RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Year 5 H2 Math Timed Practice Topical Revision: Questions

Please attempt the questions only after you have completed your revision for the topic(s). You should ***practice under time constraint*** *based on the guideline of not more than 1.8 minute for each mark. In other words, a 10 mark question should not take you more than 18 minutes to solve.*

The solutions will be released in Ivy on 30 May (Friday).

C2: Vectors

1 **ACJC Promo 9758/2022/Q7 modified**

(a) Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non parallel vectors. Point C lies on OA, between O and A, such that $OC : CA = 2 : 1$. Point D lies on OB produced such that $BD = 5OB$.

Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, the position vector of the point E where the lines AB and CD meet. [3]

(b) The position vectors of the points D and E relative to the origin O are $\mathbf{d}$ and $\mathbf{e}$ respectively, where $\mathbf{d}$ and $\mathbf{e}$ are non-zero and non parallel vectors. It is given that the length OE is 2 units and $\mathbf{d}\cdot\mathbf{e} = 3$.

The point F, with position vector $\mathbf{f}$, is the reflection of the point D in the line OE.

(i) Express $\mathbf{f}$ in terms of vectors $\mathbf{d}$ and $\mathbf{e}$. [3]

(ii) Show that the area of triangle ODF can be expressed as $k|\mathbf{d}\times\mathbf{e}|$, where k is a constant to be determined. [2]

[(a) $\frac{5}{8}\mathbf{a}+\frac{3}{8}\mathbf{b}$ (bi) $\frac{3}{2}\mathbf{e}-\mathbf{d}$ (bii) $\frac{3}{4}$]

2 **CJC Promo 9758/2022/Q9**

Relative to the origin O, the position vectors of points A, B and C are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively, where $\mathbf{b}$ is a unit vector. It is given that $\mathbf{b}$ and $\mathbf{b}-\mathbf{a}$ are perpendicular and C lies on AB such that $AC:CB = 3:1$.

Show that $|\mathbf{a}| = \sec\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$. [3]

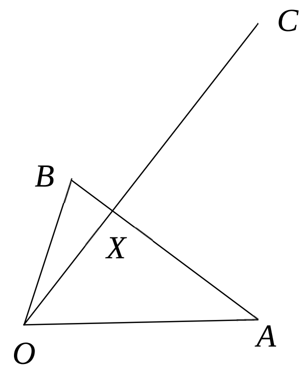
By expressing $\mathbf{c}$ in terms of $\mathbf{a}$ and $\mathbf{b}$,

find the value of $|\mathbf{c}\cdot\mathbf{b}|$ and state the geometrical interpretation of $|\mathbf{c}\cdot\mathbf{b}|$, [4]

(iii) find the value of $\dfrac{|\mathbf{b}\times\mathbf{c}|}{|\mathbf{a}\times\mathbf{b}|}$. [2]

[(ii) 1 (iii) $\frac{1}{4}$]

3 EJC Promo 9758/2022/Q7



Referred to the origin *O*, the points *A* and *B* have position vectors **a** and **b** respectively, where **a** and **b** are non-parallel vectors. The lines *AB* and *OC* intersect at point *X* such that $AX : XB = 3:1$ and $OX : XC = 3:5$ (see diagram).

(a) Express $\overrightarrow{OC}$ in terms of **a** and **b**. [2]

(b) Given that *OC* is perpendicular to *AB* and $OA:OB = 3:2$, show that $\mathbf{a}.\mathbf{b} = \frac{3}{8}|\mathbf{b}|^2$. [3]

(c) Hence, by considering $\left|\overrightarrow{OC}\right|^2$, find *OC : OB.* [3]

[(a) $\frac{2}{3}(\mathbf{a}+3\mathbf{b})$ (c) $\sqrt{6}:1$]

4 NYJC Promo 9758/2022/Q9

The line *l* has equation $1-x=\frac{z-\alpha}{2}, y=1$, where α is a constant.

(i) Given that the point of intersection between the line *l* and the *yz*-plane is $(0,1,5)$, show that $\alpha = 3$. [2]

Referred to the origin, the points *A* and *B* have position vectors $-\mathbf{j}+3\mathbf{k}$ and $12\mathbf{i}+5\mathbf{j}-\mathbf{k}$ respectively.

(ii) Find the cartesian equation of the plane π_1 which contains the point *A* and the line *l*. [3]

(iii) Find the acute angle between the line *AB* and π_1. [3]

The plane π_2 has cartesian equation $x+2z=5$.

(iv) Find the cartesian equations of the planes such that the perpendicular distance from each plane to π_2 is $2\sqrt{5}$. [4]

[(ii) $2x-y+z=4$ (iii) 24.1° (iv) $x+2z=15$ or $x+2z=-5$]

5 **HCI Promo 9758/2022/Q5**

Relative to the origin O, the points M, S and T have coordinates $(3, p, 0.03)$, $(1, 1, 0.02)$ and $(q, 0, 0.01)$ respectively, where p and q are real constants. The line l passes through the point M and is parallel to the vector $(\mathbf{i}+\mathbf{j})$.

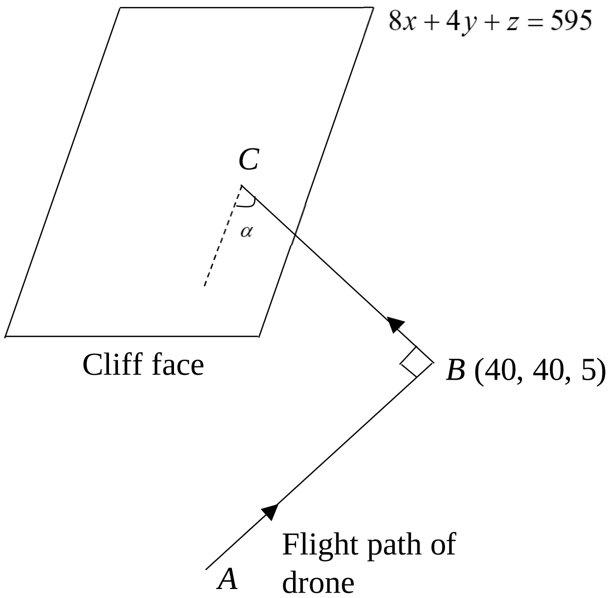
(i) Given that l is perpendicular to the line passing through S and T, show that $q=2$. [2]

(ii) Hence find the area of triangle OST, leaving your answer correct to 6 decimal places. [2]

(iii) It is further given that l and the line passing through S and T do not intersect. Determine the set of possible values of p. [3]

[(ii) 1.000125 units^2 (iii) $\{p \in \mathbb{R} \mid p \neq 5\}$]

6 **EJC Promo 9758/2022/Q12**



A drone flies in a straight line from point A to point B with coordinates (40, 40, 5). At B, it makes a sharp 90° turn and thereafter flies in a straight line in the direction of $2\mathbf{i}+\mathbf{j}+2\mathbf{k}$. Eventually, it crashes into a cliff face which is a part of the plane $8x+4y+z=595$, at point C (see diagram).

