RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Additional Practice Questions for Chapter 7A: Sequences and Series

1 Evaluate the following sum.

(a) $\sum_{r=-n}^{n-1}\left(3r+5\right)$ **(b)** $\sum_{r=n+1}^{2n}(r+n)(r-1)$ **(c)** $\sum_{r=-1}^{11}\left(r2^r\right)$

[You may use the result $\sum_{r=1}^{n} r=\frac{n}{2}(n+1)$ and $\sum_{r=1}^{n} r^2=\frac{1}{6}n(n+1)(2n+1)$]

Solution

(a)	$\sum_{r=-n}^{n-1}\left(3r+5\right)=-3n+5(n-1+n+1)=7n$
(b)	$\sum_{r=n+1}^{2n}(r+n)(r-1)=\sum_{r=n+1}^{2n}(r^2-r+nr-n)$ $=\sum_{r=1}^{2n}r^2-\sum_{r=1}^{n}r^2+(n-1)\sum_{r=n+1}^{2n}r-\sum_{r=n+1}^{2n}n$ $=\frac{1}{6}(2n)(2n+1)(4n+1)-\frac{1}{6}n(n+1)(2n+1)+(n-1)\frac{n}{2}(n+1+2n)-n(n)$ $=\frac{n}{6}[2(2n+1)(4n+1)-(n+1)(2n+1)+3(n-1)(3n+1)-6n]$ $=\frac{n}{6}[2(8n^2+6n+1)-(2n^2+3n+1)+3(3n^2-2n-1)-6n]$ $=\frac{n}{6}(23n^2-3n-2)$.
(c)	From the graphic calculator, $\sum_{r=-1}^{11}\left(r2^r\right)=40961.5$.

2 VJC Prelim 9740/2012/01/Q7b

Given that $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$, find, in terms of a, $\sum_{r=37}^{a} (2r)^2$, where a is an integer and $a \geq 37$. **[3]**

Solution

$$\begin{aligned}\sum_{r=37}^{a}(2r)^2 &= 4\left[\sum_{r=1}^{a} r^2 - \sum_{r=1}^{36} r^2\right]\\ &= 4\left[\frac{a}{6}(a+1)(2a+1) - \frac{36}{6}(36+1)(72+1)\right]\\ &= \frac{2}{3}a(a+1)(2a+1) - 64824\end{aligned}$$

3 DHS JC1 CT 9740/2014/Q3

Given that $\sum_{r=1}^{n} r = \frac{n}{2}(n+1)$ and $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$, show that $\sum_{r=1}^{n} 3r(r-1) = n^3 - n$.**[3]**

Hence find $\sum_{r=n+1}^{2n} \left[n(r+1) - 3r(r-1)\right]$, simplifying your answer. **[3]**

Solution

$$\begin{aligned}\sum_{r=1}^{n} 3r(r-1) = 3\sum_{r=1}^{n} r^2 - 3\sum_{r=1}^{n} r &= \frac{3}{6}n(n+1)(2n+1) - \frac{3n}{2}(n+1)\\ &= \frac{1}{2}n(n+1)(2n+1-3)\\ &= n(n+1)(n-1) = n^3 - n \text{ (shown)}\end{aligned}$$

$$\begin{aligned}\sum_{r=n+1}^{2n}\left[n(r+1) - 3r(r-1)\right] &= \sum_{r=n+1}^{2n} n(r+1) - \sum_{r=n+1}^{2n} 3r(r-1)\\ &= \frac{n^2}{2}(n+2+2n+1) - \left[\sum_{r=1}^{2n} 3r(r-1) - \sum_{r=1}^{n} 3r(r-1)\right]\\ &= \frac{n^2}{2}(3n+3) - \left[(2n)^3 - 2n - (n^3 - n)\right]\\ &= \frac{n^2}{2}(3n+3) - (7n^3 - n)\\ &= \frac{n}{2}(3n^2 + 3n - 14n^2 + 2)\\ &= \frac{n}{2}(-11n^2 + 3n + 2)\end{aligned}$$

4 CJC JC1CT 9740//2013/Q3

Using $\sum_{r=1}^{n} r = \frac{n}{2}(n+1)$, find $\sum_{r=50}^{100} \ln\left[2\left(3^r\right)\right]$, leaving your answer in the form $A\ln 2 + B\ln 3$, where A and B are constants to be found. **[5]**

Solution

$$\sum_{r=50}^{100} \ln\left[2\left(3^r\right)\right] = \sum_{r=50}^{100}(\ln 2 + r\ln 3) = \sum_{r=50}^{100}(\ln 2) + \ln 3\left(\sum_{r=1}^{100} r - \sum_{r=1}^{49} r\right)$$

$$= (100-50+1)\ln 2 + \ln 3\left(\sum_{r=1}^{100} r - \sum_{r=1}^{49} r\right) = 51(\ln 2) + \ln 3\left(\frac{100(101)}{2} - \frac{49(50)}{2}\right)$$

$$= 51(\ln 2) + 3825(\ln 3) \qquad \therefore A = 51, B = 3825$$

5 VJC Promo 9758/2024/Q4

(a) The sum, S_n, of the first n terms of a sequence $u_1, u_2, u_3, \ldots$ is given by $S_n = 75 - \frac{3^{n+1}}{5^{n-2}}$.

