RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Tutorial 1: Basics

Section A (Basic Questions)

1 Find the range of values of k for the line $y = x - 2k$ to intersect the curve $2y^2 - x^2 = 8$ at two distinct points.

[$\mathbb{R}$]

2 Sketch the graph of $y = \ln(x+2)$ for $x > -2$, showing clearly the equations of any asymptotes and the coordinates of any intersections with the axes. By drawing a suitable straight line graph on the same diagram, determine the number of solutions to the equation $e^{2x+1} - x = 2$.

[2]

3 Express $2\sqrt{2}\sin\theta + 2\sqrt{2}\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

[$4\sin\left(\theta + \frac{\pi}{4}\right)$]

4 Given that $\cos A = p$ and $270^\circ \le A \le 360^\circ$, express each of the following in terms of p.

(a) $\sin A$, **(b)** $\sin 2A$, **(c)** $\cos\frac{A}{2}$.

[**(a)** $-\sqrt{1-p^2}$ **(b)** $-2p\sqrt{1-p^2}$ **(c)** $-\sqrt{\frac{p+1}{2}}$]

5 $ABCD$ is a parallelogram with $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Coordinates of points A and C are $(1,1)$ and $(8,8)$ respectively.

(i) Find the coordinates of point D.

(ii) AC and BD intersect at the point E. Find the coordinates of E.

(iii) Find the length of the two sides of the parallelogram AB and BC. Deduce the geometrical relationship between A, B, C and D, and state the relationship between the two diagonals.

[**(i)** (5,4) **(ii)** (4.5,4.5) **(iii)** 5]

6 Solve the equation $|2x+3| = 5$ using 2 different methods, namely:

(i) numerically, **(ii)** graphically.

[$x = -4$ or 1]

7 Given that $|x| = 4$, find the possible values of $|1-3x|$.

[11, 13]

8 Draw the graphs of $y=|x+1|$ and $y=2-\frac{1}{2}x$ on the same diagram. Hence, solve the equation $|x+1|=2-\frac{1}{2}x$.

[$x=\frac{2}{3}$ or -6]

Section B (Discussion Questions)

1 By expressing $-\cos^2 x+\cos x+a$ in the form $-(b-\cos x)^2+c$, where $a,b,c\in\mathbb{R}$, find the range of values of a such that $-\cos^2 x+\cos x+a$ is always negative for all real values of *x*.

$\left[\left(-\infty,-\frac{1}{4}\right)\right]$

2 The line with equation $y=mx$ is a tangent to the curve with equation $(x+8)^2+(y-14)^2=52$.

(a) Show that *m* satisfies the equation $3m^2+56m+36=0$.

A and *B* are points on the curve. The tangent at *A* and the tangent at *B* intersect at the origin.

(b) Find the coordinates of *A* and *B*.

[**(b)** $(-12,8)$ and $(-0.8,14.4)$]

3 **(i)** Determine constants *A*, *B*, *C* and *D* such that

$$\frac{4x^3+6x^2+1}{(1-2x)(1+x)^2}=A+\frac{B}{1-2x}+\frac{C}{1+x}+\frac{D}{(1+x)^2}.$$

(ii) Hence find the exact value of $\int_2^4 \frac{4x^3+6x^2+1}{(1-2x)(1+x)^2}\,\mathrm{d}x$.

[**(i)** $A=-2, B=\frac{4}{3}, C=\frac{2}{3}, D=1$ **(ii)** $\frac{2}{3}\ln\frac{5}{7}-\frac{58}{15}$]

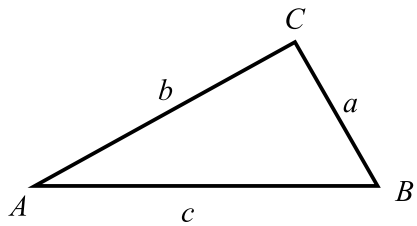
4 On a certain date, 80 cases of smallpox was recorded in a small city. This number increased with time and after *t* days the number of recorded cases was *N*. It is believed that *N* can be modelled by the formula $N=\frac{ae^{kt}}{19+e^{kt}}$, where *a* and *k* are real constants.

(i) Given that the number of cases of smallpox was recorded as 1296 after 4 days, find the value of *k* in the form ln *b*, where *b* is a real number.

(ii) Find an expression for *t* in terms of *N* and hence estimate the number of days it takes for the number of cases of smallpox in the small city to reach 1482, giving your answer to the nearest integer.

