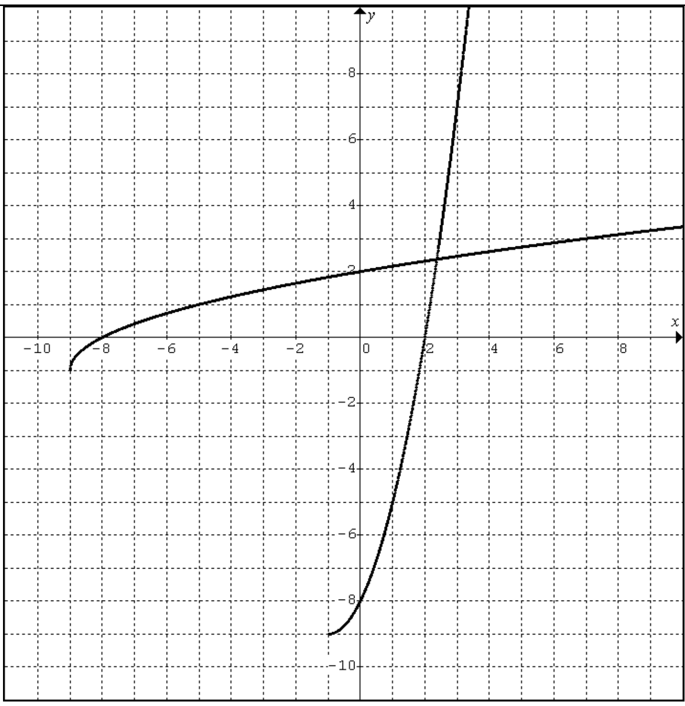
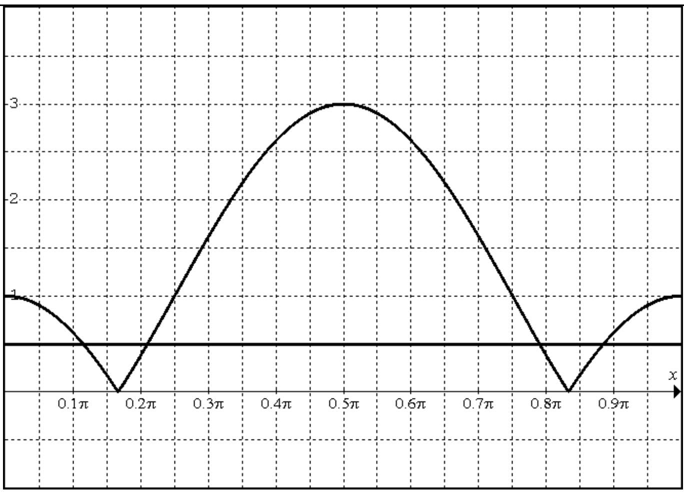
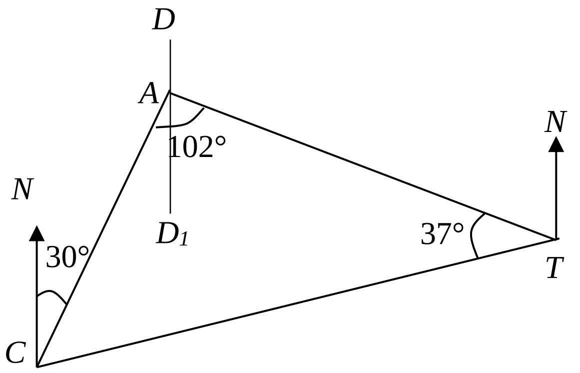
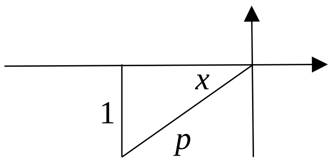
Paper D Sec 3 IP Exam Solutions for students