(a) Find the coordinates of C. [3]

Determine the acute angle, α, that the path of the drone makes with the cliff face at C. [2]

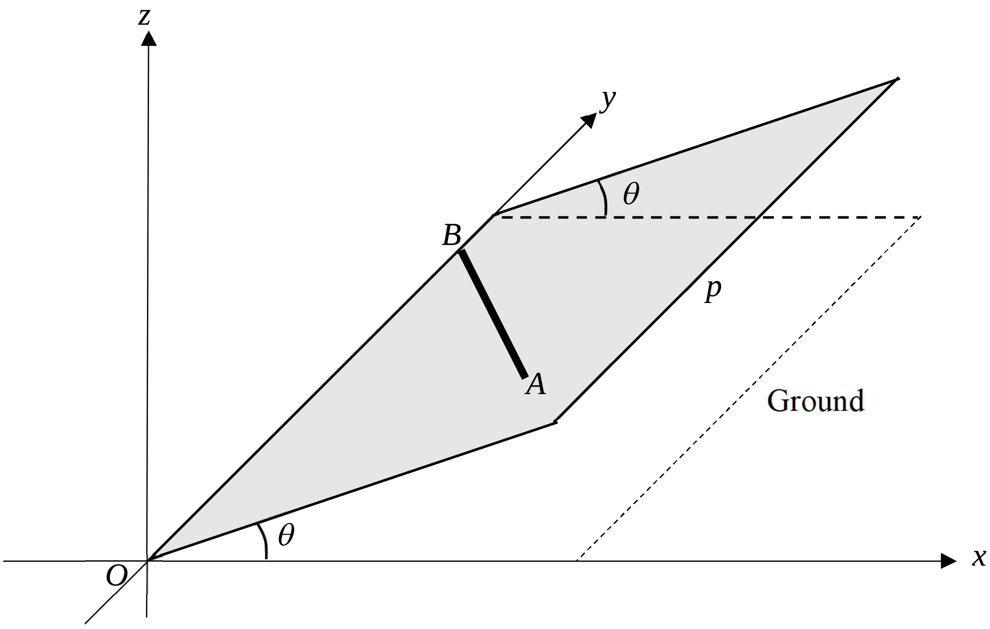
(c) Given that the coordinates of A are (q, 120, 1), find the value of q. [3]

Max is trying to track the drone using his radar. From point D with coordinates (36, 36, 0), his radar emits constant signals in the directions of $\cos\theta\,\mathbf{i}+\sin\theta\,\mathbf{j}+\mathbf{k}$ for **all** values of θ. Assuming that signals travel indefinitely along straight lines, show that none of the signals emitted succeed in hitting the drone as it flies from B to C. [5]

[(a) (50, 45, 15) (b) 54.6° (c) 4]

7 **TJC Promo 9758/2022/Q12**

As part of a structure for a marble race, a plane p is placed at an angle θ to the horizontal ground. The ground is taken to be the x-y plane and the line of intersection between p and the ground is the y-axis (see diagram).



(i) Given that a point A with coordinates (4, 1, 2) lies on p, find a vector equation of p in the scalar product form. Hence find the value of θ, leaving your answer in degrees correct to one decimal place. [4]

(ii) A straight rod is attached to p from A to a point B on the y-axis, as shown in the diagram. Given that AB = 6 units, find the position vector of B. [4]

A marble is released from rest at A and it moves along the rod AB towards B. It is observed that at time t seconds after the marble is released, the height of the marble above the ground is $2-kt^2$ units where k is a positive constant less than 1.

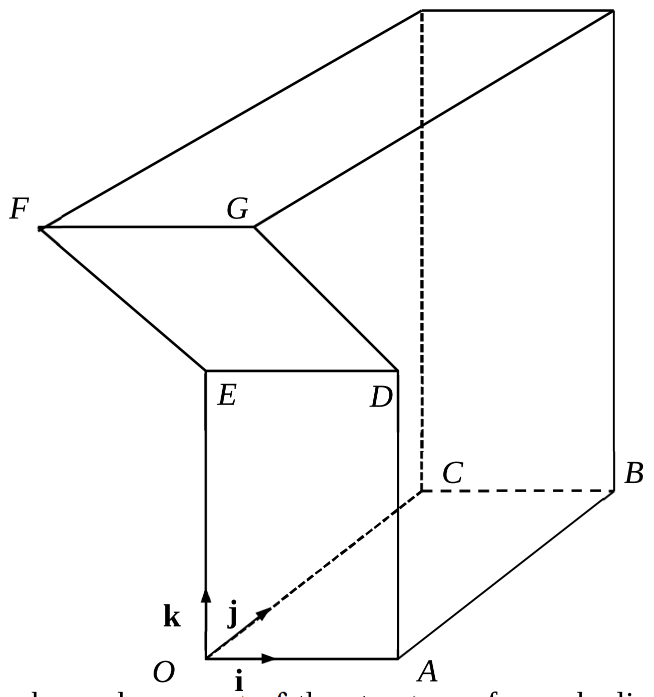
(iii) Find in terms of k,

(a) the time taken for the marble to reach B. [1]

(b) the coordinates of the marble when t = 0.5. [4]

[(i) $\mathbf{r}\cdot\begin{pmatrix}1\\0\\-2\end{pmatrix}=0$, 26.6° (ii) $\begin{pmatrix}0\\5\\0\end{pmatrix}$ (iiia) $\sqrt{\dfrac{2}{k}}$ s (iiib) $\left(4-\frac{k}{2}, 1+\frac{k}{2}, 2-\frac{k}{4}\right)$]

8 **YIJC Promo 9758/2022/Q11**



The diagram above shows part of the structure of a rock-climbing wall, with a horizontal base *OABC*, a vertical rectangular wall *OADE* and an inclined rectangular wall *DEFG*. With respect to the origin *O*, the vectors **i**, **j** and **k** each of length 1 m, are parallel to *OA*, *OC* and *OE* respectively. It is given that *OE* = 10 m.

