JURONG PIONEER JUNIOR COLLEGE
9478 H2 PHYSICS

FORCES AND MOMENTS

Content

- Types of force
- Moment and torque
- Translational and rotational equilibrium

Learning Outcomes

Students should be able to:

(a) describe the forces on a mass, charge and current-carrying conductor in gravitational, electric and magnetic fields, as appropriate.

(b) show a qualitative understanding of forces including normal force, buoyant force (upthrust), frictional force and viscous force, e.g. air resistance. (knowledge of the concepts of coefficients of friction and viscosity is not required).

(c) recall and apply Hooke's law ($F = kx$, where k is the force constant) to new situations or to solve related problems.

(d) define and apply the moment of a force and the torque of a couple.

(e) show an understanding that a couple is a pair of forces which tends to produce rotation only.

(f) show an understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity.

(g) apply the principle of moments to new situations or to solve related problems.

(h) show an understanding that, when there is no resultant force and no resultant torque, a system is in equilibrium.

(i) use free-body diagrams and vector triangles to represent forces on bodies that are in rotational and translational equilibrium.

1 INTRODUCTION

A **force** is an interaction between two objects.

- It must be acted by one object on another, and can either be a push or a pull.
- It is a vector; it has both magnitude and direction.
- The S.I. unit of force is the newton (N).
- A combination of multiple forces acting on a body can change its motion, cause it to rotate or change its shape and size by stretching, compressing or twisting.

In this topic, we will cover the *types of force*, *moment and torque* as well as *translational and rotational equilibrium*.

2 TYPES OF FORCE

At present, there are four known fundamental forces:

1. *gravitational force* – acts between all objects, and is the dominant force for shaping the large scale structure of galaxies, stars, etc,
2. *electromagnetic force* – acts between charged particles and accounts for the bonding energy of atoms and molecules,
3. *weak force* – responsible for radioactive decay, and
4. *strong force* – holds neutrons and protons together in a nucleus.

Watch this…
https://www.youtube.com/watch?v=a-6skWBuHaE
This video from YouTube gives a summary of the four fundamental forces.

Note: The weak and strong forces will **not** be covered in this syllabus. The gravitational and electromagnetic forces will be covered in greater detail in future topics.

The table below shows the two main types of forces that we cover in A Level Physics:

Non-contact force **(influences over distance)**	**Contact force** **(some)** **(requires contact to exert a force)**
Gravitational force	Pull / Push
Electric force	Tension / Compression
Magnetic force	Frictional force / Viscous force
	Normal force
	Buoyant force (Upthrust)

2.1 Concept of field

(a) describe the forces on a mass, charge and current-carrying conductor in gravitational, electric and magnetic fields, as appropriate.

A mass can exert a force on another mass, even though there may not be any physical contact between the two masses. The same can be observed for two charges or two magnets. To explain this phenomenon where two bodies can <u>interact with each other at a distance</u>, the concept of **field** can be used.

A **field** is a region in which a body experiences a force.

- A mass produces a **gravitational field** in which another mass will always experience an attractive **gravitational force** (refer to *Gravitational Field*).
  - For example, the Moon experiences a gravitational force by the Earth because it is in the gravitational field of the Earth, and vice versa.

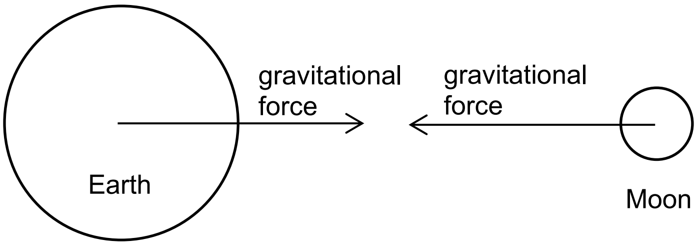


Fig. 1 Gravitational forces exerted by the Earth and Moon on each other

- A charge produces an **electric field** in which another charge will experience an **electric force** (refer to *Electric Fields*).
  - Like charges repel, while unlike charges attract.

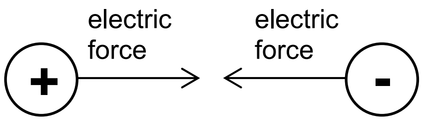


Fig. 2 Attractive electric forces between two unlike charges

Fig. 3 Repulsive electric forces between two like charges

Watch this…
https://www.youtube.com/watch?v=rPbx_XrrKLQ
This video from YouTube gives a summary of the electric field.

- Each pole of a magnet produces a **magnetic field** in which another magnetic pole will experience a **magnetic force**.
  - Two poles of the same polarity will repel each other, while those of unlike polarity will attract each other.

