RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Tutorial 2A: Vectors I
Vector Algebra, Ratio Theorem, Scalar and Vector Products

Section A (Basic Questions)

1 **(a)** The vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are such that $\mathbf{a}-2\mathbf{b}+\mathbf{c}=\mathbf{0}$. If $\mathbf{a}=4\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{b}=\mathbf{i}-2\mathbf{k}$, find a unit vector in the direction of $\mathbf{c}$.

(b) Find a vector $\mathbf{a}$ such that $\mathbf{a}$ is parallel to the vector $8\mathbf{i}+\mathbf{j}+4\mathbf{k}$ and is equal in magnitude to the vector $\mathbf{i}+2\mathbf{j}-2\mathbf{k}$.

(c) Find the position vector of P if OP is of length 5 units and is in the direction $2\mathbf{i}-\mathbf{j}+4\mathbf{k}$.

[(a) $\frac{1}{\sqrt{30}}\begin{pmatrix}-2\\1\\-5\end{pmatrix}$ **(b)** $\frac{1}{3}\begin{pmatrix}8\\1\\4\end{pmatrix}$ or $-\frac{1}{3}\begin{pmatrix}8\\1\\4\end{pmatrix}$ **(c)** $\frac{5}{\sqrt{21}}\begin{pmatrix}2\\-1\\4\end{pmatrix}$**]**

2 The points P and Q have position vectors $-3\mathbf{i}-\mathbf{j}+17\mathbf{k}$ and $8\mathbf{i}+9\mathbf{j}+2\mathbf{k}$ respectively when referred from the origin O.

(a) Evaluate $\overrightarrow{OP}\cdot\overrightarrow{OQ}$ and $\overrightarrow{OP}\times\overrightarrow{OQ}$.

(b) Find the position vector of the point R if

(i) R is the mid-point of PQ;

(ii) R lies on the line segment PQ such that $PR:RQ=2:3$;

(iii) R lies on PQ produced so that $PR:QR=3:2$.

[(a) 1, $\begin{pmatrix}-155\\142\\-19\end{pmatrix}$ **(b)(i)** $\frac{1}{2}\begin{pmatrix}5\\8\\19\end{pmatrix}$ **(ii)** $\begin{pmatrix}7/5\\3\\11\end{pmatrix}$ **(iii)** $\begin{pmatrix}30\\29\\-28\end{pmatrix}$**]**

3 **(a)** Find the value of p so that the vectors $2\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $p\mathbf{i}+\mathbf{j}+p\mathbf{k}$ are perpendicular.

(b) If $\mathbf{a}=\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $\mathbf{b}=\mathbf{j}+\mathbf{k}$, find a unit vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.

[(a) $p=-\frac{1}{3}$ **(b)** $\frac{1}{\sqrt{11}}\begin{pmatrix}3\\-1\\1\end{pmatrix}$ or $\frac{1}{\sqrt{11}}\begin{pmatrix}-3\\1\\-1\end{pmatrix}$ **]**

Section B (Discussion Questions)

1 The position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are given by $\mathbf{a} = 2p\mathbf{i} + 3p\mathbf{j} + 6p\mathbf{k}$, $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{c} = 3p\mathbf{i} - 20p\mathbf{j} + 2p\mathbf{k}$ where $p > 0$. It is given that $|\mathbf{a}| = |\mathbf{b}|$.

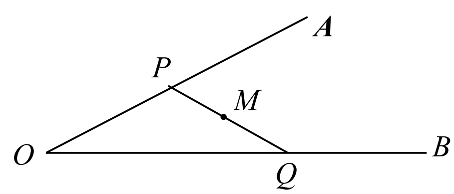
(i) Find the exact value of p. [2]

(ii) Show that $(\mathbf{a}+\mathbf{b}).(\mathbf{a}-\mathbf{b}) = 0$. [3]

(iii) The three points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively relative to the origin O. Show that A, B and C are collinear. [3]

[(i) $p = \frac{3}{7}$**]**

2



Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point P on OA is such that $OP : PA = 1 : 2$, and the point Q on OB is such that $OQ : QB = 3 : 2$. The mid-point of PQ is M (see diagram).