The plane *OADE* has equation $\mathbf{r}\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}=p$, and the plane *DEFG* has equation

$\mathbf{r}=\begin{pmatrix}0\\0\\10\end{pmatrix}+\lambda\begin{pmatrix}3\\-2\\4\end{pmatrix}+\mu\begin{pmatrix}0\\-1\\q\end{pmatrix}$, where p and q are constants, λ and μ are parameters.

(a) Give a reason why $p=0$. [1]

(b) Write down a vector equation of the line passing through the points *D* and *E*. [1]

(c) Show that $q=2$. [2]

(d) Hence, find the obtuse angle between the walls *OADE* and *DEFG*. [3]

A rock climber is at the point $H(2,-1,12)$.

(e) Show that the climber is on the plane *DEFG*. [2]

The line l passing through the points F and G has equation $\mathbf{r}=\begin{pmatrix}3\\-2\\14\end{pmatrix}+\gamma\begin{pmatrix}1\\0\\0\end{pmatrix}$, where γ is a parameter.

(f) Find the exact shortest distance from the climber to l. [3]

[(b) $\mathbf{r}=10\mathbf{k}+\lambda\mathbf{i},\ \lambda\in\mathbb{R}$ (d) 153.4° (f) $\sqrt{5}$ m]

9 2018/9758/RVHS/Prelim/01/Q4

Relative to the origin O, the points A, B and C are such that $\overrightarrow{OA}=\mathbf{a}$, $\overrightarrow{OB}=\mathbf{b}$ and $\overrightarrow{OC}=\mathbf{c}$ respectively, where $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are vectors which are mutually non-parallel. The plane Π and the line l have the following equations

$$\Pi : \mathbf{r} = \lambda\mathbf{a} + \mu\mathbf{b} \quad \text{where } \lambda, \mu \in \mathbb{R} \quad \text{and}$$
$$l : \mathbf{r} = \mathbf{a} + \beta\mathbf{c} \quad \text{where } \beta \in \mathbb{R}$$

respectively.

(i) Find the point(s) of intersection between Π and l given that the points O, A, B and C are

(a) coplanar,

(b) not coplanar [3]

(ii) State the geometrical meaning of $\frac{1}{2}\left|\mathbf{a}\times\mathbf{b}\right|$. [1]

In the rest of the question, O, A, B and C are not coplanar.

(iii) The vector $\mathbf{p}$ is a unit vector in the direction of $\mathbf{a}\times\mathbf{b}$. State the geometrical meaning of $\left|\mathbf{c}\cdot\mathbf{p}\right|$. [1]

(iv) It is given that $\overrightarrow{OA}=\begin{pmatrix}1\\1\\0\end{pmatrix}$, $\overrightarrow{OB}=\begin{pmatrix}0\\1\\1\end{pmatrix}$ and $\overrightarrow{OC}=\begin{pmatrix}0\\0\\2\end{pmatrix}$. Find the volume of the pyramid $OABC$. [3]

[(i) (a) line l, (b) point A (iv) $\frac{1}{3}$ unit^3]

C3: Equations and Inequalities

1 **DHS Prelim 9758/2022/01/Q1**

(a) Show that $-x^2+6x-14$ is always negative. [1]

(b) Without using a calculator, solve the inequality $\frac{x-5}{x-2} \geq \frac{3}{4-x}$, giving your answer in exact form. [3]

(c) Hence find the solution for the inequality $\frac{\ln x+5}{\ln x+2} \geq \frac{3}{4+\ln x}$. [3]

[(b) $x<2$ or $x>4$ (c) $x>\mathrm{e}^{-2}$ or $0<x<\mathrm{e}^{-4}$]

2 **HCI Promo 9758/2022/Q1**

(i) Solve algebraically the inequality $x^2+x>\frac{4x}{3-x}$. [3]

(ii) Hence solve the inequality $x-x^2<\frac{4x}{3+x}$. [2]

[(i) $x<0$ or $x>3$ (ii) $x>0$ or $x<-3$]

3 **CJC Promo 9758/2022/Q1**

(i) Sketch, on the same diagram, the graphs of $y=|2x+3|$ and $y=-2x^2-8x-3$.
(You are not required to label any axial intercepts and stationary points.) [2]

(ii) Solve exactly the inequality $|2x+3|>-2x^2-8x-3$. [3]

[$x>\frac{-5+\sqrt{13}}{2}$ or $x<-3$]

4 **EJC Promo 9758/2022/Q4**

(a) Sketch, on the same axes, the graphs of $y=\ln(4x)$ and $y=|x^3-2|$ for $x>0$.
Hence, solve the inequality $|x^3-2|>\ln(4x)$. [4]

(b) Using your answer in part **(a)**, find the solution of $\left|\frac{1-2x^3}{x^3}\right|+\ln x>\ln 4$ for $x>0$. [3]

(c) State the solution of the inequality $|x^3-2|<-x$ for $x>0$. [1]

[(a) $0<x<0.897$ or $x>1.57$ (b) $0<x<0.639$ or $x>1.11$ (c) $\varnothing$]

5 **NYJC Promo 9758/2022/Q1**

(i) Solve the inequality $\ln(x+1) \le \dfrac{4}{x}$. [3]

(ii) Hence solve the inequality $\cos x + 1 \le e^{4\sec x}$ for $0 < x < \pi$. [3]

6 **NJC Promo 9758/2022/Q1**

Ben, Caleb and Dylan went to the supermarket in the morning to buy salmon, tuna and swordfish. Ben paid \$246 for 2 kg of salmon, 1 kg of tuna and 3 kg of swordfish. Caleb bought 1 kg of salmon and 1 kg of swordfish. Caleb paid \$18 more than Dylan who bought 1 kg of tuna.

After 12pm, a discount of 15%, 10% and 5% is given for salmon, tuna and swordfish respectively such that the total cost of 1 kg of salmon, 1 kg of tuna and 1 kg of swordfish after discount is \$123.30.

Find the selling price of 1 kg of salmon, tuna and swordfish respectively before discount. [3]

[\$48, \$60 and \$30]

7 **VJC Promo 9758/2022/Q1**

When the polynomial $\mathrm{P}(x) = ax^3 + bx^2 + cx - 1$ is divided by $(x+1)$ and $(x-3)$, the remainders are -12 and 140 respectively. It is given that $(2x-1)$ is a factor of $\mathrm{P}(x)$.

