

Additional Mathematics Syllabus 4049

2026 Preliminary Exam Paper 4049/01

Level: Sec 4 Express/ 5 Normal Academic

Qn	Working
1(a)	$M = \lg\left(\frac{1200}{0.001}\right)$ $= 6.08 \text{ (to 3 s.f.)}$
1(b)	$4.6 = \lg\left(\frac{A}{0.001}\right)$ $\frac{A}{0.001} = 10^{4.6}$ $A = 10^{4.6} \times 10^{-3}$ $= 39.8 \text{ mm (to 3 s.f.)}$
1(c)	$M = \log_3\left(\frac{A}{9}\right)$ $M = \log_3\left(\frac{1200}{9}\right)$ $M = \frac{\lg\left(\frac{1200}{9}\right)}{\lg 3}$ $M = 4.45 \text{ (to 3 s.f.)}$
2	$y = -3x^2 + 5x - 4 \dots\dots\dots(1)$ $y = x - 2 \dots\dots\dots(2)$ $(1) = (2)$ $-3x^2 + 5x - 4 = x - 2$ $-3x^2 + 4x - 2 = 0$ $\text{Discriminant} = 4^2 - 4(-3)(-2)$ $= -8$ Since discriminant < 0 and the coefficient of x^2 is -3, which is negative (or < 0), the curve lies below the line $y = x - 2$
3(a)	Let $y = e^{2x}(3x - 1)$ $\frac{dy}{dx} = 2e^{2x}(3x - 1) + 3e^{2x}$ $\frac{dy}{dx} = 6xe^{2x} - 2e^{2x} + 3e^{2x}$ $\frac{dy}{dx} = 6xe^{2x} + e^{2x}$
3(b)	$\int (6xe^{2x} + e^{2x}) dx = e^{2x}(3x - 1)$

	$\int 4xe^{2x} dx = \frac{4}{6} \int (6xe^{2x} + e^{2x}) dx - \frac{2}{3} \int e^{2x} dx$ $\int 4xe^{2x} dx = \frac{2}{3} [e^{2x} (3x-1)] - \frac{2}{3} \left(\frac{e^{2x}}{2} \right) + c$ $\int 4xe^{2x} dx = 2xe^{2x} - \frac{2}{3} e^{2x} - \frac{1}{3} e^{2x} + c$ $\int 4xe^{2x} dx = 2xe^{2x} - e^{2x} + c$
4(a)	$\text{LHS} = \frac{1 - \cos 2A}{\sin 2A} + \frac{\sin 2A}{1 + \cos 2A}$ $= \frac{1 - (1 - 2\sin^2 A)}{2\sin A \cos A} + \frac{2\sin A \cos A}{1 + (2\cos^2 A - 1)}$ $= \frac{2\sin A \sin A}{2\sin A \cos A} + \frac{2\sin A \cos A}{2\cos A \cos A}$ $= \tan A + \tan A$ $= 2 \tan A$
4(b)	$\frac{1 - \cos 2A}{\sin 2A} + \frac{\sin 2A}{1 + \cos 2A} = \frac{2}{7} \cot A$ $2 \tan A = \frac{2}{7} \left(\frac{1}{\tan A} \right)$ $\tan^2 A = \frac{1}{7}$ $\tan A = \pm \sqrt{\frac{1}{7}}$ Basic angle, $\alpha = \tan^{-1} \sqrt{\frac{1}{7}}$ $\alpha = 20.70^\circ$ (to 2 d. p.) Therefore A = -20.7° (to 1 d.p.), 20.7° (to 1 d. p.)
5(a)	$y = k^{x+h}$ $\lg y = \lg k^{x+h}$ $\lg y = (x+h) \lg k$ $\lg y = x \lg k + h \lg k$ Gradient of straight line = $\frac{23-3}{10-0}$ = 2 $\lg k = 2$ $k = 10^2$ $k = 100$ $h \lg k = 3$ $h = \frac{3}{2}$

5(b)	$\lg y = 2x + \frac{3}{2} \quad (2)$ $4x = 2x + 3$ $2x = 3$ $x = \frac{3}{2}$
6(a)	$\frac{x^2}{2} + (y - k)^2 = 2 \dots\dots\dots(1)$ $y = 2x \dots\dots\dots(2)$ Substitute (2) into (1) $\frac{x^2}{2} + (2x - k)^2 = 2$ $x^2 + 2(4x^2 - 4xk + k^2) - 4 = 0$ $x^2 + 8x^2 - 8xk + 2k^2 - 4 = 0$ $9x^2 - 8xk + 2k^2 - 4 = 0$ For the curve to meet the line at 1 point or intersect at two distinct points, Discriminant ≥ 0 $(8k)^2 - 4(9)(2k^2 - 4) \geq 0$ $64k^2 - 72k^2 + 144 \geq 0$ $-8k^2 + 144 \geq 0$ $-8k^2 \geq -144$ $k^2 \leq 18$ $\therefore -\sqrt{18} \leq k \leq \sqrt{18} \quad \text{or} \quad -3\sqrt{2} \leq k \leq 3\sqrt{2} \quad \text{O.E.}$
6(b)	$k^2 = 18$ $k = \pm\sqrt{18}$ $k = \pm 3\sqrt{2}$