Qn	Solutions
1(a)	$\frac{1}{1+\sqrt{3}}+\frac{1}{1-\sqrt{3}}=\frac{1-\sqrt{3}+1+\sqrt{3}}{\left(1+\sqrt{3}\right)\left(1-\sqrt{3}\right)}$ $=\frac{2}{1-3}=-1$
1(b)	$\frac{\frac{2}{x^2+3x+9}}{x-3}-\frac{x-3}{(x-3)(x^2+3x+9)}$ $=\frac{2}{(x-3)\ (x^2+3x+9)}-\frac{x-3}{(x-3)(x^2+3x+9)}$ $=\frac{5-x}{(x-3)(x^2+3x+9)}$
1(c)	Sum of roots = $20+\sqrt{17}+20-\sqrt{17}=40$ Product of roots = $\left(20+\sqrt{17}\right)\left(20-\sqrt{17}\right)=400-17=383$ The equation is $x^2-40x+383=0$ Or $(x-20+\sqrt{17})(x-20-\sqrt{17})=0$ $x^2-20x-\sqrt{17}x-20x+400+20\sqrt{17}+\sqrt{17}x-20\sqrt{17}-17=0$ $x^2-40x+383=0$
2(a)	$2(x-y)=x+y-1\ \Rightarrow x=3y-1$ ……….(1) $x+y-1=2x^2-11y^2$ ……….(2) Sub (1) into (2): $(3y-1)+y-1=2(3y-1)^2-11y^2$ $7y^2-16y+4=0\ \Rightarrow y=2\ \ or\ \ y=\frac{2}{7}$ $\Rightarrow x=5\ \ or\ \ x=-\frac{1}{7}\Rightarrow x=5\ ,\ y=2\ \ or\ \ x=-\frac{1}{7}\ ,\ y=\frac{2}{7}$
2 (b)	$y=x-m$ meets the curve $y=mx^2+7x\Rightarrow\Delta\geq 0$ $mx^2+6x+m=0$ $Meet\Rightarrow 6^2-4m^2\geq 0,\ \ \Rightarrow m^2\leq 9$ $\Rightarrow -3\leq m\leq 3$
3(i)	$g(x)=(x+4)(x-2)$ or $=\left(x+1\right)^2-9$ $a=\frac{-4+2}{2}=-1$

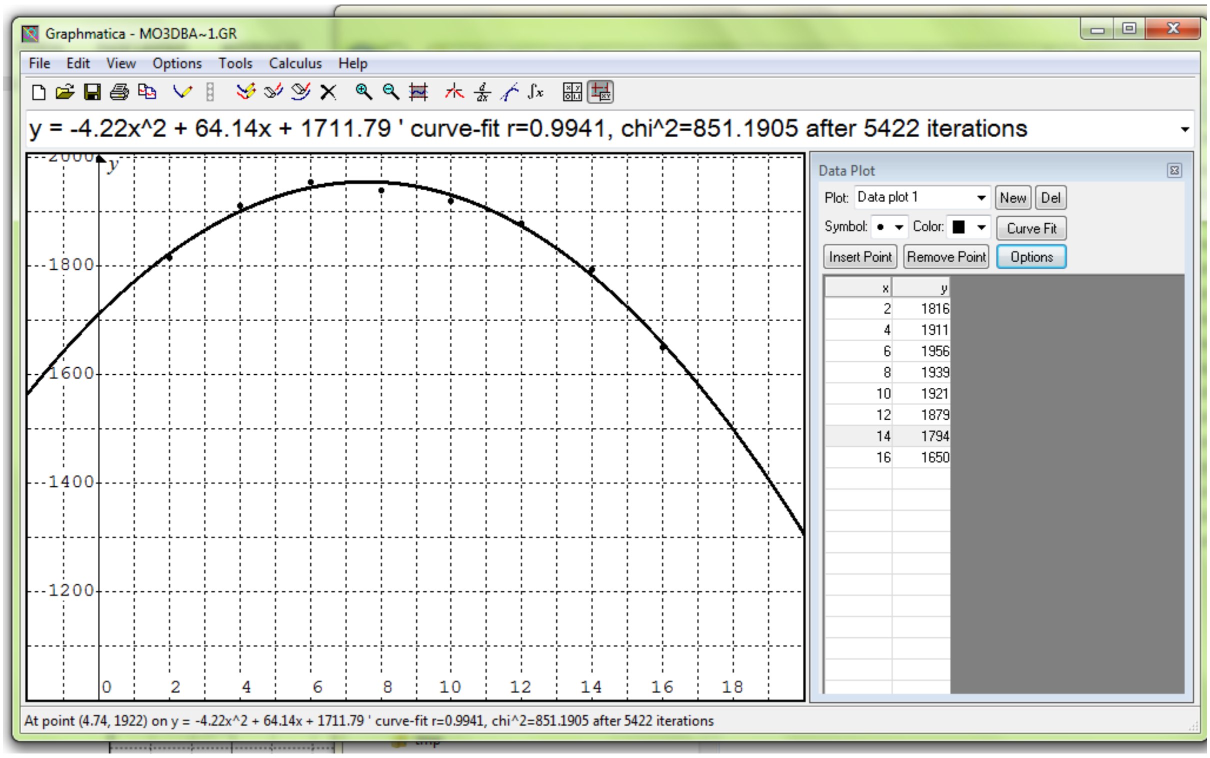
3(ii)	
3(iii)	*Let* $y = x^2 + 2x - 8 = (x+1)^2 - 9$. $\Rightarrow y + 9 = (x+1)^2 \Rightarrow x + 1 = \pm\sqrt{y+9}$ $\Rightarrow x = \pm\sqrt{y+9} - 1,$ $x \geq -1 \Rightarrow$ accept $x = \sqrt{y+9} - 1 \;\; \therefore g^{-1} : x \mapsto \sqrt{x+9} - 1, x \geq -9$
4(i)	$\text{p}(1) = 0 \;\Rightarrow a = 5 - b$ ……….(1) $\text{p}(-2) = 0 \Rightarrow -2a + b + 4 = 0$ ……….(2) Sub (1) into (2), $a = 3, \;\; b = 2$
4(ii)	*let* $p(x) = x^4 + 3x^3 + 2x^2 - 2x - 4 = (x-1)(x+2)(x^2 + Ax + 2)$ Comparing coefficient of x, $2 + (-2) + (-2A) = -2$ $\Rightarrow A = 2$ Third factor is $(x^2 + 2x + 2)$
4(iii)	$(x^2 + 2x + 2) = (x+1)^2 + 1 \geq 1 > 0$ Hence, this factor is positive for all value of x. Alternative mtd: Discriminant =-4 < 0
4(iv)	For $p(x) > 0$, $(x-1)(x+2) > 0 \;\; \Rightarrow \;\; x < -2$ or $x > 1$
5(a)	$\log_5 (x-1)(x-2) = \log_5 6$ $x^2 - 3x + 2 = 6 \Rightarrow x^2 - 3x - 4 = 0$ $(x+1)(x-4) = 0 \Rightarrow x = -1(\textit{rejected}), x = 4$ $\therefore x = 4$
5(b)	$\log_a x + \log_a 3x = \log_a 6 + 2,$ $\log_a x(3x) - \log_a 6 = 2 \Rightarrow \log_a \dfrac{3x^2}{6} = 2$