[**(i)** k=ln3 **(ii)** $t=\frac{1}{\ln 3}\ln\frac{19N}{1600-N}$; 5]

5 **Do not use a calculator in answering this question.**



In triangle ABC, the ratio of the sides are such that $a : b : c = 2 : 3 : 4$.

(i) Find the ratio of $\sin A : \sin B : \sin C$.

(ii) Calculate the value of $\cos C$ and hence find the value of $\sin C$.

(iii) Given that the area of the triangle is $\sqrt{540}$ cm^2, find the length of the shortest side and deduce that the perpendicular distance of A to BC is $\sqrt{\frac{135}{2}}$ cm.

[**(i)** 2:3:4 **(ii)** $-\frac{1}{4}, \frac{\sqrt{15}}{4}$ **(iii)** $4\sqrt{2}$ cm]

6 Find an expression for $\tan\frac{5\pi}{12}$ in the form $a+b\sqrt{c}$, where a, b and $c \in \mathbb{Z}$.

[$2+\sqrt{3}$]

7 Find all possible exact values of $\dfrac{\cos\left(\frac{\pi}{3}\right)-\cos\left((2n+1)\left(\frac{\pi}{6}\right)\right)}{2\sin\left(\frac{\pi}{4}\right)}$, where n is a positive integer.

[$\frac{\sqrt{2}+\sqrt{6}}{4}, \frac{\sqrt{2}}{4}, \frac{\sqrt{2}-\sqrt{6}}{4}$]

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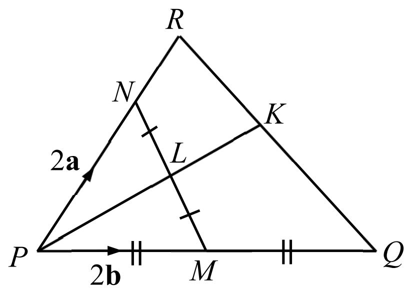

In triangle PQR, the point N on PR is such that $PN = \frac{2}{3}PR$. M is the mid-point of PQ, L is the mid-point of MN and PL produced meets RQ at K. $4\overrightarrow{RK} = 3\overrightarrow{KQ}$, $\overrightarrow{PL} = \frac{7}{12}\overrightarrow{PK}$, $\overrightarrow{PN} = 2\mathbf{a}$ and $\overrightarrow{PM} = 2\mathbf{b}$.

(a) Express, as simply as possible, in terms of **a** and /or **b**

(i) $\overrightarrow{NM}$, **(ii)** $\overrightarrow{NL}$, **(iii)** $\overrightarrow{PK}$, **(iv)** $\overrightarrow{PR}$, **(v)** $\overrightarrow{PQ}$.

(b) Express $\overrightarrow{RQ}$ as simply as possible, in terms of **a** and **b**.

(c) Calculate the value of $\frac{KR}{QR}$.

(d) Show that $\overrightarrow{KR} = \frac{3}{7}(3\mathbf{a} - 4\mathbf{b})$.

(e) Calculate the value of

(i) $\frac{\text{the area of } \Delta PKR}{\text{the area of } \Delta PQR}$,

(ii) $\frac{\text{the area of } \Delta PKN}{\text{the area of } \Delta PQR}$.

[**(a)(i)** $-2\mathbf{a}+2\mathbf{b}$ **(ii)** $-\mathbf{a}+\mathbf{b}$ **(iii)** $\frac{12}{7}(\mathbf{a}+\mathbf{b})$ **(iv)** $3\mathbf{a}$ **(v)** $4\mathbf{b}$

(b) $-3\mathbf{a}+4\mathbf{b}$ **(c)** $\frac{3}{7}$ **(e)(i)** $\frac{3}{7}$ **(ii)** $\frac{2}{7}$]

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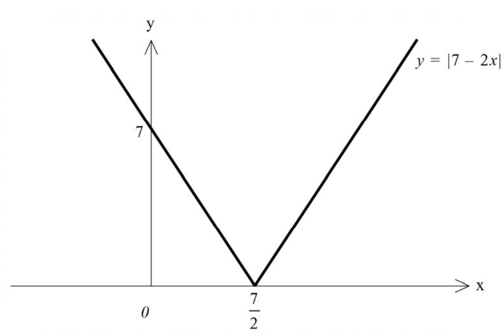

The diagram shows part of the graph of $y = |7 - 2x|$. The line $y = mx - 2$, where m is a constant, cuts $y = |7 - 2x|$ at two distinct points. Explain why $\frac{4}{7} < m < 2$.