7(a)	$\left(2 - \frac{ax}{4}\right)^5$ $= \binom{5}{0} 2^5 \left(-\frac{ax}{4}\right)^0 + \binom{5}{1} 2^4 \left(-\frac{ax}{4}\right)^1 + \binom{5}{2} 2^3 \left(-\frac{ax}{4}\right)^2 + \binom{5}{3} 2^2 \left(-\frac{ax}{4}\right)^3 + \dots$ $= 32 - 20ax + 5a^2x^2 - \frac{5}{8}a^3x^3 + \dots$	B4	
7(b)	$(5 - 2x)(32 - 20ax + 5a^2x^2 - \frac{5}{8}a^3x^3 + \dots)$ $= -64x - 100ax + \dots$ No term in x, $-64 - 100a = 0$ $-100a = 64$ $a = -\frac{64}{100}$ $a = -\frac{16}{25}$	M1 M1 A1	FT (7a) but must state “=0”
		7	
8(a)	$PQ^2 = (2x)^2 + 7^2$ $PQ^2 = 4x^2 + 49$ $PQ = \sqrt{4x^2 + 49}$	B1	
8(b)	Let distance PQ be D $D = (4x^2 + 49)^{\frac{1}{2}}$ $\frac{dD}{dx} = \frac{1}{2}(4x^2 + 49)^{-\frac{1}{2}}(8x)$ $= \frac{4x}{\sqrt{4x^2 + 49}}$ Rate of change, $\frac{dD}{dt} = \frac{dD}{dx} \times \frac{dx}{dt}$ $\frac{dD}{dt} = \frac{4x}{\sqrt{4x^2 + 49}} \times 5$ $= \frac{20x}{\sqrt{4x^2 + 49}}$ At $x = 12$, $\frac{dD}{dt} = \frac{20(12)}{\sqrt{4(12)^2 + 49}}$ $= 9.6 \text{ m/s}$	M1 M1 M1 A1	Differentiate composite ftn State correct chain rule for connected rate of change Substitute $x = 12$
8(c)	Area of triangle PQR, A $= \frac{1}{2}(4x^2 + 49)\sin 60^\circ$ $= \frac{\sqrt{3}}{4}(4x^2 + 49) \text{ O.E.}$ $\frac{dA}{dx} = 2\sqrt{3}x$ Rate of change of area,	M1 M1	Convert $\sin 60^\circ = \frac{\sqrt{3}}{2}$

	$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ $= 2\sqrt{3}x \times 5$ $= 10\sqrt{3}x$ $10\sqrt{3}x \geq 250$ $x \geq \frac{250}{10\sqrt{3}}$ x is increasing at a constant rate of 5 m/s, Therefore the shortest time = $\frac{250}{10\sqrt{3}} \div 5$ = 2.89 s (to 3 s.f.)	M1	
		M1	
		A1	
		10	
9(a)	$y = 5x^{\frac{1}{2}} + x - 2$ $\frac{dy}{dx} = -\frac{5}{2}x^{-\frac{3}{2}} + 1$ For stationary point, $\frac{dy}{dx} = 0$ $-\frac{5}{2\sqrt{x^3}} + 1 = 0$ $\frac{5}{2\sqrt{x^3}} = 1$ $2\sqrt{x^3} = 5$ $x^{\frac{3}{2}} = \frac{5}{2}$ $x = \left(\frac{5}{2}\right)^{\frac{2}{3}}$ $x = \sqrt[3]{\frac{25}{4}}$	M1	
		M1	
		A1	

