^2 y}{dx^2} = x \frac{d^2 z}{dx^2} + \frac{dz}{dx} + \frac{dz}{dx} = x \frac{d^2 z}{dx^2} + 2 \frac{dz}{dx}$ Substitute into given DE $x^2 \frac{d^2 z}{dx^2} - 2z = xg(x)$: $x \left(\frac{d^2 y}{dx^2} - 2 \frac{dz}{dx} \right) - 2z = xg(x)$ $x \frac{d^2 y}{dx^2} - 2 \left(x \frac{dz}{dx} + z \right) = xg(x)$ $x \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} = xg(x)$
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(c)(i)	$x^2 \frac{d^2 z}{dx^2} - 2z = x^3 e^x$ By part (b), $x \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} = x^3 e^x$ Let $\frac{dy}{dx} = u$ $x \frac{du}{dx} - 2u = x^3 e^x$ From part (a),
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	$u = x^2 e^x + Cx^2$ $\frac{dy}{dx} = x^2 e^x + Cx^2$ $y = \int x^2 e^x + Cx^2 \, dx$ $= x^2 e^x - \int 2xe^x \, dx + \frac{Cx^3}{3}$ $= x^2 e^x - 2xe^x + \int 2e^x \, dx + \frac{Cx^3}{3}$ $= x^2 e^x - 2xe^x + 2e^x + \frac{Cx^3}{3} + D$ $= (x^2 - 2x + 2)e^x + \frac{Cx^3}{3} + D$ $xz = (x^2 - 2x + 2)e^x + \frac{Cx^3}{3} + D$ $z = \left(x - 2 + \frac{2}{x}\right)e^x + \frac{Cx^2}{3} + \frac{D}{x}$
(c)(ii)	$x^2 \frac{d^2 z}{dx^2} - 2z = 1$ By part (b), $x \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} = 1$ Let $\frac{dy}{dx} = u$ $x \frac{du}{dx} - 2u = 1$ $\frac{du}{dx} - \frac{2u}{x} = \frac{1}{x}$ $\frac{du}{dx} = \frac{2u+1}{x}$ $\int \frac{1}{2u+1} \, dx = \int \frac{1}{x} \, dx$ $\frac{1}{2} \ln 2u+1 = \ln x + C$ $\ln 2u+1 = 2 \ln x + 2C$ $\ln \left \frac{2u+1}{x^2} \right = 2C$ $\left \frac{2u+1}{x^2} \right = e^{2C}$ $\frac{2u+1}{x^2} = \pm e^{2C}$ $2u+1 = Ax^2, \text{ where } A = \pm e^{2C}$

$$2 \frac{dy}{dx} + 1 = Ax^2$$

$$\frac{dy}{dx} = \frac{Ax^2 - 1}{2}$$

$$y = \frac{Ax^3}{6} - \frac{x}{2} + B$$

$$xz = \frac{Ax^3}{6} - \frac{x}{2} + B$$

$$z = \frac{Ax^3}{6} - \frac{1}{2} + \frac{B}{x}$$

Section B: Probability and Statistics [50 marks]

- 6 A publisher is interested in testing whether the number of printing errors per 100 pages follows a Poisson distribution. The observed frequencies from examining 400 randomly selected sets of 100 pages are as follows:

Number of errors	0	1	2	3	4	5	6	7	8
Frequency	80	100	86	67	33	25	6	2	1

- (a) Test at the 5% level of significance whether the Poisson distribution provides a good fit for the number of printing errors per 100 pages. [5]
- (b) Referring to the two largest contributions to the test statistic, discuss what the test indicates about the printing errors. [1]

Suggested Solution

(a) $\lambda = \frac{788}{400} = 1.97$

H_0 : The number of printing errors can be modelled by a Poisson distribution.

H_1 : The number of printing errors cannot be modelled by a Poisson distribution.