Find the values of *a*, *b* and *c*. [4]

[$a = 6$, $b = -3$, $c = 2$]

8 **TJC Prelim 9758/2022/02/Q1**

A movie theatre supports a charity event by providing discounted movie tickets at the following prices based on age group.

Age group	Discounted ticket price
≤ 12 years	\$6
13 to 49 years	\$9
≥ 50 years (senior citizen)	\$4

The event organiser plans to spend \$700 to sponsor 100 participants of which at least 35 of them are senior citizens. Find the possible number of participants for each age group. [4]

9 RVHS Prelim 9758/2022/01/Q1

In a game show, each contestant earns points by performing consecutive basketball free-throws within a time limit. If the basketball goes through the hoop, the contestant earns x points; if the basketball merely hits the hoop without going through it, the contestant earns y points. Otherwise, the contestant earns no point. The number of basketballs that went through the loop and that which hit the loop without going through it for contestants A, B and C are shown in the following table.

Contestant	Number of basketballs that went through the loop	Number of basketballs that hit the loop without going through it
A	4	9
B	2	10
C	9	8

For the second part of the game, each of the contestants takes turn to choose from 2 envelopes, one with the number 'n' written on it while the other with nothing written on it. The contestant who chooses the envelope with 'n' written on it will have his total points earned increased by n times. Otherwise, the points remain the same. Both contestants A and B chose the envelope with 'n' written on it, while contestant C chose the one with nothing written on it.

The final points earned by contestants A, B and C are 762, 480 and 498 respectively.

Find x, y and n. [4]

Using the value of n found above and suppose contestant C also chooses the envelope with 'n' written on it, how many more points will he have in comparison to the combined points of contestants A and B? [2]

[$x = 50$, $y = 6$ and $n = 2$; 252 more points]

C4: Functions

1 **YIJC Prelim 9758/2022/02/Q3 modified**

Functions f and g are defined by

$$\mathrm{f} : x \mapsto x^2 + 3x - 1,\ x \in \mathbb{R},\ x \le k,$$

$$\mathrm{g} : x \mapsto \sqrt{x+5},\ x \in \mathbb{R},\ x \ge -5.$$

(a) Given that f^{-1} exists, state the largest possible value of k. Using this value of k, find $\mathrm{f}^{-1}(x)$. [3]

For the rest of this question, let $k = -2$.

(b) Determine whether the composite functions fg and gf exist. If the composite function exists, give a definition (including the domain) of the function. [3]

(c) Hence find the exact range of the composite function that exists. [1]

[(a) -1.5, $\mathrm{f}^{-1}(x) = -1.5 - \sqrt{x+3.25}$ (b) $\mathrm{gf}(x) = \sqrt{x^2+3x+4}$, $(-\infty, -2]$ (c) $[\sqrt{2}, \infty)$]]

2 **VJC Prelim 9758/2022/02/Q2**

(a) Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2 - \mathrm{e}^{x+a}, \quad \text{for } x \in \mathbb{R},\ x > -2,$$

$$\mathrm{g} : x \mapsto x^2 + 2x, \quad \text{for } x \in \mathbb{R},\ x < -1,$$

where a is a constant.

(i) Find $\mathrm{g}^{-1}(x)$. [2]

(ii) Explain why the composite function fg exists. [2]

(iii) Find, in terms of a, an expression for $\mathrm{fg}(x)$ and write down the domain of fg. [2]

(iv) Find the range of fg, giving your answer in terms of e and a. [2]

(b) The function h is defined by $\mathrm{h} : x \mapsto \mathrm{e}^{|x+\lambda|}$, $x \in \mathbb{R}$, where λ is a constant. Does h have an inverse? Justify your answer. [2]

[(ai) $\mathrm{g}^{-1}(x) = -1 - \sqrt{1+x}$ (iii) $\mathrm{fg}(x) = 2 - \mathrm{e}^{x^2+2x+a}$, $\mathrm{D_{fg}} = (-\infty, -1)$ (iv) $\left(-\infty, 2 - \mathrm{e}^{a-1}\right)$]

3 SAJC Prelim 9758/2022/01/Q3 modified

The function f is defined by $\mathrm{f}: x \mapsto ax^3 + bx^2 + cx + d$, where $x \in \mathbb{R}$ and a, b, c and d are constants.

The graph of f intersects the y-axis at $y = -3$ and passes through the points $(-1, 0)$ and $(2, 0)$.

(i) Explain why f does not have an inverse. [1]

(ii) Given also that the tangent to the graph of f at $x = 1$ is a horizontal line, find f(x). [3]

(iii) Given that the function f has an inverse if its domain is restricted to $x \geq k$, state the smallest possible value of k. [1]

For the rest of the question, use the domain given and value of *k* found in part (iii).

(iv) Sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the same diagram. [3]

(v) Show that the solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ satisfies the equation $3x^3 - 11x - 6 = 0$. Hence, find the solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [3]

(vi) It is given that $\mathrm{g}(x) = \ln(x+5)$, where $x > -5$. A student attempts to find the composite function gf. The student's solution is shown below:

$\mathrm{g}(x) = \ln(x+5)$
$\mathrm{D_{gf}} = \mathrm{D_f} = [k, \infty)$
$\therefore \mathrm{gf}(x) = \ln\left(ax^3 + bx^2 + cx + d + 5\right),\ x \in \mathbb{R},\ x \geq k.$

Comment on the validity of the student's solution. [1]

[(ii) $\mathrm{f}(x) = \frac{3}{2}x^3 - \frac{9}{2}x - 3$ (iii) k=1 (v) 2.14]

4 **MI PU2 P1 Promo 9758/2022/Q6**

The function f is defined by

$$\mathrm{f}: x \mapsto x^2 - 4x + 5 \quad \text{for} \quad x \in \mathbb{R}, \quad x \leq 3.$$

(i) Find the range of f. [1]

The function g is defined by

$$\mathrm{g}: x \mapsto \ln(2x - a) \quad \text{for} \quad x \in \mathbb{R}, \quad x > \frac{a}{2}.$$

(ii) Given that $a < 2$, explain why gf exists. [2]

Use $a = 1$ for the rest of this question.