9(b)	$\frac{dy}{dx} = -\frac{5}{2}(x)^{-\frac{3}{2}} + 1$ $\frac{d^2y}{dx^2} = \frac{15}{4}(x)^{-\frac{5}{2}}$ $\frac{d^2y}{dx^2} = \frac{15}{4\sqrt{x^5}}$ For $x > 0$, $\sqrt{x^5} > 0$, $\frac{d^2y}{dx^2} > 0$ Therefore A is a minimum point. Alternative solution Using first derivative test <table border="1" data-bbox="236 857 1007 1205"> <tr> <td>x</td><td>$\left(\sqrt[3]{\frac{25}{4}}\right)^{-}$</td><td>$\sqrt[3]{\frac{25}{4}}$</td><td>$\left(\sqrt[3]{\frac{25}{4}}\right)^{+}$</td></tr> <tr> <td>$\frac{dy}{dx}$</td><td>$< 0$ or negative</td><td>0</td><td>> 0 or positive</td></tr> <tr> <td>Sketch of tangent</td><td></td><td></td><td></td></tr> <tr> <td>Outline of tangent</td><td colspan="3"></td></tr> </table> Therefore A is a minimum point.	x	$\left(\sqrt[3]{\frac{25}{4}}\right)^{-}$	$\sqrt[3]{\frac{25}{4}}$	$\left(\sqrt[3]{\frac{25}{4}}\right)^{+}$	$\frac{dy}{dx}$	< 0 or negative	0	> 0 or positive	Sketch of tangent				Outline of tangent				M1 M1 A1	Accept substituting $x = \sqrt[3]{\frac{25}{4}}$ to find $\frac{d^2y}{dx^2} = 2.76$ and conclude $\frac{d^2y}{dx^2} > 0$ M1 state the three x values M1 state the value of $\frac{dy}{dx}$ to be negative (< 0), 0, positive (> 0) for the corresponding x values A1
x	$\left(\sqrt[3]{\frac{25}{4}}\right)^{-}$	$\sqrt[3]{\frac{25}{4}}$	$\left(\sqrt[3]{\frac{25}{4}}\right)^{+}$																
$\frac{dy}{dx}$	< 0 or negative	0	> 0 or positive																
Sketch of tangent																			
Outline of tangent																			
		6																	
10(a)	$\frac{dy}{dx} = 4 \cos 3x(-\sin 3x)(3) - 4b \cos 4bx$ $= -12 \cos 3x \sin 3x - 4b \cos 4bx$ When $x = 0$, $\frac{dy}{dx} = -4b$ Gradient of normal = $\frac{1}{4b}$ $y = \frac{x}{4b} + c$ Substitute (0, 4) $y = \frac{x}{4b} + 4$	M1 B1 A1 M1 M1 A1	Chain Rule: $4 \cos 3x(-\sin 3x)(3)$ $4b \cos 4bx$ Substitution “$x = 0$” Product of $m_1 m_2 = -1$																

	$= \ln \frac{8}{5} + \frac{7}{4}$	A2	
		10	
12ai	When $y = 0$, $kx = 2\pi \quad \text{or} \quad kx = 3\pi$ $x = \frac{2\pi}{k} \quad \text{or} \quad x = \frac{3\pi}{k}$ Therefore the coordinates of A are $(\frac{2\pi}{k}, 0)$ and the coordinates of B are $(\frac{3\pi}{k}, 0)$.	B1 B1	
12aii	Coordinates of $C = \left(\frac{\frac{2\pi}{k} + \frac{3\pi}{k}}{2}, a \right)$ $= \left(\frac{5\pi}{2k}, a \right)$ $\tan \frac{\pi}{6} = \frac{a}{\frac{\pi}{2k}}$ $\frac{1}{\sqrt{3}} = a \div \left(\frac{\pi}{2k} \right)$ $\frac{1}{\sqrt{3}} = \frac{2ka}{\pi}$ $a = \frac{\pi \times \sqrt{3}}{2\sqrt{3}k \times \sqrt{3}}$ $a = \frac{\sqrt{3}\pi}{6k} \quad (\text{Shown})$		Award 3 marks regardless of the working

12b	$a = \frac{\sqrt{3}\pi}{6k}$ $k = \left[\frac{\sqrt{3}\pi}{6} \right] \div 3^{\frac{3}{2}}$ $k = \left[\frac{\pi}{18} \right]$ $\text{Period} = \frac{2\pi}{k}$ $= \frac{2\pi}{\left[\frac{\pi}{18} \right]}$ $= 36$	M1	
		M1	
		A1	
		8	

13a	Gradient of $AP = 2$ Midpoint of $AP = (0, 6)$ Equation of perpendicular bisector of AP: $y - 6 = -\frac{1}{2}(x - 0)$ $y = -\frac{1}{2}x + 6$	M1 A1	Use of $m_1 m_2 = -1$, gradient of $AP = -\frac{1}{2}$
13b	$y = -\frac{1}{2}x + 6 \dots\dots(1)$ Gradient of line $AB = \frac{3}{4}$ Equation of AB : $y - 2 = \frac{3}{4}(x + 2)$ $y - 2 = \frac{3}{4}x + \frac{3}{2}$ $y = \frac{3}{4}x + \frac{7}{2} \dots\dots (2)$ (1) = (2) $-\frac{1}{2}x + 6 = \frac{3}{4}x + \frac{7}{2}$ $x = 2$ Substitute $x = 2$ into (1), $y = 5$ Centre of circle = (2, 5) Radius of circle = $\sqrt{(2 - (-2))^2 + (5 - 2)^2}$ $= 5$ Equation of circle is $(x - 2)^2 + (y - 5)^2 = 25$	M1 M1 A1 M1 A1	Using gradient = $\frac{3}{4}$ or the point A to find equation of line AB Equate both equations of line AP and AB Find centre of circle Apply distance formula
13c	Coordinates of $B = (6, 8)$ Substitute (6, 8) into $3y + 4x = k$ $k = 3(8) + 4(6)$ $k = 48$	M1 A1	
		9	