Number of errors	0	1	2	3	4	5	6	7	8
Frequency	80	100	86	67	33	25	6	2	1
Expected	55.8	109.9	108.3	71.1	35	13.8	4.5	1.3	0.3

Since the expected value for number of errors between 6 and 8 inclusive are less than 5, combine the three cells

Number of errors	0	1	2	3	4	5	$x \geq 6$
Frequency	80	100	86	67	33	25	9
Expected	55.8	109.9	108.3	71.1	35	13.8	6.1

Degree of freedom = $7 - 1 - 1 = 5$

Test statistic $\chi^2 = \sum \frac{(O - E)^2}{E} \sim \chi^2_5$

$\chi^2 = 26.8$ $p\text{-value} = 6.24 \times 10^{-5}$

Since the p -value is less than 0.05, reject H_0 at 5% level of significance. There is sufficient evidence to conclude that the number of errors per 100 pages is not consistent with a Poisson distribution.

(b)

Number of errors	0	1	2	3	4	5	$x \geq 6$
Frequency	80	100	86	67	33	25	9
Expected	55.8	109.9	108.3	71.1	35	13.8	6.1
Contributions	10.5	0.892	4.59	0.236	0.114	9.10	1.38

Poisson distribution underestimates the number of pages without any error and the number of pages with 5 errors.

The two largest contributions to the test statistics are 10.5 and 9.10 which come from 0 error and 5 errors. The

- 7 The probability density function (p.d.f) of a normal distribution is given by

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad x \in \mathbb{R}$$

where μ and σ^2 are the mean and variance of the normal distribution respectively.

- (a) Using the given p.d.f, show that $\int_0^\infty e^{-2x^2} dx = \sqrt{\frac{\pi}{8}}$. [2]

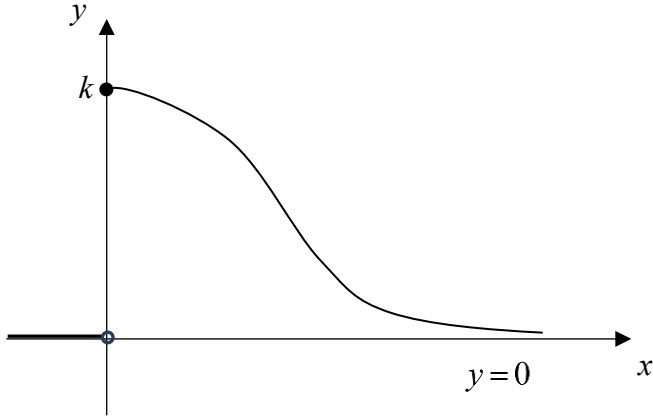
A continuous random variable X has its probability distribution given by

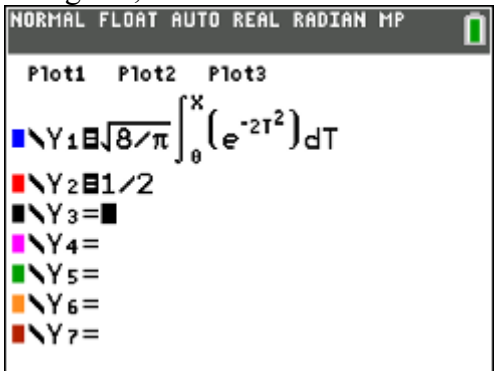
$$g(x) = \begin{cases} ke^{-2x^2}, & x \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

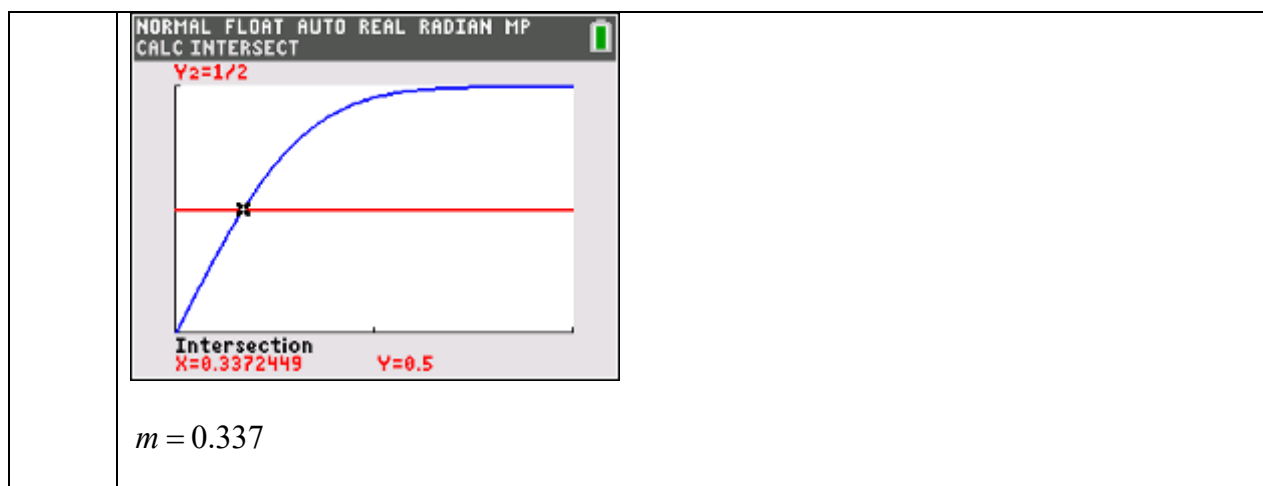
where k is a constant.