- A <u>current-carrying conductor</u> or a <u>moving charge</u> in a magnetic field will also experience a **magnetic force** (refer to *Electromagnetism*). The direction of the force is perpendicular to the directions of both the current or moving charge and the magnetic field, as shown in Fig. 4.

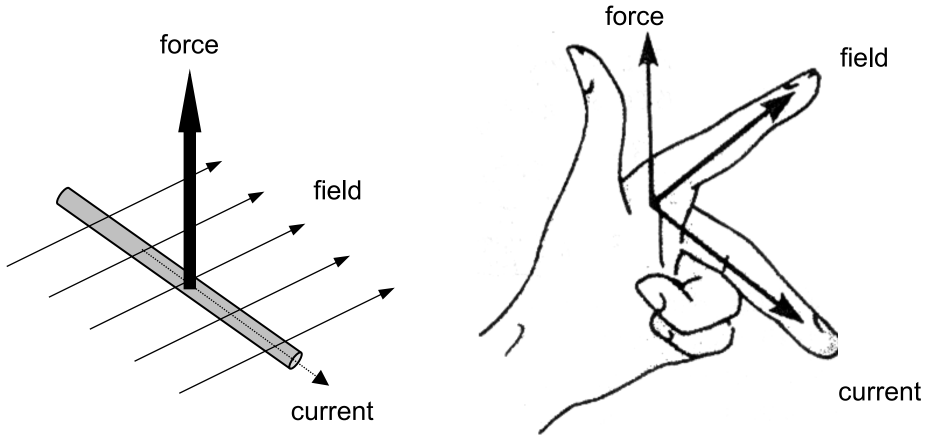


Fig. 4 Fleming's Left-Hand Rule representing the magnetic force acting on a current-carrying conductor in a magnetic field

2.2 Contact forces

When atoms of one object are too close to the atoms of another object, there will be a **contact force** between them.

- All contact forces are *electromagnetic forces* in nature.
- Examples of contact forces are pull, push, tension, compression, normal force, buoyant force (upthrust), frictional force and viscous force.

	Watch this… **https://www.youtube.com/watch?v=yE8rkG9Dw4s** This video from YouTube aims to address the question on when we touch an object, are we really in contact with that object?

2.2.1 Tension and compression

Under the influence of applied forces, every material deforms to some extent. The shape and/or volume of a material changes when external forces act on it.

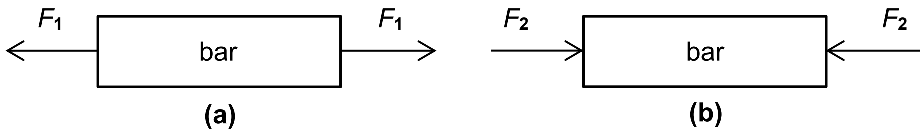


Fig. 5 (a) A bar in tension, and (b) the bar under compression

<u>Hooke's Law</u>

(c) recall and apply Hooke's law (F = kx, where k is the force constant) to new situations or to solve related problems.

Hooke's Law states that the extension (or compression) of a material is directly proportional to the force required to extend (or compress) it, <u>provided the limit of proportionality is not exceeded.</u>

Mathematically, it is written as

$$F \propto x \qquad \text{or} \qquad \boxed{F = kx}$$

where F: load or force applied to the material
x: extension in material
k: proportionality constant, or force constant (S.I. unit: N m^{-1})

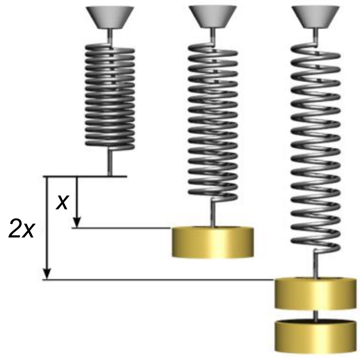


Fig. 6 A spring loaded vertically

The behaviour of stretching and compressing a material can be represented using a force-extension graph, as shown in Fig. 7.