(i) Find $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ and show that the area of triangle OMP can be written as $k|\mathbf{a}\times\mathbf{b}|$, where k is a constant to be found. [6]

(ii) The vectors $\mathbf{a}$ and $\mathbf{b}$ are now given by

$$\mathbf{a} = 2p\mathbf{i} - 6p\mathbf{j} + 3p\mathbf{k} \quad \text{and} \quad \mathbf{b} = \mathbf{i} + \mathbf{j} - 2\mathbf{k},$$

where p is a positive constant. Given that $\mathbf{a}$ is a unit vector,

(a) find the exact value of p, [2]

(b) give a geometrical interpretation of $|\mathbf{a}\cdot\mathbf{b}|$, [1]

(c) evaluate $\mathbf{a}\times\mathbf{b}$. [2]

[(i) $k = \frac{1}{20}$ **(ii)(a)** $p = \frac{1}{7}$ **(ii)(c)** $\frac{1}{7}\begin{pmatrix} 9 \\ 7 \\ 8 \end{pmatrix}$**]**

3 **(i)** Given that $\mathbf{a}\times\mathbf{b}=\mathbf{0}$, what can be deduced about the vectors **a** and **b**? [2]

(ii) Find a unit vector **n** such that $\mathbf{n}\times(\mathbf{i}+2\mathbf{j}-2\mathbf{k})=\mathbf{0}$. [2]

(iii) Find the cosine of the acute angle between $\mathbf{i}+2\mathbf{j}-2\mathbf{k}$ and the z-axis. [1]

[(ii) $\mathbf{n}=\frac{1}{3}\begin{pmatrix}1\\2\\-2\end{pmatrix}$ or $\mathbf{n}=-\frac{1}{3}\begin{pmatrix}1\\2\\-2\end{pmatrix}$ **(iii)** $\frac{2}{3}$**]**

4 Find the cosine of the angle between the vectors $\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{i}-\mathbf{j}-2\mathbf{k}$.

Hence, find the length of projection of $\mathbf{i}-\mathbf{j}-2\mathbf{k}$ on $\mathbf{i}+\mathbf{j}+\mathbf{k}$.

[$-\frac{2}{\sqrt{18}}$, $\frac{2}{\sqrt{3}}$**]**

5 The angle between the unit vectors **a** and **b** is θ.
By expanding the scalar product, show that $(3\mathbf{a}-\mathbf{b})\cdot(\mathbf{a}+3\mathbf{b})=8\cos\theta$.

Given that $\theta=60°$, show that the length of projection of $(3\mathbf{a}-\mathbf{b})$ onto $(\mathbf{a}+3\mathbf{b})$ is $\frac{4}{\sqrt{13}}$.

6 It is given that **a** and **b** are non-zero vectors.

(a) Given that $|\mathbf{a}+\mathbf{b}|=|\mathbf{a}-\mathbf{b}|$, by considering suitable scalar product, comment on the relationship between **a** and **b**. [2]

(b) Given that $\frac{\mathbf{a}}{|\mathbf{a}|}=\frac{\mathbf{b}}{|\mathbf{b}|}$, what can you say about the relationship between **a** and **b**? [1]

(c) Given that $2\mathbf{a}+\mathbf{b}=-5\mathbf{j}+\mathbf{k}$, $\mathbf{c}=\mathbf{i}-2\mathbf{j}+2\mathbf{k}$, $\mathbf{a}.\mathbf{c}=5$ and **b** is a unit vector,

(i) find the value of **b.c**. [2]

(ii) find the sine of the angle between **b** and **c**. [2]

(iii) find $|\mathbf{b}\times\mathbf{c}|$, and state the geometrical meaning of this result. [2]