(iii) Find $\mathrm{gf}(x)$ and state the domain of gf. [2]

(iv) Find the range of gf. [2]

(v) A function h is said to be self-inverse if $\mathrm{h}(x) = \mathrm{h}^{-1}(x)$ for all x in the domain of h.
A self-inverse function h is such that the composite function $\mathrm{h}^{-1}\mathrm{g}^{-1}$ exists and $\mathrm{h}^{-1}(1) = 2$. Without finding an expression for $\mathrm{g}^{-1}(x)$, find the exact value of the constant k such that $\mathrm{h}^{-1}\mathrm{g}^{-1}(k) = 1$. [3]

[(i) $[1, \infty)$ (iii) $\ln\left(2x^2 - 8x + 9\right); (-\infty, 3]$ (iv) $[0, \infty)$ (v) $\ln 3$]

5 **RVHS Prelim 9758/2022/01/Q6**

The function f is defined by

$$\mathrm{f}: x \mapsto \begin{cases} \mathrm{e}^x - 1, & x \in \mathbb{R}, x \leq 0, \\ 1 - \mathrm{e}^{-x}, & x \in \mathbb{R}, x > 0. \end{cases}$$

(i) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(ii) Explain why f^{-1} exists and define f^{-1} in a similar form. [4]

The function g is defined for $\mathbb{R}$ such that its domain and range are as follows.

Function	Domain	Range
g	$(-4, \infty)$	$(2, \infty)$

Find the exact domain and range of the function fg. [3]

[(ii) $\mathrm{f}^{-1}: x \mapsto \begin{cases} \ln(x+1), x \in \mathbb{R}, -1 < x \leq 0 \\ -\ln(1-x), x \in \mathbb{R}, 0 < x < 1 \end{cases}$; $\mathrm{D}_{\mathrm{fg}} = (-4, \infty)$, $\mathrm{R}_{\mathrm{fg}} = \left(1 - \mathrm{e}^{-2}, 1\right)$]

6 **HCI Prelim 9758/2022/01/Q6**

The function f is defined by

$$\mathrm{f}: x \mapsto \begin{cases} 5+\ln x & \text{for } 0<x<1, \\ -x^2+6x & \text{for } 1 \le x \le a, \end{cases}$$

where a is a constant.

(i) Given that f has an inverse, determine the range of values of a. [2]

(ii) Given that a satisfies the range of values found in part **(i),** sketch the graph of f, indicating clearly the coordinates of the end point(s) in terms of a. [2]

(iii) Find f^{-1} in similar form. [3]

[(i) $1 \le a \le 3$ (iii) $\mathrm{f}^{-1}: x \mapsto \begin{cases} \mathrm{e}^{x-5}, & x<5 \\ 3-\sqrt{9-x}, & 5 \le x \le -a^2+6a \end{cases}$]

C5: Complex Numbers

1 **2018/9758/EJC/Prelim/01/Q5(a)**

Showing your working clearly, find the complex numbers z and w which satisfy the simultaneous equations

$$4\mathrm{i}z - w = 9\mathrm{i} - 13 \ ,$$
$$(4+2\mathrm{i})w^* = z + 3\mathrm{i}. \qquad [4]$$

[(a) $w = 1-\mathrm{i}$, z=2+3i]

2 **2018/9758/NJC/Prelim/02/Q4(b)**

The complex variables u and v satisfy the equations

$$\mathrm{i}u - v = 3 \quad \text{and} \quad u^* + (1-\mathrm{i})v = 7 + 4\mathrm{i}.$$

Find the values of u and v, giving your answers in the form x + iy. [4]

[$u = 1-8\mathrm{i}$, $v = 5+\mathrm{i}$ **]**

3 **2018/9758/RVHS/Prelim/01/Q8 (modified)**

Given that $\mathrm{f}(z) = z^4 + 2z^3 + (\pi^2 + \mathrm{e}^2)z^2 + pz + q$, where $p \geq 0$ and $q \geq 0$, and that $\mathrm{i}\pi$ satisfies the equation $\mathrm{f}(z) = 0$.

(i) Find the exact values of p and q. [2]

(ii) Hence find the remaining roots of $\mathrm{f}(z) = 0$ in the form $\alpha + \beta\mathrm{i}$, where α and β are exact real numbers. [4]

[(i) $q = \mathrm{e}^2\pi^2$ and $p = 2\pi^2$ (ii) $-\mathrm{i}\pi$, $-1+\mathrm{i}\sqrt{\mathrm{e}^2-1}$, $-1+\mathrm{i}\sqrt{\mathrm{e}^2-1}$ **]**

4 **2018/9758/SRJC/Prelim/02/Q4**

Given that the equation $\mathrm{f}\left(z\right) = 2z^3 + az^2 + \left(2a-4\right)z + \left(2a-8\right) = 0$ has no real solution, explain clearly why a is not a real number. [2]

(i) It is known that f has a factor $\left(2z+\mathrm{i}\right)$, find a. Hence find all the roots of $\mathrm{f}\left(z\right) = 0,$ showing your workings clearly. [4]

(ii) Deduce the roots of the equation $2 - \dfrac{a}{w} + \dfrac{\left(2a-4\right)}{w^2} - \dfrac{2a-8}{w^3} = 0$, where a takes the value obtained in **(i)**. [2]

[(i) $4+\mathrm{i}$; $-1+\mathrm{i}$, $-1-\mathrm{i}$ and $-\dfrac{\mathrm{i}}{2}$ (ii) $1-\mathrm{i}$, $1+\mathrm{i}$ and $\dfrac{\mathrm{i}}{2}$ **]**

5 EJC JC2 Prelim 9758/2019/02/Q1

(a) Find the complex numbers z and w that satisfy the equations

$$\frac{z}{w} = 2 + 2\text{i},$$
$$(1-2\text{i})z = 39 - (11\text{i})w.$$ [3]

(b) It is given that $(1+\text{i}c)^3$ is real, where c is also real. By first expressing $(1+\text{i}c)^3$ in Cartesian form, find all possible values of c. [3]

[(a) $z = 10 - 2\text{i}, w = 2 - 3\text{i}$ (b) 0, $\pm\sqrt{3}$]

6 ASRJC JC2 Prelim 9758/2019/01/Q7

Do not use a calculator in answering this question.

The roots of the equation $z^2 + (2 - 2\text{i})z = -3 - 2\text{i}$ are z_1 and z_2.

