St. Patrick's School – Preliminary Examination 2026
Mathematics 4052 Paper 2 – Worked Solutions

Qn	Answer	Mark
1(a)	$$\frac{5}{x-2}-\frac{3}{2x+1}=\frac{5(2x+1)-3(x-2)}{(x-2)(2x+1)}=\frac{10x+5-3x+6}{(x-2)(2x+1)}=\frac{7x+11}{(x-2)(2x+1)}$$	[2]
1(b)	$$Vr^2=h(R^2+r^2)$$ $$Vr^2-hr^2=hR^2$$ $$r^2(V-h)=hR^2$$ $$r^2=\frac{hR^2}{V-h}$$ $$r=R\sqrt{\frac{h}{V-h}}$$	[2]
1(c)	$$\frac{2x^2-xy-6x+3y}{9-x^2}=\frac{x(2x-y)-3(2x-y)}{(3-x)(3+x)}=\frac{(2x-y)(x-3)}{-(x-3)(x+3)}=\frac{y-2x}{x+3}$$	[3]
1(d)	$$(2x^3y^{-2})^4\div(8x^5y^{-3})=\frac{16x^{12}y^{-8}}{8x^5y^{-3}}=2x^7y^{-5}=\frac{2x^7}{y^5}$$	[2]
2(a)(i)	$$\text{Deposit}=0.20\times 2400=480$$ $$\text{Instalments}=24\times 92=2208$$ $$\text{Total HP price}=480+2208=\$2688$$	[2]
2(a)(ii)	$$\text{Difference}=2688-2400=288$$ $$\text{Percentage}=\frac{288}{2400}\times 100\%=12\%$$	[2]

Qn	Answer	Mark
2(b)	$$\begin{aligned} 85\% \text{ of price} &= 1734 \\ \text{Original price} &= \frac{1734}{0.85} \\ &= \$2040 \end{aligned}$$	[2]
2(c)(i)	$$\begin{aligned} \text{Value} &= 6666 \times 1.08^2 \\ &= 6666 \times 1.1664 \\ &= \$7775.2224 \\ &= \$7775.22 \text{ (nearest cent)} \end{aligned}$$	[2]
2(c)(ii)	Continue year by year from the year 3 value: $$\begin{aligned} &\text{End of year 3: } 6666 \times 1.08^3 = 8397.24 \quad (< 8888) \\ &\text{End of year 4: } 6666 \times 1.08^4 = 9069.02 \quad (> 8888) \end{aligned}$$ The value first exceeds \$8888 after 4 complete years. $$\begin{aligned} \text{Profit} &= 9069.02 - 6666 \\ &= \$2403.02 \end{aligned}$$	[3]
3(a)	$$\begin{aligned} AC^2 &= 11^2 + 8^2 - 2(11)(8)\cos 58^\circ \\ &= 185 - 176\cos 58^\circ \\ &= 91.73 \\ AC &= 9.58 \text{ cm} \end{aligned}$$	[3]
3(b)	$$\begin{aligned} \frac{\sin\angle ADC}{9.578} &= \frac{\sin 42^\circ}{7} \\ \sin\angle ADC &= \frac{9.578\sin 42^\circ}{7} \\ &= 0.9155 \\ \angle ADC &= 113.7^\circ \text{ (obtuse)} \\ \angle CAD &= 180^\circ - 42^\circ - 113.7^\circ \\ &= 24.3^\circ \end{aligned}$$	[3]
3(c)	$$\begin{aligned} \frac{CD}{\sin\angle CAD} &= \frac{7}{\sin 42^\circ} \\ CD &= \frac{7\sin 24.3^\circ}{\sin 42^\circ} \\ &= 4.30 \text{ cm} \end{aligned}$$	[2]
3(d)	$$\begin{aligned} \text{Area } ABC &= \tfrac{1}{2}(11)(8)\sin 58^\circ \\ &= 37.31 \\ \text{Area } ACD &= \tfrac{1}{2}(9.578)(7)\sin 24.3^\circ \\ &= 13.79 \\ \text{Area } ABCD &= 37.31 + 13.79 \\ &= 51.1 \text{ cm}^2 \end{aligned}$$	[2]
4(a)(i)	Median $\approx$ 4.1 kg (read at cumulative frequency = 60)	[1]