	$\Rightarrow a^2 = \frac{1}{2}x^2 \;\Rightarrow 2a^2 = x^2 \;\Rightarrow x = \pm\sqrt{2}a$ Since $a > 0$, $x > 0$, $\Rightarrow x = \sqrt{2}a$
6(a)	$\lvert 2-\frac{x}{2}\rvert = 3+x$ $2-\frac{x}{2} = 3+x$ or $2-\frac{x}{2} = -3-x$ $-\frac{3x}{2} = 1$ $\quad -\frac{x}{2} = 5$ $x = -\frac{2}{3}$ $\quad x =$ -10 (reject)
6(b)	$8^x - 3\left(2^{2x}\right) = 2^x - 3 \Rightarrow 2^{3x} - 3\left(2^{2x}\right) - 2^x + 3 = 0$ Let 2^x be y. $\quad \Rightarrow y^3 - 3\left(y^2\right) - y + 3 = 0$ Let $f(y)$=$y^3 - 3y^2 - y + 3$. $f(3)$=$27-27-3+3=0$ $\Rightarrow$ $y-3$ is a factor of $y^3 - 3\left(y^2\right) - y + 3 = 0$ Let $f(y) = (y-3)(y^2 - Ay - 1) = y^3 - 3y^2 - y + 3$. Comparing coeff of y^2, $A = 0$ $\therefore f(y) = (y-3)(y^2-1) = (y-3)(y-1)(y+1)$ $\Rightarrow y = 3, y = 1, y = -1 \Rightarrow 2^x = 3, \quad 2^x = 1, \quad 2^x = -1$ $\Rightarrow x = \frac{\ln 3}{\ln 2}$ *or* $x = 0$ *or* *na* $\therefore x = 1.58$ *or* 0
7(i) 7(ii) 7(iii)	 4
8(a)	RHS = sec² x cosec² x = (tan² x+1)(1+ cot² x) = (tan² x+1)+(1+ cot² x) == sec² x+ cosec² x *or* LHS= sec² x+ cosec² x = $\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$ $= \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} = \frac{1}{\cos^2 x \sin^2 x} = \sec^2 x \cos ec^2 x$