(i) Find z_1 and z_2 in cartesian form $x + \text{i}y$, showing clearly your working. [5]

(ii) The complex numbers z_1 and z_2 are also roots of the equation

$$z^4 + 4z^3 + 14z^2 + 4z + 13 = 0.$$

Find the other roots of the equation, explaining clearly how the answers are obtained. [2]

(iii) Using your answer in part **(i)**, solve $z^2 + (2 + 2\text{i})z = 3 + 2\text{i}$. [2]

[(i) $-\text{i}$, $-2 + 3\text{i}$ (ii) i, $-2 - 3\text{i}$ (iii) 1, $-3 - 2\text{i}$]

7 HCI JC2 Prelim 9758/2019/02/Q3

The complex numbers p and q are given by $\dfrac{a}{1+\sqrt{3}\text{i}}$ and $-\dfrac{a}{2}\text{i}$ respectively, where a is a positive real constant.

(i) Find the modulus and argument of p. [2]

(ii) Illustrate on an Argand diagram, the points P, Q and R representing the complex numbers p, q and $p+q$ respectively. State the shape of $OPRQ$.
Hence, find the argument of $p+q$ in terms of π and the modulus of $p+q$ in exact trigonometrical form. [6]

~~**(iii)** Find the smallest positive integer n such that $(p+q)^n$ is purely imaginary. [2]~~

[(i) $\dfrac{a}{2}$, $-\dfrac{\pi}{3}$ (ii) $a\cos\dfrac{\pi}{12}, -\dfrac{5\pi}{12}$ ~~(iii) 6~~]

C6: Differentiation Techniques and Applications

1 **DHS Prelim 9758/2022/01/Q5**

A curve C has equation $y^2 = 2\sin x + 2xy$, where $0 \le x \le 2\pi$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{\cos x + y}{y - x}$. [2]

(b) Hence find the coordinates of the stationary points and use the second derivative test to determine its nature. [6]

[(b) $\min(0.874, -0.642), \max(4.49, 0.223)$]

2 **PJC Prelim 9758/2022/01/Q7**

The equation of a curve C is $2x^3 + y^3 - 3xy = k$, where k is a constant.

(i) Find $\dfrac{dy}{dx}$ in terms of x and y. [2]

It is given that C has a tangent which is parallel to the y-axis.

(ii) Show that the y-coordinate of the point of contact of the tangent with C must satisfy $ay^6 + by^3 - k = 0$, where the constants a and b are to be determined. [3]

(iii) Hence, find the values of k when the line $x = 1$ is a tangent to the curve C. [3]

[(i) $\dfrac{dy}{dx} = \dfrac{y - 2x^2}{y^2 - x}$ (ii) $2y^6 - 2y^3 - k = 0$ (iii) $k = 0$ and 4]

3 **HCI Prelim 9758/2022/01/Q9**

A curve C is given by the equation $e^{x^2} y = \ln x^2$, where $x \ne 0$.

(i) Show that $2x\ln x^2 + e^{x^2}\dfrac{dy}{dx} = \dfrac{2}{x}$. [2]

(ii) Find the equation of the tangent to C at the point P where $x = 1$. Given that the tangent at P meets C again at the point Q, find the coordinates of Q, leaving your answer correct to 5 significant figures. [4]

(iii) Determine the acute angle, in degrees, between the tangents to C at the points P and Q. [3]

[(ii) $y = \dfrac{2}{e}(x - 1)$; $Q(-0.49516, -1.1001)$ (iii) $66.9°$]

4 **EJC Promo 9758/2022/Q3**

A curve C_1 has parametric equations

$$x = at(t+2), \quad y = a(t^2+1)$$

where a is a positive constant.

(a) Find, in terms of a, the equation of the normal to the curve at the point where $t = \frac{1}{2}$. [3]

Another curve C_2 has equation $e^{3x} + e^{y} = 4$.

(b) Given that the normal in part **(a)** is also tangential to C_2 at a point, find the exact value of a. [4]

[(a) $y = -3x + 5a$ (b) $\frac{2\ln 2}{5}$]

5 **JPJC Promo 9758/2022/Q11 modified**

A curve C has parametric equations

$x = 3t^2$, $y = 6t^3$, where t is a real parameter.

(i) Find the exact coordinates of the point P on C where the tangent is parallel to the line $y = 4 - 2x$. [3]

(ii) Show that the equation of the tangent to C at the point $Q\left(\frac{4}{3}, \frac{16}{9}\right)$ is $9y = 18x - 8$. [2]

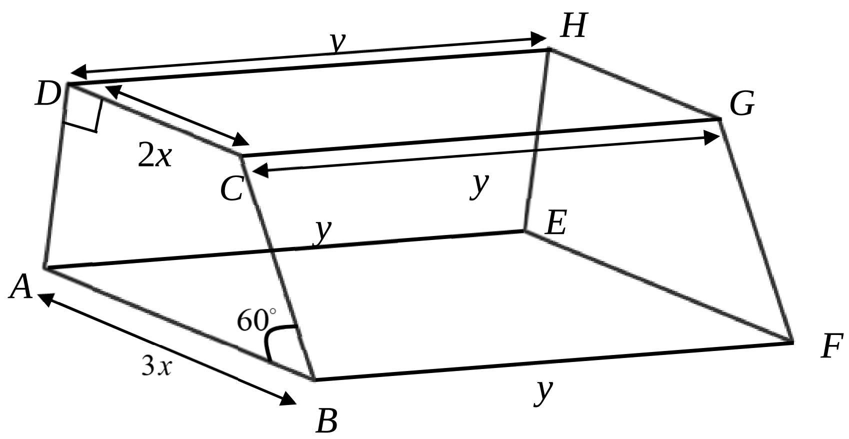
(iii) The tangent at Q cuts the x-axis at the point R. Find the area of triangle PQR. [2]

(iv) The tangent at Q cuts C again at the point S. Find the coordinates of S. [3]

[(i) $\left(\frac{4}{3}, -\frac{16}{9}\right)$ (iii) $\frac{128}{81}$ (iv) $\left(\frac{1}{3}, -\frac{2}{9}\right)$]

6 **ACJC Promo 9758/2022/Q12**

Mr See wants to build a fish tank with a fixed capacity $\left(5\sqrt{3}\right)k$ m^3. The following diagram shows the dimensions of the fish tank he desires, with variables x and y.