Qn	Answer	Mark
4(a)(ii)	$Q_1 = 3.5$ kg (CF = 30), $Q_3 = 5.0$ kg (CF = 90) $\text{IQR} = Q_3 - Q_1 = 5.0 - 3.5 = 1.5$ kg	[2]
4(b)	Below 3 kg: CF at 3.0 kg ≈ 13 Above 6.5 kg: 120 − CF(6.5) = 120 − 116 = 4 $\text{Discounted} = 13 + 4 = 17$ $\text{Percentage} = \frac{17}{120} \times 100\% = 14.2\%$	[3]
4(c)	Five-number summary for shipment 2: minimum 3, lower quartile 4, median 5, upper quartile 6.5, maximum 7.5 (range 4.5). Box drawn from 4 to 6.5 with median line at 5; whiskers to 3 and 7.5.	[3]
4(d)	Shipment 1: within 3–6.5 kg ≈ 103/120 ≈ 86%, so ≈14% discounted. Shipment 2: $Q_3 = 6.5$ kg, so 75% lie within 3–6.5 kg; 25% (above 6.5 kg) discounted. Shipment 1 is more desirable: a larger proportion (≈86% vs 75%) falls within the acceptable 3–6.5 kg range, so fewer melons are discounted.	[2]
5(a)	$d = 0.1(3)^3 - 1.2(3)^2 + 3.3(3) + 4 = 5.8$	[1]
5(b)	Plot the nine points from the table. Join them with a single smooth curve.	[3]
5(c)(i)	Draw a tangent to the curve at t = 5. Gradient ≈ −1.2 (accept −1.0 to −1.4)	[2]
5(c)(ii)	Minimum depth ≈ 2.2 m (at t ≈ 6.2)	[1]
5(d)	Depth is decreasing for 1.8 < t < 6.2 (approx).	[1]
5(e)	Draw the line d = 0.2t + 2 on the grid. It meets the curve on the falling part at t = 5. t = 5 is 5 hours after 6 am, so the ship must leave by 11 am.	[2]
6(a)(i)	$\frac{1}{2}\left(\frac{1}{3}\pi r^2 h\right) = 48\pi$ $\frac{1}{2}\left(\frac{1}{3}\pi (6)^2 h\right) = 48\pi$ $6\pi h = 48\pi$ $h = 8$ cm	[2]

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<tr><th>Qn</th><th>Answer</th><th>Mark</th></tr>
<tr><td>6(a)(ii)</td><td>

$$l = \sqrt{6^2 + 8^2}$$
$$= 10 \text{ cm}$$

$$\begin{aligned} \text{Half curved surface} &= \tfrac{1}{2}\pi r l \\ &= \tfrac{1}{2}\pi(6)(10) \\ &= 30\pi \\ \text{Half base} &= \tfrac{1}{2}\pi r^2 \\ &= \tfrac{1}{2}\pi(6)^2 \\ &= 18\pi \\ \text{Triangular face} &= \tfrac{1}{2}(12)(8) \\ &= 48 \\ \text{Total surface area} &= 30\pi + 18\pi + 48 \\ &= 199 \text{ cm}^2 \end{aligned}$$

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<tr><td>6(b)</td><td>

$$\begin{aligned} \text{Detergent} &= 3.20 \times \left(\tfrac{3}{2}\right)^3 \\ &= 3.20 \times \tfrac{27}{8} \\ &= \$10.80 \\ \text{Material} &= 0.80 \times \left(\tfrac{3}{2}\right)^2 \\ &= 0.80 \times \tfrac{9}{4} \\ &= \$1.80 \\ \text{Total} &= 10.80 + 1.80 + 0.15 \\ &= \$12.75 \end{aligned}$$

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<tr><td>7(a)</td><td>

∠BCA = 32° (AC bisects ∠BCD)

$$\angle BOA = 2 \times 32^\circ$$
$$= 64^\circ$$

(∠ at centre = 2 × ∠ at circumference)

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<tr><td>7(b)</td><td>

O and M are the midpoints of AC and DC, so OM ∥ AD (midpoint theorem).
∠OMC = ∠ADC = 90° (corr. ∠s, OM ∥ AD; ∠ in semicircle)
∠OCM = ∠ACB = 32° (AC bisects ∠BCD)
∴ △COM is similar to △CAB (2 pairs of equal angles)

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<tr><td>7(c)</td><td>

In △ABC and △ADC:
∠ABC = ∠ADC = 90° (∠ in semicircle)
∠BCA = ∠DCA = 32° (AC bisects ∠BCD)
AC is common.
∴ △ABC ≅ △ADC (AAS)
∴ BC = DC

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<tr><td>7(d)</td><td>

$$BC = 18.4 \cos 32^\circ$$
$$= 15.60 \text{ cm}$$

The perpendicular from O bisects BC, so the half-chord = 7.802 cm.