- (b) Sketch the graph of g . [1]
- (c) Using the result found in part (a), find the value of k . [1]
- (d) With the value of k found in part (c), find $\text{Var}(X)$, leaving your answer in exact form. [3]
- (e) Find the median value of X . [2]

	Suggested Solution
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(a)	Note that for the normal distribution with mean 0 and variance $\frac{1}{4}$, its probability function is $\frac{1}{\sqrt{2\pi(\frac{1}{4})}} e^{-\frac{x^2}{2(\frac{1}{4})}} = \sqrt{\frac{2}{\pi}} e^{-2x^2}$, and we have $\int_{-\infty}^{\infty} \sqrt{\frac{2}{\pi}} e^{-2x^2} dx = 1$ Since the normal distribution is symmetrical about 0, $\sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-2x^2} dx = \frac{1}{2}$ $\int_0^{\infty} e^{-2x^2} dx = \sqrt{\frac{\pi}{8}} \quad (\text{Shown})$
(b)	
(c)	$\int_{-\infty}^{\infty} g(x) dx = 1$ $0 + \int_0^{\infty} k e^{-2x^2} dx = 1$ $k \int_0^{\infty} e^{-2x^2} dx = 1$ From (a), $k \left(\sqrt{\frac{\pi}{8}} \right) = 1$ Therefore $k = \sqrt{\frac{8}{\pi}}$

(d)	$E(X) = \sqrt{\frac{8}{\pi}} \int_0^{\infty} x e^{-2x^2} dx$ $= \sqrt{\frac{8}{\pi}} \left[\frac{e^{-2x^2}}{-4} \right]_0^{\infty}$ $= \sqrt{\frac{8}{\pi}} \left(0 + \frac{1}{4} \right)$ $= \sqrt{\frac{1}{2\pi}}$ $E(X^2) = \sqrt{\frac{8}{\pi}} \int_0^{\infty} x^2 e^{-2x^2} dx$ $= \sqrt{\frac{8}{\pi}} \left(\left[x \frac{e^{-2x^2}}{-4} \right]_0^{\infty} + \frac{1}{4} \int_0^{\infty} e^{-2x^2} dx \right)$ $= \sqrt{\frac{8}{\pi}} \left([0 - 0] + \frac{1}{4} \sqrt{\frac{\pi}{8}} \right)$ $= \frac{1}{4}$ $\text{Var}(X) = E(X^2) - [E(X)]^2$ $= \frac{1}{4} - \left(\sqrt{\frac{1}{2\pi}} \right)^2$ $= \frac{1}{4} - \frac{1}{2\pi}$
(e)	Let m be the median of X. $\sqrt{\frac{8}{\pi}} \int_0^m e^{-2x^2} dx = \frac{1}{2}$ Using GC, 



- 8 A biased blue die is rolled. The probability that “1” is obtained is 0.4. Let X be the number of attempts to obtain the first “1” on the blue die.

(a) Find the probability that at least 3 attempts are required to obtain a “1”. [1]

(b) Given that no “1” was obtained in the first 6 attempts, find the probability that the first “1” is obtained on the 10th attempt. [2]

Another biased red die with the same probability of obtaining “1” is rolled independently.