8(b)	$3 \sin x \tan x + 8 = 0 \Rightarrow 3\sin x \dfrac{\sin x}{\cos x} = -8$ $\Rightarrow 3\sin^2 x = -8\cos x \quad \Rightarrow 3(1-\cos^2 x) = -8\cos x$ $\Rightarrow 3\cos^2 x - 8\cos x - 3 = 0$ $\Rightarrow (3\cos x + 1)(-\cos x + 3) = 0 \Rightarrow \cos x = -\dfrac{1}{3}$ or $\cos x = 3$ (na) $\Rightarrow x = 109.5^\circ$ or 250.5°
9(i)	 Angle CAD1 = 30° (alt angle) Angle D1AT = 102°−30° = 72° Bearing of T from A = 180°−72° = 108°
9(ii)	$\dfrac{150}{\sin 41^\circ} = \dfrac{AC}{\sin 37^\circ}$ $AC = 137.6m$
9(iii)	$\tan 70.5^\circ = \dfrac{58-h}{20}$ $\Rightarrow h = 58 - 20\tan 70.5^\circ = 1.52$
10(i)	 By Pythagoras theorem, Adjacent side = $\sqrt{p^2 - 1}$

	$\cos(-x)$ $=\cos x$ $=\dfrac{\sqrt{p^2-1}}{p}$
10(ii)	$\sec\left(90^o-x\right)=\dfrac{1}{\cos\left(90^o-x\right)}=\dfrac{1}{\sin x}$ $=p$
11(i) 11(ii) 11(iii) 11(iv)	(i) $\begin{pmatrix}0 & 8 & 12 & 10\\ 8 & 0 & 4 & 7\\ 12 & 4 & 0 & 9\\ 10 & 7 & 9 & 0\end{pmatrix}$ (ii) $\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$ (iii) $\begin{pmatrix}30\\19\\25\\26\end{pmatrix}$ school B (iv) $\begin{pmatrix}3460\\1750\\2330\\2860\end{pmatrix}$ school B
12(i) 12(ii)	(a) general formula, to find solution of $0=-0.43x^2+1.712x+5.183$. ie. $x=\dfrac{-1.712\pm\sqrt{(1.712)^2-4(-0.43)(5.183)}}{2(-0.43)}$ (b) checking of discriminant, if $\Delta\geq 0$, then it will have intercepts. Or (b) Graphing using Desmo, Visually see if there is an x intercepts or alternatives
13(i)	Gradient of AD = $\dfrac{7-0}{9-2}=1$ $y=\lvert x-2\rvert$ *or* $y=\lvert -x+2\rvert$
13(ii)	Radius = $\sqrt{(9-5)^2+(7-11)^2}=\sqrt{32}$ or $4\sqrt{2}$ *units* $(x-5)^2+(y-11)^2=32$ *or* $x^2+y^2-10x-22y+114=0$
13(iii)	Area of triangle CAB= $\dfrac{1}{2}\begin{vmatrix}5 & 9 & 15 & 5\\ 11 & 7 & 18 & 11\end{vmatrix}=$ $\dfrac{1}{2}(35+162+165-99-105-90)=\dfrac{1}{2}(68)=34\,\text{unit}^2$
13(iv)	$AB=\sqrt{6^2+11^2}=\sqrt{157}$ units or 12.53 units $\dfrac{1}{2}(C\text{A})(\text{AB})\sin C\hat{\text{A}}\text{B}=34\Rightarrow\sin C\hat{\text{A}}\text{B}=\dfrac{34\times 2}{\sqrt{32}\sqrt{157}}$ $\Rightarrow C\hat{\text{A}}\text{B}=73.6^\circ$

	Alternative mtd : Cosine rule Using $BC=\sqrt{149}$, $BA=\sqrt{157}$, $AC=\sqrt{32}$ BC = 12.21 units, BA =12.53 units, AC = 5.65 units
14	$y=(x+3)(x-1)(x-2)$ $=x^3-7x+6$
15(i)	$y=ax^2+bx+c \qquad \Rightarrow \dfrac{y-1712}{x}=ax+b$
15(ii)	Refer to graph Paper Line of best fit Points plotted, Label Axes Table of Points
15(iii)	$y=-4.2x+64$ Accept $a=-4.2\pm0.4$ and $b=64\pm5$ *Note: Deduct only 1M (iii)&(iv) for inaccurate range due to poor line of best fit if correct method use for these three answers*
15(iv)	Accept 1585 ± 40