$$d = \sqrt{9.2^2 - 7.802^2}$$
$$= 4.88 \text{ cm (3 s.f.)}$$

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Qn	Answer	Mark
8(a)(i)	$$\overrightarrow{AB} = 2\mathbf{b} - 3\mathbf{a}$$	[1]
8(a)(ii)	$$\begin{aligned}\overrightarrow{BP} &= \overrightarrow{BA} + \overrightarrow{AP} \\ &= (3\mathbf{a} - 2\mathbf{b}) + (1.5\mathbf{a} + \mathbf{b}) \\ &= 4.5\mathbf{a} - \mathbf{b}\end{aligned}$$	[1]
8(b)	$$\begin{aligned}\overrightarrow{OQ} &= \overrightarrow{OA} + \overrightarrow{AP} + \overrightarrow{PQ} \\ &= 3\mathbf{a} + (1.5\mathbf{a} + \mathbf{b}) + k\mathbf{b} \\ &= 4.5\mathbf{a} + (1 + k)\mathbf{b}\end{aligned}$$ $$\begin{aligned}\overrightarrow{BQ} &= \overrightarrow{OQ} - \overrightarrow{OB} \\ &= 4.5\mathbf{a} + (k - 1)\mathbf{b}\end{aligned}$$ BR = h a is parallel to a, and B, R, Q are collinear, so BQ is parallel to a. ∴ coefficient of b = 0: $$\begin{aligned}k - 1 &= 0 \\ k &= 1\end{aligned}$$	[2]
8(c)	$$\begin{aligned}\overrightarrow{OP} &= 3\mathbf{a} + (1.5\mathbf{a} + \mathbf{b}) \\ &= 4.5\mathbf{a} + \mathbf{b} \\ \overrightarrow{OR} &= \overrightarrow{OB} + \overrightarrow{BR} \\ &= h\mathbf{a} + 2\mathbf{b}\end{aligned}$$ $$\begin{aligned}\overrightarrow{PR} &= \overrightarrow{OR} - \overrightarrow{OP} \\ &= (h - 4.5)\mathbf{a} + \mathbf{b}\end{aligned}$$ Since PR ∥ AB, PR = λ AB: $$(h - 4.5)\mathbf{a} + \mathbf{b} = \lambda(2\mathbf{b} - 3\mathbf{a})$$ Comparing coefficients of b: $$\begin{aligned}1 &= 2\lambda \\ \lambda &= \tfrac{1}{2}\end{aligned}$$ Comparing coefficients of a: $$\begin{aligned}h - 4.5 &= -3\lambda \\ &= -1.5 \\ h &= 3\end{aligned}$$	[3]
8(d)	With k = 1: $$\begin{aligned}\overrightarrow{RQ} &= \overrightarrow{BQ} - \overrightarrow{BR} \\ &= 4.5\mathbf{a} - 3\mathbf{a} \\ &= 1.5\mathbf{a}\end{aligned}$$ $$\begin{aligned}BR : RQ &= 3 : 1.5 \\ &= 2 : 1\end{aligned}$$ R lies on BQ, two-thirds of the way from B to Q (BR is twice RQ).	[2]

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<tr><td>8(e)</td><td>

$$\begin{aligned}\overrightarrow{AR} &= \overrightarrow{AB} + \overrightarrow{BR}\\ &= (2\mathbf{b} - 3\mathbf{a}) + 3\mathbf{a}\\ &= 2\mathbf{b}\end{aligned}$$

Since AR = 2b = OB and BR = 3a = OA, OARB is a parallelogram.
Diagonal AB divides it into two equal triangles, △AOB and △ABR.

$$\begin{aligned}\text{Area of } \triangle ABR &= \text{Area of } \triangle AOB\\ &= 60 \text{ units}^2\end{aligned}$$

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<tr><td>9(a)</td><td>

x = 1: tickets sold = 2300; x = 2: tickets sold = 2200.
Row x: new price = 8 + x, tickets sold = 2400 − 100x,
revenue R = (8 + x)(2400 − 100x).

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<tr><td>9(b)</td><td>

$$\begin{aligned}(8+x)(2400-100x) &= 23100\\ 19200 + 1600x - 100x^2 &= 23100\\ 100x^2 - 1600x + 3900 &= 0\\ x^2 - 16x + 39 &= 0\\ (x-3)(x-13) &= 0\\ x &= 3\\ \text{or} \quad x &= 13\end{aligned}$$

x = 3: price = \$11, tickets = 2100 ($\geq$ 2000 ✓)
x = 13: price = \$21, tickets = 1100 ($<$ 2000 ×)
∴ ticket price = \$11.

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<tr><td>9(c)</td><td>

R = (8 + x)(2400 − 100x) = 0 when x = −8 or x = 24.

$$\begin{aligned}\text{Line of symmetry: } x &= \frac{-8+24}{2}\\ &= 8\end{aligned}$$

Maximum revenue is at x = 8, so price = 8 + 8 = \$16.

$$\begin{aligned}R_{\text{max}} &= (16)(2400-800)\\ &= \$25\,600\end{aligned}$$

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<tr><td>9(d)</td><td>

Sponsor A (\$15 000 if $\geq$ 1900 tickets): 2400 − 100x $\geq$ 1900, so x $\leq$ 5.

$$\begin{aligned}\text{Best } R &= (13)(1900)\\ &= 24700\\ \text{Total } (A) &= 24700 + 15000\\ &= \$39\,700\end{aligned}$$

Sponsor B (50c per \$1 of sales = 1.5R): maximise R at x = 8.

$$\begin{aligned}\text{Total } (B) &= 1.5 \times 25600\\ &= \$38\,400\end{aligned}$$

\$39 700 > \$38 400, so accept Sponsor A and set the price at \$13.

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