Let Y be the number of attempts to obtain the first “1” on the red die respectively. Let W be the minimum of X and Y .

(c) Find $P(W = w)$, in terms of w . Hence or otherwise, find $P(W = 2)$. [4]

(d) Let $T = X + Y$. Find $P(T = 4)$. [2]

(e) Using your result from parts (c) and (d), show that T and W are not independent. [2]

Suggested Solution	
(a)	Let X be the number of attempts needed for the first “1” to be obtained. $P(X \geq 3) = P(X > 2) = (0.6)^2 = 0.36$ Alternatively, Let X be the number of trials needed for the first “1” to be obtained. $X \sim \text{Geo}\left(\frac{2}{5}\right)$ $P(X \geq 3) = 1 - P(X \leq 2) = 0.36$
(b)	Let X be the number of attempts needed for the first “1” to be obtained. $X \sim \text{Geo}\left(\frac{2}{5}\right)$ $P(X = 10 X > 6) = P(X = 4) = 0.0864$

(c)	$ \begin{aligned} P(W \geq w) &= P(X \geq w)P(Y \geq w) \\ &= (0.6)^{w-1} (0.6)^{w-1} \\ &= (0.36)^{(w-1)} \\ P(W \geq w+1) &= (0.36)^w \\ P(W = w) &= P(W \geq w) - P(W \geq w+1) \\ &= (0.36)^{(w-1)} - (0.36)^w \\ &= \frac{16}{9} (0.36)^w \end{aligned} $ Alternative method: $ \begin{aligned} P(W \geq w) &= P(X = w \cap Y > w) + P(X > w \cap Y = w) \\ &\quad + P(X = w \cap Y = w) \\ P(X = w \cap Y > w) &= P(X > w \cap Y = w) \\ P(X = w \cap Y > w) &= \left(\frac{3}{5}\right)^{w-1} \left(\frac{2}{5}\right) \left(\frac{3}{5}\right)^w \\ &= \left(\frac{2}{5}\right) \left(\frac{3}{5}\right)^{2w-1} \\ P(X = w \cap Y = w) &= \left(\frac{3}{5}\right)^{w-1} \left(\frac{2}{5}\right) \times \left(\frac{3}{5}\right)^{w-1} \left(\frac{2}{5}\right) \\ &= \left(\frac{2}{5}\right)^2 \left(\frac{3}{5}\right)^{2(w-1)} \\ P(W = w) &= 2 \left(\frac{2}{5}\right) \left(\frac{3}{5}\right)^{2w-1} + \left(\frac{2}{5}\right)^2 \left(\frac{3}{5}\right)^{2(w-1)} \\ &= \left(\frac{2}{5}\right) \left(\frac{3}{5}\right)^{2(w-1)} \left[2 \left(\frac{3}{5}\right) + \left(\frac{2}{5}\right) \right] \\ &= \left(\frac{16}{25}\right) \left(\frac{3}{5}\right)^{2(w-1)} \end{aligned} $ Hence $ \begin{aligned} P(W = 2) &= \left(\frac{16}{25}\right) \left(\frac{3}{5}\right)^{2(2-1)} \\ &= \frac{144}{625} \end{aligned} $

(d)	$P(T = 4) = P(X = 1 \cap Y = 3) + P(X = 2 \cap Y = 2) + P(X = 3 \cap Y = 1)$ $= 2 \left[\left(\frac{2}{5}\right) \times \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right) \right] + \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^2$ $= \frac{108}{625}$
(e)	$P(W = 2 \cap T = 4) = P(X = 2 \cap Y = 2)$ $= \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^2$ $= \frac{36}{625}$ But $P(W = 2) \times P(T = 4) = \frac{144}{625} \times \frac{108}{625} = 0.0398 \neq \frac{36}{625}$ Therefore T and W are not independent.