Graphmatica - MO3DBA~1.GR
File Edit View Options Tools Calculus Help
y = -4.22x^2 + 64.14x + 1711.79 ' curve-fit r=0.9941, chi^2=851.1905 after 5422 iterations
1800
1600
1400
1200
y
0
2
4
6
8
10
12
14
16
18
Data Plot
Plot: Data plot 1
New
Del
Symbol:
Color:
Curve Fit
Insert Point
Remove Point
Options
x
y
2 1816
4 1911
6 1956
8 1939
10 1921
12 1879
14 1794
16 1650
At point (4.74, 1922) on y = -4.22x^2 + 64.14x + 1711.79 ' curve-fit r=0.9941, chi^2=851.1905 after 5422 iterations

(i) $y = ax^2 + bx + 1712$

$\frac{y-1712}{x} = ax + b$ [B1]

(ii) [G1]

x	2	4	6	8	10	12	14	16
y	1816	1911	1956	1939	1921	1879	1794	1650
$\frac{y-1712}{x}$	52	49.75	40.7	28.4	20.9	13.9	5.9	-3.9

(iii) From the y-intercept of graph,

$b = 63.5$. [1]

$a = \frac{63.5-0}{0-15.2}$

$= -4.18$. [1]

(iv) From the graph, $x=17$,

$\frac{y-1712}{x} = -7.5$

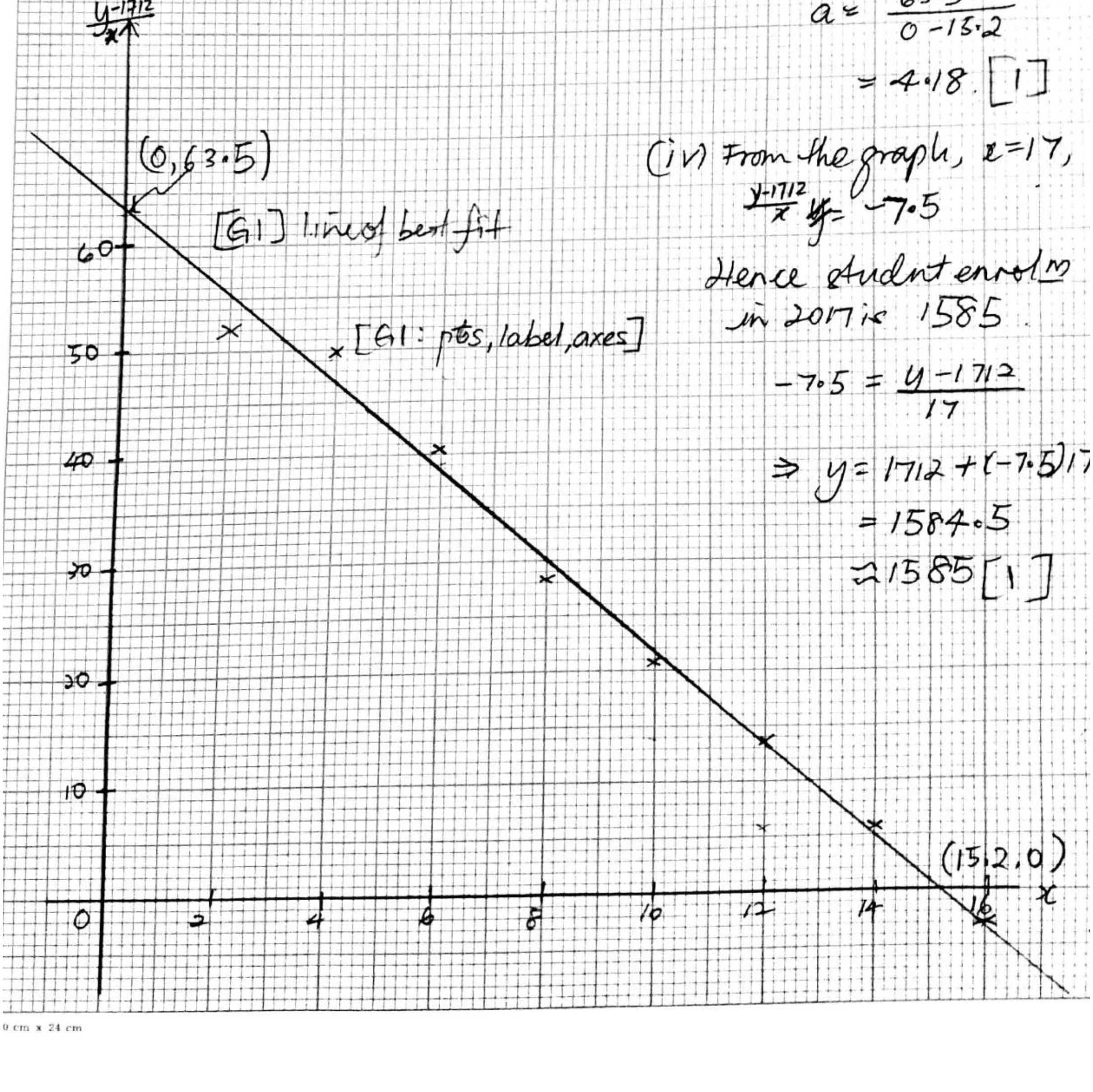
Hence student enrolm in 2017 is 1585.