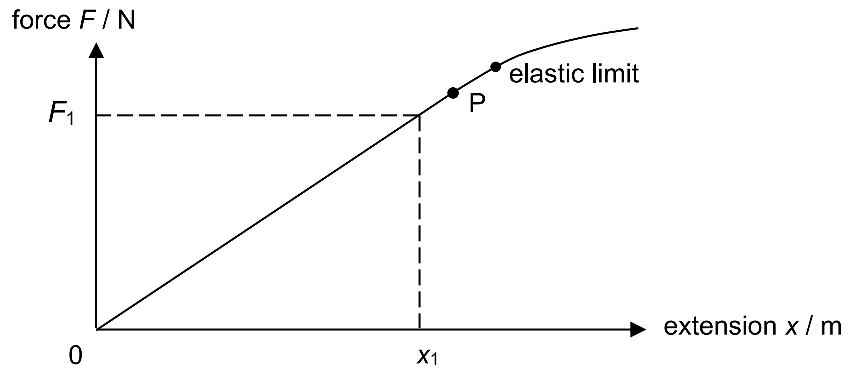


Fig. 7 A force-extension graph representing the spring-mass system in Fig. 6

Note:

- Hooke's Law applies only within the **limit of proportionality**, indicated by the point P in Fig. 7.
    - The material extends linearly and elastically with increasing force within this limit. The material returns to zero extension when the force or load is removed.
    - Beyond this limit, force is no longer proportional to extension, i.e. the material does not extend linearly. However, when the force or load is removed, the material still returns to its original length or shape if the elastic limit is not exceeded.
    - The **elastic limit** is the limit beyond which the material undergoes plastic deformation and there is a permanent extension when the load is removed.

- The **force constant *k*** can be determined from the gradient of the graph within the limit of proportionality.
    - The larger the value of k, the greater the **stiffness** of the material. This is shown in Fig. 8.

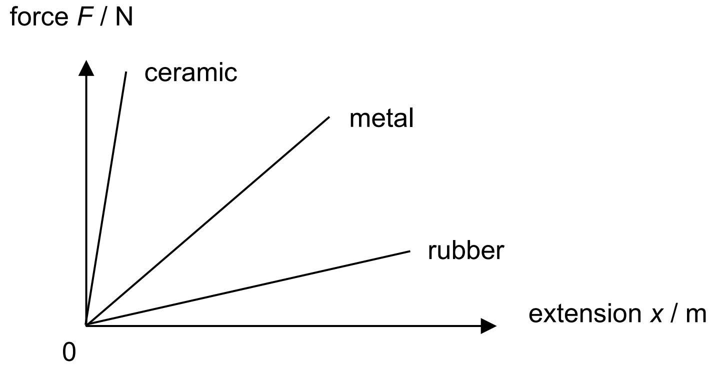
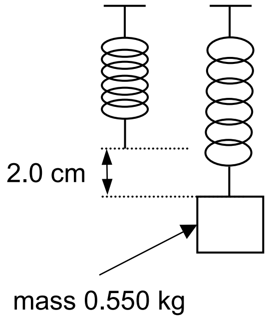


Fig. 8 The force-extension graphs of various materials, within their limits of proportionality

EXAMPLE 1
A spring is hung vertically, and an object of mass 0.550 kg is attached to the lower end of the spring. As a result, the spring is stretched by 2.0 cm.

Calculate the force constant of the spring.



Solutions

Using Hooke's law,

$F = kx$

$mg = kx$

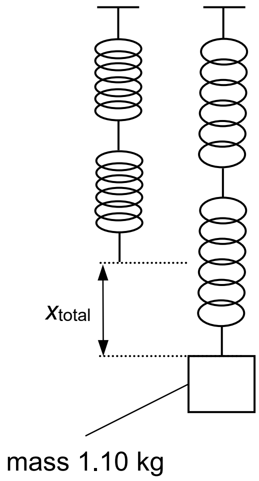
$k = \frac{mg}{x} = \frac{0.550 \times 9.81}{0.020}$

$= 270\ \mathrm{N\,m^{-1}}$

EXAMPLE 2
Two light identical springs, each of force constant 270 N m^{-1}, are connected in series. One end of the combination is secured to the ceiling and a load of mass 1.10 kg is attached to the lower end.

Assuming the limits of proportionality of the springs have not been exceeded, determine

(a) the total extension, and
(b) the force constant of the combination.



Solutions

(a) Consider one spring loaded with mass 1.10 kg, using Hooke's law,

$F = kx$

$mg = kx$

$x = \frac{mg}{k} = \frac{1.10 \times 9.81}{270}$

$= 0.0400\ \mathrm{m}$

For two identical springs connected in series, total extension, x_{total} = 0.0800 m

(b) Let k' be the force constant of the combination.

Using Hooke's law,

$F = k'x_{total}$

$mg = k'x_{total}$

$k' = \frac{mg}{x_{total}} = \frac{1.10 \times 9.81}{0.0800}$

$= 135\ \mathrm{N\ m^{-1}}$

(b) show a qualitative understanding of forces including normal force, buoyant force (upthrust), frictional force and viscous force, e.g. air resistance (knowledge of the concepts of coefficients of friction and viscosity is not required).

