Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

25 August 2025 Monday 2 hours 15 min

2025 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES
Write your name, index number and class in the spaces provided above.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided below the questions.
Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

	For Examiner's Use													
Qn	1	2	3	4	5	6	7	8	9	10	11	12	13	*Marks Deducted*
Marks														

TOTAL MARKS
/ 90

Category	Accuracy	Units	Notations	Others
Question No.				

Setter: Mr Tan Lip Sing
Vetter: Ms Sabrina Tan

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\displaystyle \binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Differentiate $\ln\left(\dfrac{3x}{x^2+1}\right)$ with respect to x. [3]

(b) Hence find $\displaystyle\int \frac{2x}{x^2+1}\,\mathrm{d}x$. [2]

2 It is given that $\cos A = \frac{1}{2}$ and $\sin B = -\frac{1}{\sqrt{2}}$ where $0° < A < 90°$ and $180° < B < 270°$.

Find, without using a calculator, the exact value of $\cos(A - B)$, leaving your answer in the form $p\sqrt{2} + q\sqrt{6}$, where p and q are real numbers. [4]

3 Baking powder is poured onto a flat surface at a constant rate of 2π cm^{3}s^{-1}, forming a right circular cone. The radius of the cone is always $\frac{1}{18}$ of its height. Find the rate of change of the radius of the cone after 3 seconds of pouring.

$\left[\text{Volume of cone} = \frac{1}{3}\pi r^2 h\right]$ [5]

4 A and B are the points of intersection of the line $4y = 2x + 1$ and the curve $3y - x = 4xy$.

(a) Find the coordinates of A and of B. [4]

(b) Henry says that the line $2y - 4x = 5$ is perpendicular to the line AB.
Is he correct? Justify your answer with workings. [3]

5 **(a)** Write down and simplify the first three terms in the expansion, in descending powers of x, of $\left(2-\frac{3}{x}\right)^8$. [2]

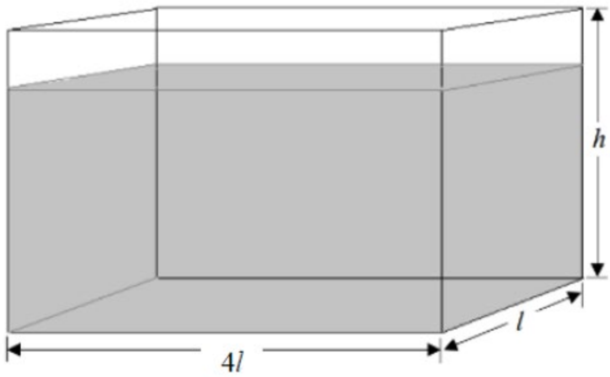
(b) Given that there is no x term in the expansion of $\left(1-2x-kx^2\right)\left(2-\frac{3}{x}\right)^8$, find the constant term in the expansion. [4]

6 **(a)** Express $y = 4x - 4x^2 - 3$ in the form $p(x+q)^2 + r$ where p, q and r are constants. [2]

(b) Hence, explain whether $4x - 4x^2 - 3 = 0$ has any real solutions. [2]

TURN OVER FOR QUESTION 7

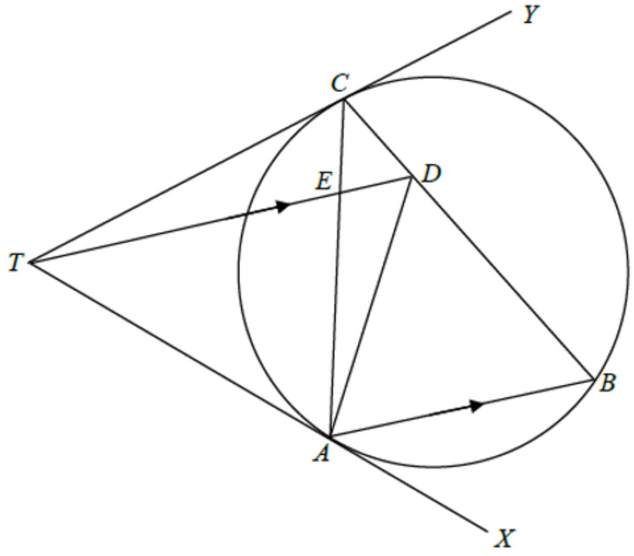
7 Peter constructed an open fish tank with a rectangular base of length $4l$ m, breadth l m, and height h m. He wanted the total outer surface area of the fish tank to be $5\ \text{m}^2$.