The cross-section of the fish tank $ABCD$ is a right trapezoid with AB being $3x$ m long, DC being $2x$ m long, and $\angle ABC = 60^\circ$. The cross-section $ABCD$ is perpendicular to both the rectangular base $ABFE$ and the rectangular opening $DCGH$, with $AE = BF = DH = CG = y$ m.

(i) Show that the total surface area $A\,\text{m}^2$ of the fish tank in contact with water when it is fully filled is $A = 5\sqrt{3}x^2 + \dfrac{2\left(5+\sqrt{3}\right)k}{x}$. [4]

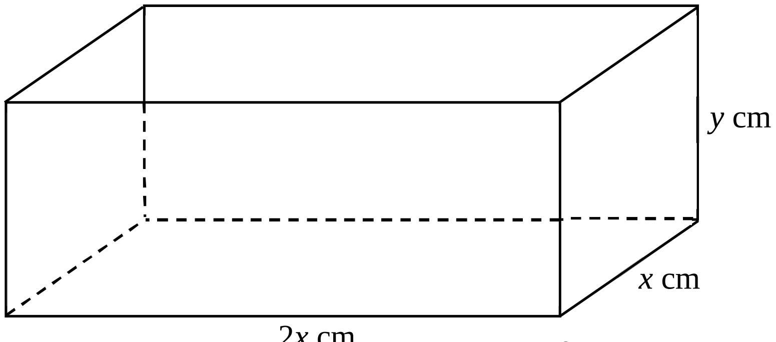
(ii) Mr See would like to minimise the amount of material required to construct the walls and base of the fish tank. Using differentiation, find the value of x in terms of k when A is minimum. [4]

(iii) Mr See decided to build his tank with $k = \dfrac{3}{160}$ and $y = 0.6$. After which, he used a hose to fill it with water at a constant rate of $0.015\,\text{m}^3$ per minute. Find the rate at which the depth of the water is increasing when the fish tank is 50% filled with water. [5]

[(ii) $\left(\dfrac{\left(5+\sqrt{3}\right)k}{5\sqrt{3}}\right)^{\frac{1}{3}}$ (iii) 0.0392 m per minute]

7 **TJC Promo 9758/2022/Q11**

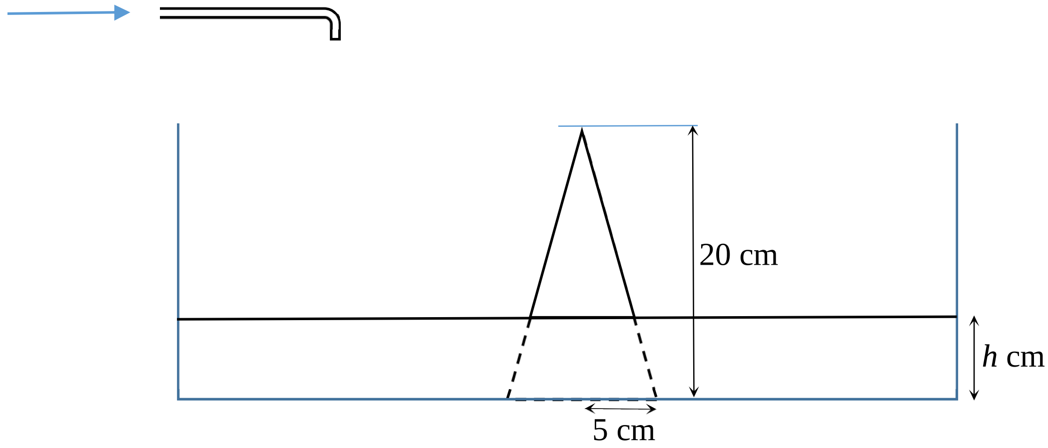
A fish tank manufacturer makes an open fish tank in a shape of a cuboid with internal breadth, length and height of x cm, $2x$ cm and y cm respectively (see diagram).



The manufacturer determines that the internal volume of the tank must be 36 000 cm^3 and the total internal surface area of the tank, A cm^2, should be as small as possible.

(i) Find a formula for A in terms of x. Hence show that for A to be minimum, the breadth, length and height of the tank are 30 cm, 60 cm and 20 cm respectively. [6]

(ii) A fish tank of dimension found in part **(i)** is built and is placed on a horizontal shelf. A solid cone with height 20 cm has its circular base with radius 5cm attached to the internal rectangular base of the fish tank (see diagram).



Water is poured at a constant rate of 1000 cm^3 per second into the tank. At time t seconds after the start, the height of the water level in the tank is h cm and the volume of water is V cm^3.

[The volume of a cone of base radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.]

(a) Show that for $h < 20$, $V = 1800h - \frac{\pi}{48}\left[8000 - (20-h)^3\right]$. [3]

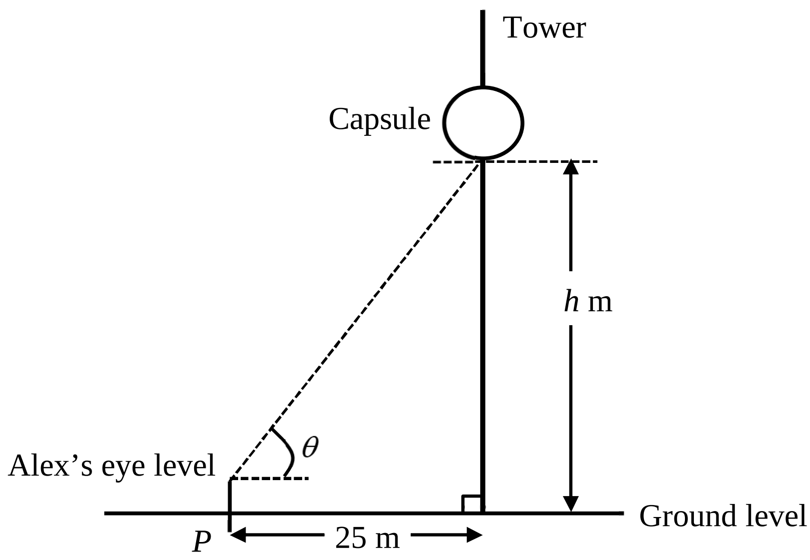
(b) Find the rate of increase of the water level when it is at a height of 10 cm. [3]

[(i) $\frac{108000}{x} + 2x$ (iib) 0.562 cm/s]

