



FM Statistics 2 – Hypothesis Testings and Confidence Intervals

Instruction: Attempt the questions marked with *.

- 1* (a) In a test, students were asked to interpret the phrase "95% confidence interval".
A student wrote that "95% confidence interval for $\mu = (77, 83)$ means that there is a 95% chance that the true population mean falls between 77 and 83."
Explain why his answer is incorrect and give a correct interpretation. [2]
- (b) Prior to an election contested by 2 candidates A and B, a news agency conducted an online survey amongst its subscribers. Of 21,356 respondents, 16,331 indicated that they will vote for A. Calculate the 95% confidence interval for the population proportion that will vote for A. Show your workings clearly. [3]
- (c) For a confidence interval for population proportion (a, b) , $M = \frac{b-a}{2}$ is defined as the margin of error. Before collecting any sample, it is a common industrial practice to have a target $M_{\max}$ and calculate the minimum required sample size to achieve a confidence interval that is guaranteed to have $M \leq M_{\max}$.
Find the minimum sample size so that $M_{\max} = 0.03$ for a 95% confidence interval. [4]

- 2* An investigation was carried out into the effectiveness of two well-known drugs, A and B, in relieving pain for adult arthritis sufferers. Ten adult arthritis sufferers each volunteered to record the number of hours of relief from pain gained when taking drug A and when taking drug B. The adults agreed to take one of the drugs, A or B, on one day and the other drug on another day. The order of taking the drugs was randomly assigned. The recorded times, in hours, are given in the table.

Adult	1	2	3	4	5	6	7	8	9	10
Drug A	2.0	3.6	2.6	2.6	7.2	3.4	6.5	2.5	6.1	8.5
Drug B	3.5	5.7	2.8	2.3	9.8	3.3	5.9	3.7	9.1	11.9

- (a) Explain the purpose of randomly assigning the order of taking drug A or drug B. [2]
- (b) It was believed that drug B is more effective in relieving pain gained by arthritis sufferers than drug A. Determine if the data support this belief, at the 1% level of significance, using
- (i) the Wilcoxon matched-pairs signed rank test, [4]
 - (ii) a t -test, given that the corresponding populations are normally distributed. [4]
 - (iii) State, giving a reason, which of the two tests is more appropriate. [1]
- (c) Give a reason why your conclusions in part (b) might not apply to all adult arthritis sufferers. [1]

The investigator claims that the average number of hours of relief from pain gained by arthritis sufferers when taking drug A is 4.5 hours.

- (d) Use a non-parametric test, at the 10% significance level, to test this claim against the alternative hypothesis that the average number of hours is less than 4.5 hours. [3]

- 3 A To explore the effectiveness of a newly launched Mathematics remedial programme, Ms Chan randomly selected six JC2 students who were identified as underperforming in the subject. She implemented an assessment to these six students before the remedial programme. After each student had gone through 10 hours of remedial programme, they were given a second assessment. Their scores were recorded in the table below.

Student	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
1 st assessment (%)	42	48	36	27	30	45
2 nd assessment (%)	39	48	41	31	31	51

- (a) Carry out a suitable test, at the 2% level of significance, to determine whether there is any difference in the students' scores, on average, after the remedial programme. State the assumptions necessary for your test. [6]
- (b) To ensure that the two assessments are set at comparable standards, Ms Chan invited another Math tutor Ms Soh to examine the two assessments. Ms Soh commented that the first assessment was too easy and recommended that each score for the first assessment to be reduced by a constant positive integer k .
Find the least value of k for which at the 2% significance level, the students' scores have, on average, improved. [4]
- (c) A 98% confidence interval for the population mean score difference for the two assessments is obtained as $(-2.55, 6.88)$. Explain whether this result is consistent with the test's conclusion in (a)? [1]
- (d) Another teacher uses the same data to determine another confidence interval for this difference and he obtains $(-1.23, 5.56)$. Determine the confidence level of this interval, to the nearest integer. [3]
- 4 Customers were asked which choice of three brands of coffee, *A*, *B*, or *C*, they prefer. For a random sample of 80 male customers and 60 female customers, the numbers preferring each brand are shown in the following table.

	<i>A</i>	<i>B</i>	<i>C</i>	Total
Male	32	36	12	80
Female	18	30	12	60
Total	50	66	24	140

- (a) Test, at the 10% level of significance, whether there is a difference between coffee preferences of male and female customers. [5]
- (b) A larger random sample is taken, with $80n$ male customers and $60n$ female customers, where n is a positive integer. The numbers preferring each brand are shown in the following table.

	<i>A</i>	<i>B</i>	<i>C</i>	Total
Male	$32n$	$36n$	$12n$	$80n$
Female	$18n$	$30n$	$12n$	$60n$
Total	$50n$	$66n$	$24n$	$140n$

Find the least possible value of n that would lead to a different conclusion from part (a) at the 10% level of significance. [3]