(a) Show that the volume of the tank, $V\ \text{m}^3$, is given by $V = \frac{2}{5}\left(5l - 4l^3\right)$. [3]

(b) Determine the area of the rectangular base for the tank to contain the maximum amount of water when filled to the brim. [4]

8 In the diagram below, TAX and TCY are tangents to the circle at A and C respectively. AC meets TD at E and D is on BC such that TD is parallel to AB.



(a) Prove that angle ACB is equal to angle ATD. [2]

(b) Using the result from **part (a)**, explain whether a circle can be drawn passing through the points T, A, D and C. [2]

(c) Prove that $CE \times EA = DE \times TE$. [3]

9 **(a)** A square has an area $\left(17-8\sqrt{2}\right)$ cm^2. The length of each side of the square can be expressed in the form $\left(a+b\sqrt{2}\right)$ cm, where a and b are integers. Show that $2b^4-17b^2+16=0$. [4]

(b) [The area of a sector is $\frac{1}{2}r^2\theta$ and the arc length of a sector is $r\theta$.]

The sector of a circle with radius, r, has an arc length of $\left(\sqrt{15}-\sqrt{3}\right)$ cm and an area of $\left(3\sqrt{3}-\sqrt{15}\right)$ cm^2. Show that $r=\dfrac{6\sqrt{3}-2\sqrt{15}}{\sqrt{15}-\sqrt{3}}$ and hence express r in the form $\left(p+q\sqrt{5}\right)$ cm, where p and q are integers. [5]

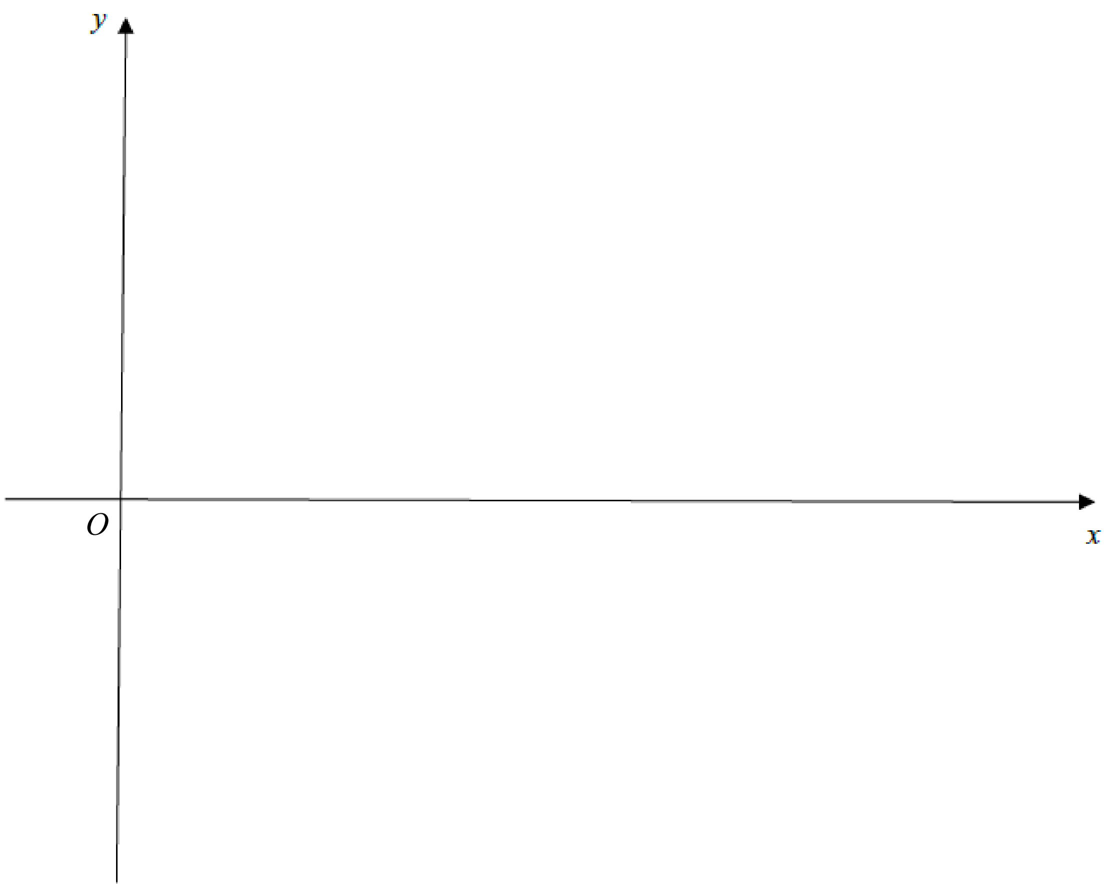
10 It is given that $f(x) = 2\sin\frac{x}{2}$ and $g(x) = 3\cos x + 1$, where $0 \le x \le 2\pi$.