8 **MI PU2 P1 Promo 9758/2022/Q11**

A tourist attraction, Sky Lift, has just opened for business. Visitors can enjoy panoramic views of Singapore as they sit in a spherical capsule and travel up and down a tower. The ride starts and ends at the ground level. The duration of the ride is α minutes. At t minutes after the start of the ride, the height, h m, from the horizontal ground to the bottom of the capsule is given by

$$h=-\frac{1}{6}t^3+\frac{1}{2}t^2+8t,\ \ 0\le t\le\alpha\ .$$



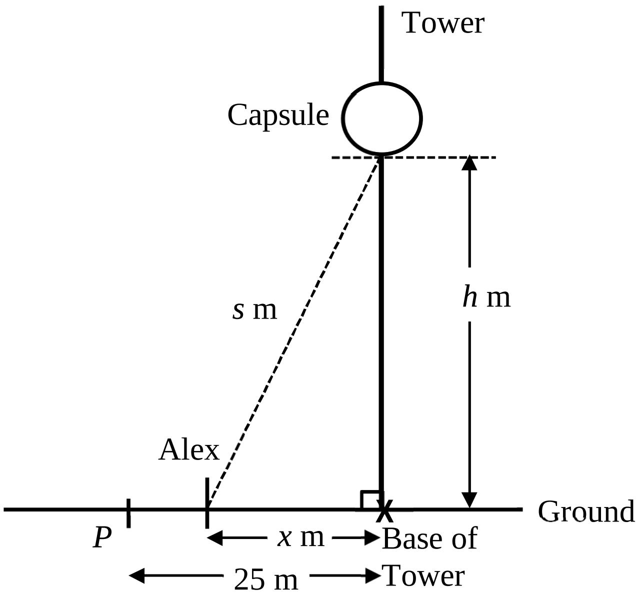
(i) Find the value of α . [1]

(ii) Find the time taken for the capsule to reach its highest point. [1]

Alex stood at the point P which was situated 25 m away from the base of the tower and looked up at capsule as it travelled upwards. His eye level was 1.6 m above the ground. The angle of elevation from his eye level to the bottom of the capsule is denoted by θ radians.

(iii) By expressing $\tan\theta$ in terms of h, find the rate of increase of θ when t =4. [5]

As soon as the capsule started to descend from the highest point, Alex started walking in a straight line from point P towards the base of the tower, at a constant speed of 20 m/min. Let x m be Alex's distance away from the base of the tower and let s m be the distance between Alex and the bottom of the capsule.



(iv) Form an equation involving s, x and h and show that

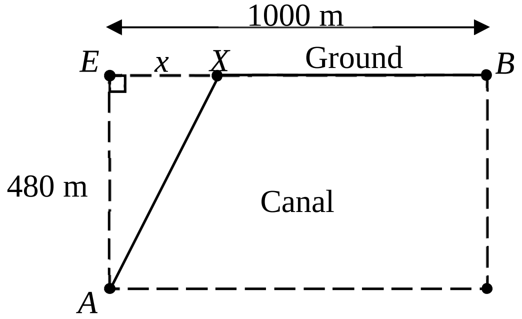
$$s\frac{\mathrm{d}s}{\mathrm{d}t} = x\frac{\mathrm{d}x}{\mathrm{d}t} + h\frac{\mathrm{d}h}{\mathrm{d}t}.$$ [2]

(v) Using the results in parts **(ii)** and **(iv)**, find $\frac{\mathrm{d}s}{\mathrm{d}t}$ at the instant when Alex had walked for 1 minute. [3]

[(i) 8.59 (ii) 5.12 minutes (iii) 0.0717 rad/min (v) -7.90 m / min]

9 **NJC Promo 9758/2022/Q11**

Engineers need to lay pipes to connect two factories A and B that are separated by a canal of uniform width 480 m as shown in the diagram. They plan to lay the pipes under the canal in a line from A to X and then under the ground in a line from X to B. The cost of laying the pipes under the canal is five times the cost of laying the pipes under the ground.



The point E is such that AE is perpendicular to the line XB, where EB has a length of 1000 m. X is between E and B such that EX has a length of x m. The cost per metre of laying the pipes under the ground is 180 dollars.

(i) Show that the total cost C, in dollars, of laying the pipes from A to B is given by $C = 900\sqrt{230400 + x^2} + 180(1000 - x)$. [2]

(ii) In the context of the question, find the range of values of x such that $\frac{dC}{dx} > 0$. [3]

Let the angle at which the pipes are joined, $\angle AXB$, be θ. For engineering reasons, $\frac{2\pi}{3} \le \theta \le \frac{3\pi}{4}$.

(iii) State an equation relating θ and x. Hence find the range of values of x when $\frac{2\pi}{3} \le \theta \le \frac{3\pi}{4}$. [3]

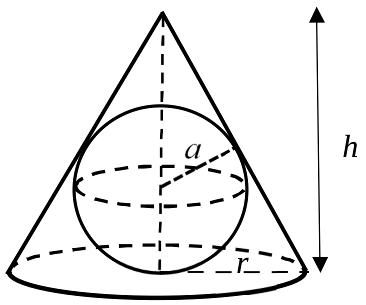
(iv) By considering part **(ii)** and part **(iii)**, show that $\frac{dC}{d\theta} > 0$ for $\frac{2\pi}{3} \le \theta \le \frac{3\pi}{4}$. [3]

(v) Hence, deduce the value of θ which gives the minimum total cost. [1]

[(ii) $98.0 < x < 1000$ (iii) $x = -480\cot\theta$; $\frac{480}{\sqrt{3}} \le x \le 480$ (v) $\frac{2\pi}{3}$]

10 **2019/9758/ASRJC/Promo/Q11**

(a)



A decorative artifact, as shown in the diagram above, is in the shape of a cone with radius *r* and height *h* with a sphere of fixed radius *a* inscribed in it.

(i) Show that the volume of the cone, *V*, is given by $V = \dfrac{\pi a^2 h^2}{3(h-2a)}$. [3]

(ii) Use differentiation to find, in terms of *a*, the minimum value of *V*. Leave your answer in exact form. [5]

(b) In a triangle PQR, $PQ = 3$ cm and $PR = 2$ cm. If angle QPR is increasing at a constant rate of 0.1 radians per second, find the rate of increase of the length QR at the instant when angle QPR is $\dfrac{\pi}{3}$ radians. [4]

[(aii) $\dfrac{8\pi a^3}{3}$ (b) 0.196 cm/s]

<u>The END</u>