Show that $u_n = k\left(\frac{3}{5}\right)^{n-1}$, where k is a constant to be determined. Find S_∞. [4]

(b) Another sequence of numbers $v_1, v_2, v_3, \ldots$ is such that $\sum_{i=1}^{n} v_i = \frac{2n}{2n+1}$.

Find the exact value of $\sum_{i=11}^{\infty} v_i$. [3]

Solution

(a)

$$S_n - S_{n-1} = 75 - \frac{3^{n+1}}{5^{n-2}} - \left[75 - \frac{3^n}{5^{n-3}}\right]$$

$$u_n = \frac{3^n}{5^{n-3}} - \frac{3^{n+1}}{5^{n-2}} = \frac{3^{n-1}}{5^{n-1}}\left(\frac{3}{5^{-2}} - \frac{3^2}{5^{-1}}\right) = \left(\frac{3}{5}\right)^{n-1}(75-45) = 30\left(\frac{3}{5}\right)^{n-1}$$

$$S_n = 75 - \frac{3^{n+1}}{5^{n-2}} = 75 - 27\left(\frac{3}{5}\right)^{n-2}$$

As $n \to \infty$, $\left(\frac{3}{5}\right)^{n-2} \to 0$. Hence, $S_\infty = 75$.

(b) $\sum_{i=1}^{n} v_i = \frac{2n}{2n+1} = 1 - \frac{1}{2n+1}$

As $n \to \infty$, $\frac{1}{2n+1} \to 0$. Hence, $\sum_{i=1}^{\infty} v_i = 1$.

$$\sum_{i=11}^{\infty} v_i = \sum_{i=1}^{\infty} v_i - \sum_{i=1}^{10} v_i = 1 - \frac{20}{21} = \frac{1}{21}.$$

6 NYJC Promo 9758/2024/Q4

During a clinical trial, the concentration U, of a specific blood agent is measured at one-hour intervals following the initial administration of the trial drug to a patient.
The following readings were obtained

$$U_3 = 88,\ U_4 = 76 \text{ and } U_5 = 70,$$

where U_t denotes the reading t hours after the drug was first administered.
It is believed that U satisfies the relationship

$$U_{t+1} = p + qU_t,\ t \geq 0,$$

where p and q are constants.

(a) Find the values of p and q. [2]

(b) Determine the initial concentration of the blood agent when the drug was initially administered. [3]

(c) Describe how the concentration behaves over a long period of time. [1]

Solution

(a)

$U_4 = p + qU_3 \Rightarrow 76 = p + 88q ---(1)$

$U_5 = p + qU_4 \Rightarrow 70 = p + 76q ---(2)$

Solving equation (1) and (2):

$p = 32, q = 0.5$

(b)

$U_3 = p + qU_2 \Rightarrow 88 = 32 + (0.5)U_2 \Rightarrow U_2 = 112$

$U_2 = p + qU_1 \Rightarrow 112 = 32 + (0.5)U_1 \Rightarrow U_1 = 160$

$U_1 = p + qU_0 \Rightarrow 160 = 32 + (0.5)U_0 \Rightarrow U_0 = 256$

Alternatively,

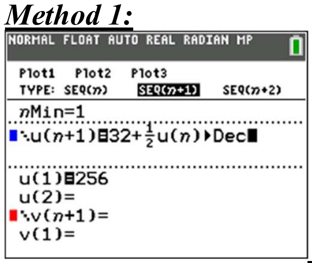
$U_3 = 32 + \frac{1}{2}U_2 = 32 + \frac{1}{2}\left(32 + \frac{1}{2}U_1\right) = 32 + \frac{1}{2}\left(32 + \frac{1}{2}\left(32 + \frac{1}{2}U_0\right)\right)$

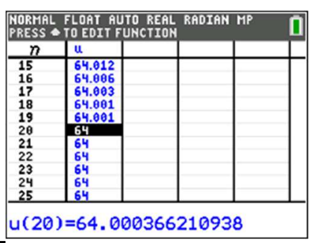
$88 = 32 + \frac{1}{2}\left(32 + \frac{1}{2}\left(32 + \frac{1}{2}U_0\right)\right)$

$\therefore U_0 = 256$

(c)

Method 1:





Method 2:

$As\ t \to \infty, U_t \to L,\ U_{t+1} \to L$ Note : This method assumes that the series is convergent.

$L = 32 + \frac{1}{2}L$

$2L = 64 + L$

$L = 64$

The reading decreases and converges to 64.

THE END