(a) State the period of $f(x)$. [1]

(b) State the smallest value of $f(x)$. [1]

(c) State the largest value of $g(x)$. [1]

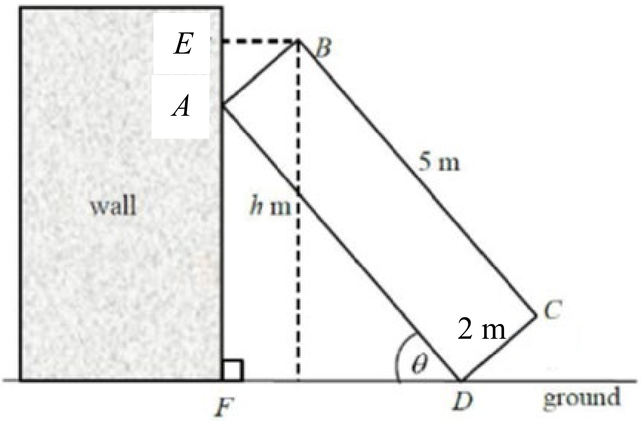
(d) Sketch on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0 \le x \le 2\pi$.
Label your graphs clearly. [4]



(e) The solutions to the equation $f(x) = g(x)$ for $0 \le x \le 2\pi$ are a and b, where $a < b$.
State, in terms of a and b, the range of values of x for which $f(x) > g(x)$. [1]

11 The diagram shows a rectangular wooden plank *ABCD*, 5 m by 2 m, leaning against a vertical wall. *AD* makes an acute angle θ with the horizontal ground.

E is a point on the vertical wall such that *EB* is parallel to the ground and *F* is a point on the ground such that *AF* is perpendicular to *FD*.
The point *B* is *h* m vertically above the ground.



(a) Show that $h = 5\sin\theta + 2\cos\theta$. [2]

(b) Express *h* in the form $R\sin(\theta + \alpha)$, where $R > 0$, and $0° < \alpha < 90°$. [3]

(c) Find the greatest possible value of h and the value of θ at which it occurs. [2]

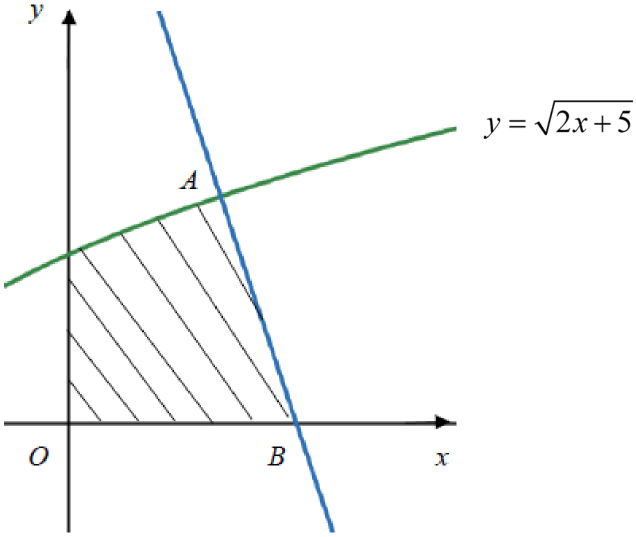
(d) Find the value of θ for which $h = \sqrt{15}$ m. [2]

12 **(a)** By using long division, divide $4x^3+5x^2+x-1$ by $x^2(x+1)$. [1]

(b) Hence, express $\dfrac{4x^3+5x^2+x-1}{x^2(x+1)}$ in partial fractions. [5]

(c) Hence, find $\int \frac{4x^3 + 5x^2 + x - 1}{x^2\left(x+1\right)}\mathrm{d}x$. [3]

13 The diagram shows part of the curve $y = \sqrt{2x+5}$. A is a point on the curve and the x-coordinate of A is 2. The normal to the curve at A meets the x-axis at B.



(a) Find the equation of the normal to the curve at A. [5]

(b) Find the area of the shaded region bounded by the normal AB, the curve, the x-axis and the y-axis. [5]

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