- 9 A sugar company produces bags of sugar with a target weight of 1 kg per bag. The mass of the packaging is negligible. A quality control team randomly selects 40 bags and records the mass of the sugar in each bag in grams. The data are summarized as follows.

$$\sum x = 39889.4 \quad \sum x^2 = 39779879.2$$

- (a) Construct a 95% confidence interval for the mean mass of the sugar based on the data, leaving your answer correct to two decimal places. [4]
- (b) State, with a reason, what conclusion the control team should reach. [2]
- After receiving feedback from this initial assessment, the company makes changes to the production line.
- (c) The quality control team calculated the 95% confidence interval for the mass of the sugar after the change in production line. The confidence interval is (998.01, 1000.59). By comparing it with the confidence interval found in part (a), state if it can be concluded that there is a change in the mean mass of sugar. [1]
- (d) Subsequently, another 40 bags of sugar are randomly sampled, and their masses are recorded in grams. The sample mean mass is 999.3 grams while the sample variance is 16.799 grams². Using this sample and the data from part (a), test at 5% level of significance whether there is evidence that the mean mass of sugar differs before and after the production line change. [4]

	Suggested Solution
(a)	Let X be the mass of the sugar in grams $\bar{x} = 997.235$, $s^2 = \frac{1}{39} \left[39779879.2 - \frac{(39889.4)^2}{40} \right]$ $= 19.830538$ Since $n = 40$ is large, by Central Limit Theorem $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ approximately. $Z = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \sim N(0,1) \text{ approximately}$ Critical value: 1.95996 Confidence interval for population mass of sugar is $\left(997.235 - 1.95996 \times \sqrt{\frac{19.830538}{40}}, 997.235 + 1.95996 \times \sqrt{\frac{19.830538}{40}} \right) = (995.85, 998.62)$
(b)	Since 1kg does not lie within the confidence interval, there is sufficient evidence at 5% level of significance that the mean mass of the sugar is not 1kg.
(c)	No, it cannot be concluded that there is a change in the mean mass of sugar. When the confidence interval overlap, it is inconclusive if the mean mass is different at 5% level of significance. It is only conclusive when the confidence intervals do not overlap.
(d)	Let μ_1 and μ_2 be the population mean mass of the sugar before and after the change respectively Let Y be the mass of the sugar after change in production line $H_0: \mu_1 - \mu_2 = 0$ $H_1: \mu_1 - \mu_2 \neq 0$ $\bar{y} = 999.3 \quad s_y^2 = \frac{40}{39} (16.799) = 17.23$ $n_1 = n_2 = 40$ $\bar{x} = 997.235, s_x^2 = 19.83$ Since $n_1 = 40$ and $n_2 = 40$ are large, by Central Limit Theorem $\bar{X} - \bar{Y} \sim N\left(\mu_1 - \mu_2, \frac{\sigma_x^2}{n_1} + \frac{\sigma_y^2}{n_2}\right) \text{ approximately.}$ Under H_0, $Z = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{S_x^2}{n_1} + \frac{S_y^2}{n_2}}} \sim N(0,1) \text{ approximately}$ z - value = -2.15, p - value = 0.0319 < 0.05

Therefore reject H_0 . There is sufficient evidence at the 5% level of significance that the population mean mass of the sugar after a change in the production line is different from the population mean mass of the sugar before the change
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- 10** AceDrink and BrightDrink are two popular drinks. A company carried out a survey to determine which drink is preferred by consumers. Ten consumers who took part in a taste test were asked to rate the two drinks on a scale of 1 to 10, 10 being the best. The following scores were recorded.

AceDrink	9	4	7	8	3	6	9	10	10	5
BrightDrink	3	6	4	4	8	5	2	2	1	5

- (a) Give one reason why a paired-sample t -test may be inappropriate in testing whether consumers prefer one drink to the other. [1]
- (b) Use the Wilcoxon signed rank test to determine whether consumers prefer AceDrink to BrightDrink at the 5% significance level. [4]
- (c) Explain why it is generally better to use a Wilcoxon matched pair signed rank test than a sign test. Give an instance where a sign test has to be used instead of a Wilcoxon signed rank test. [2]
- (d) Suppose that the company carried out a similar survey on n participants, and 20 of the participants gave a higher score for AceDrink as compared to BrightDrink. The company used a one-tailed sign test at the 5% level of significance and was able to conclude that consumers prefer AceDrink. Assuming no ties, find the largest possible value of n . [3]
- (e) In a separate survey, 20 participants were asked to indicate which of the two beverages they prefer. 13 of them indicated a preference for AceDrink, and 5 indicated a preference for BrightDrink, and 2 of them have no preference. Use a sign test to determine at the 5% significance level if this shows a significant difference in the preference for either drink. [3]