$-7.5 = \frac{y-1712}{17}$

$\Rightarrow y = 1712 + (-7.5)17$

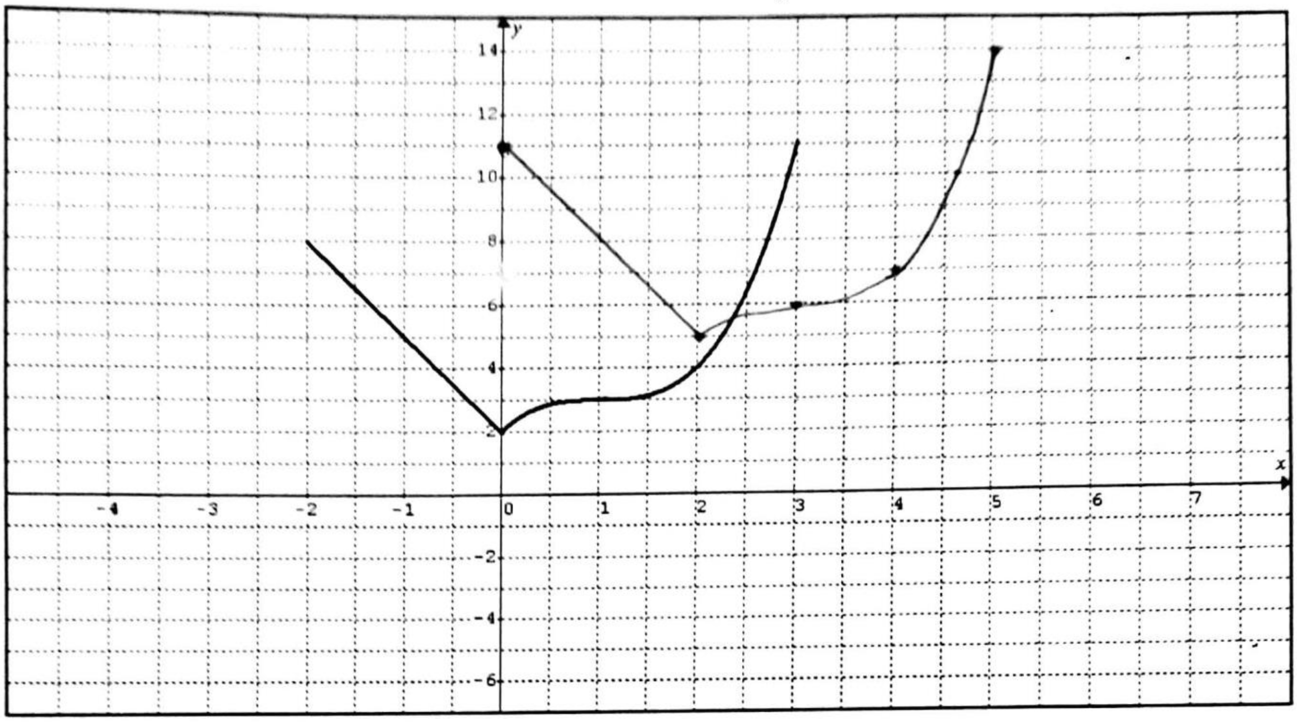
$= 1584.5$

≈ 1585 [1]



(i)	Correct translation of 2 units to the left	1M
	Correct translation of 3 units up	1M
(ii)	Correct reflection about the x-axis	1M
	Correct stretch along x axis by factor ½ or correct compression along x-axis by factor 1/2	1M

Answer for (i)



Answer for (ii)