[(c)(i) 2 **(c)(ii)** $\frac{\sqrt{5}}{3}$ **(c)(iii)** $\sqrt{5}$ **]**

7 With respect to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and not parallel.

(i) It is given that B lies on the line segment AC, such that $\overrightarrow{BC} = 3\mathbf{b} - k\mathbf{a}$, where k is a constant. State, with a reason, the value of k. Hence find $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

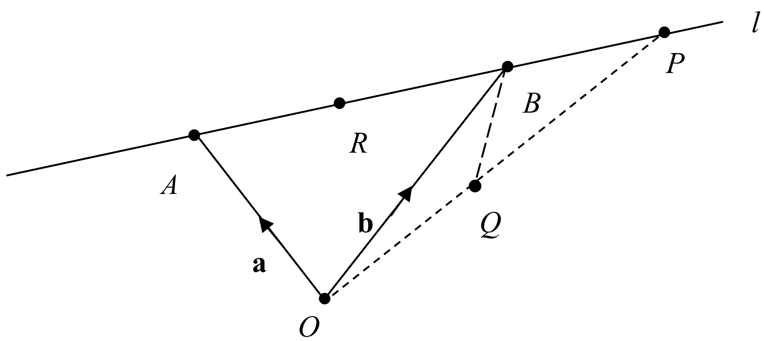
The point N divides the line OC in the ratio $1-\lambda : \lambda$.

(ii) Given that $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$, Show that $\overrightarrow{BN}.\overrightarrow{OC}$ can be written as $p|\mathbf{a}|^2 + q|\mathbf{b}|^2$, where p and q are constants to be found in terms of λ. [4]

(iii) Given that $\overrightarrow{BN}.\overrightarrow{OC} = 0$ and $|\mathbf{a}| = \frac{2}{3}|\mathbf{b}|$, find the value of λ. [2]

[(i) $k = 3$, $\overrightarrow{OC} = 4\mathbf{b} - 3\mathbf{a}$ **(ii)** $p = 9(1-\lambda)$, $q = 4(3-4\lambda)$ **(iii)** $\frac{4}{5}$ **]**

8 The diagram below shows a straight line l passing through the points A and B. With reference to the origin O, the position vectors of A and B are $\mathbf{a}$ and $\mathbf{b}$ respectively. It is further given that $\mathbf{a}$ is a unit vector, $|\mathbf{b}| = 2$ and $\angle AOB = 60°$.



(i) State the values of $\mathbf{a}\bullet\mathbf{b}$ and $|\mathbf{a} \times \mathbf{b}|$. [2]

(ii) The point P lies on the line l and is such that $AB : BP = 2:1$. The point Q on the line OP is such that $\overrightarrow{OQ} = \lambda\overrightarrow{OP}$ where $0 < \lambda < 1$. Determine the value of λ such that the area of triangle OBQ is $\frac{1}{2\sqrt{3}}$ of the area of triangle OAB. [3]

(iii) It is further given that the point R on the line l is such that $\angle AOR = \angle ROB$. Show that R has position vector $\mathbf{a} + \mu(\mathbf{b} - \mathbf{a})$ for some $\mu \in \mathbb{R}$ and hence find this value of μ. [3]

[(i) $\mathbf{a}\bullet\mathbf{b} = 1$, $|\mathbf{a}\times\mathbf{b}| = \sqrt{3}$ **(ii)** $\frac{1}{\sqrt{3}}$ **(iii)** $\frac{1}{3}$ **]**

9 The points P and Q have position vectors $\mathbf{a}+\mathbf{b}$ and $3\mathbf{a}-2\mathbf{b}$ respectively relative to the origin O. Given that $OPQR$ is a parallelogram, express the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

By evaluating two scalar products, show that if $OPQR$ is a square, then $|\mathbf{a}|^2 = 2|\mathbf{b}|^2$.

[$\overrightarrow{PQ} = 2\mathbf{a}-3\mathbf{b}$, $\overrightarrow{PR} = \mathbf{a}-4\mathbf{b}$ **]**

THE END