	Suggested Solution
(a)	The difference in the consumers' preference may not be normally distributed.
(b)	Let D = score for AceDrink – score for BrightDrink. Let m be the actual median of D . $H_0: m = 0$ $H_1: m > 0$ Level of significance: 5%

	<table><tr><td>AceDrink</td><td>9</td><td>4</td><td>7</td><td>8</td><td>3</td><td>6</td><td>9</td><td>10</td><td>10</td><td>5</td></tr><tr><td>BrightDrink</td><td>3</td><td>6</td><td>4</td><td>4</td><td>8</td><td>5</td><td>2</td><td>2</td><td>1</td><td>5</td></tr><tr><td>d</td><td>6</td><td>-2</td><td>3</td><td>4</td><td>-5</td><td>1</td><td>7</td><td>8</td><td>9</td><td>0</td></tr><tr><td>d</td><td>6</td><td>2</td><td>3</td><td>4</td><td>5</td><td>1</td><td>7</td><td>8</td><td>9</td><td>0</td></tr><tr><td>Rank</td><td>6</td><td>2</td><td>3</td><td>4</td><td>5</td><td>1</td><td>7</td><td>8</td><td>9</td><td></td></tr><tr><td>Signed rank</td><td>6</td><td>-2</td><td>3</td><td>4</td><td>-5</td><td>1</td><td>7</td><td>8</td><td>9</td><td></td></tr></table> $S_+ = 38$ and $S_- = 7$ So test statistic $t = 7$ Since $t = 7 < 8$, we reject H_0 at the 5% significance level. Hence there is sufficient evidence to conclude that consumers prefer AceDrink to BrightDrink.	AceDrink	9	4	7	8	3	6	9	10	10	5	BrightDrink	3	6	4	4	8	5	2	2	1	5	d	6	-2	3	4	-5	1	7	8	9	0	$ d $	6	2	3	4	5	1	7	8	9	0	Rank	6	2	3	4	5	1	7	8	9		Signed rank	6	-2	3	4	-5	1	7	8	9	
AceDrink	9	4	7	8	3	6	9	10	10	5																																																									
BrightDrink	3	6	4	4	8	5	2	2	1	5																																																									
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Signed rank	6	-2	3	4	-5	1	7	8	9																																																										
(c)	The Sign Test ignores the magnitude of differences in the data. A sign test has to be used if - the magnitudes of the differences are unknown - there are too many tied ranks in the Wilcoxon signed rank test																																																																		
(d)	Let D = score for AceDrink – score for BrightDrink. Let m be the actual median of D. $H_0: m = 0$ $H_1: m > 0$ Level of significance: 5% Under H_0, $S \sim B(n, 0.5)$, $p\text{-value} = P(S \geq 20)$ Since H_0 is rejected, $p\text{-value} < 0.05$, so $P(S \geq 20) < 0.05$ $1 - P(S \leq 19) < 0.05$ From GC, <table><tr><td>n</td><td>$1 - P(S \leq 19)$</td></tr><tr><td>30</td><td>$0.0494 < 0.05$</td></tr><tr><td>31</td><td>$0.0748 > 0.05$</td></tr></table> So largest n is 30.	n	$1 - P(S \leq 19)$	30	$0.0494 < 0.05$	31	$0.0748 > 0.05$																																																												
n	$1 - P(S \leq 19)$																																																																		
30	$0.0494 < 0.05$																																																																		
31	$0.0748 > 0.05$																																																																		
(e)	Let d be the difference in the preference of the drinks. $H_0: d = 0$ $H_1: d \neq 0$ Level of significance: 5% Sample size = 18 Under H_0, $S \sim B(18, 0.5)$, $p\text{-value} = P(S \leq 5 \text{ or } S \geq 13) = 2 \times P(S \leq 5) = 0.0963$ Since $p\text{-value} = 0.0963 > 0.05$, we do not reject H_0 at the 5% significance level. There is insufficient evidence to conclude that there is a difference in the preference for either drink.																																																																		