2.2.2 Normal force

The **normal force** is the perpendicular* force exerted by the surface of one object on the surface of another when they are in physical contact and it prevents the objects from passing through each other (See Fig. 9).

- It only exists when an object is in contact with another object.
- It is always perpendicular* to the contact surface.
- Its magnitude can change; it is not always equal to the weight of the object.

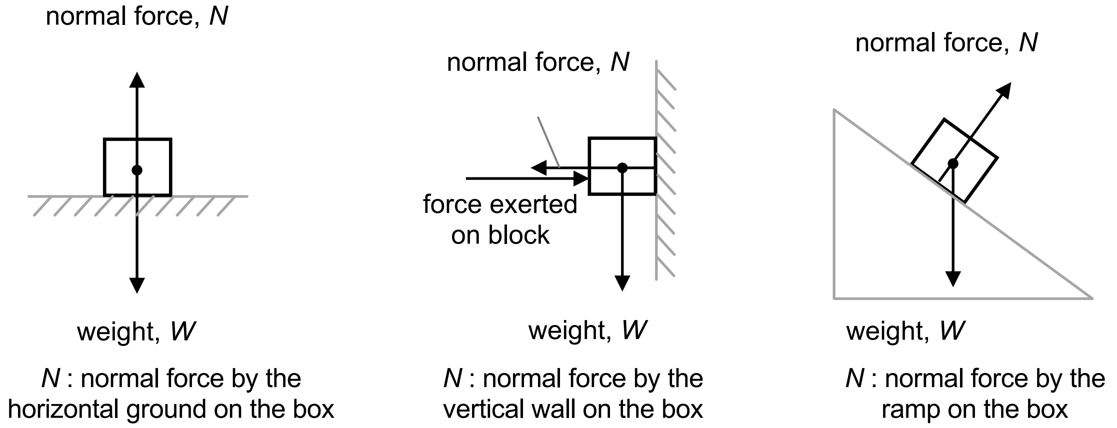


Fig. 9 Normal forces

2.2.3 Buoyant force (Upthrust)

A body that is immersed (partially or fully) in a fluid will experience a buoyant force (upthrust). The following statements describe the buoyant force (upthrust):

- It is a net upward force acting on a body when it is immersed in a fluid.
- It is exerted by the fluid.
- It is due to the difference in pressure at the top and bottom of the immersed portion of the body.

Consider a submarine submerged in water, as shown in Fig. 10.

- The arrows represent the forces which act at right angles to the surface of the submarine.
- Sideways forces cancel out.
- Upward forces on the bottom surface are larger than the downward forces on the top surface due to higher hydrostatic pressure at greater depth.
- The resultant of all of these forces is an upward force – the buoyant force (upthrust).

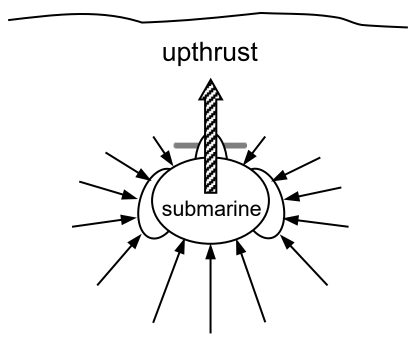


Fig. 10 A submarine submerged in water

2.2.4 Frictional force and viscous force

Frictional forces are those that resist motion, and are dissipative (i.e. to cause to lose energy irreversibly) in nature.

- The friction for an object against a surface is proportional to the normal contact force exerted by the surface on the object.
- The direction of the friction is opposite to the actual motion (kinetic friction) or the impending motion (static friction) of the object relative to the surface.
- The magnitude of frictional force experienced is independent of the area of contact between the surfaces.

Frictional forces are essential in our everyday lives. Friction makes it possible to grip and hold things, drive a car, walk and run. However, frictional forces are also responsible for the wear and tear of surfaces in contact.

When an object moves in a **fluid (i.e. liquid or gas),** it will experience resistance. This is called **viscous force or drag force**. The magnitude of this force depends on

- the speed of the object
- the shape and size of the object
- the viscosity of the fluid
- the density of the fluid

* The word ‘perpendicular’ is for two lines at right angles to each other.
The word ‘normal’ should be used for a line perpendicular to a surface.

Comparison between frictional force and viscous force

Similarities	Differences
• Both are dissipative in nature i.e. work is done in overcoming these forces. • Both are contact forces.	• Frictional forces exist even when solids are at rest, whereas viscous forces exist only when there is relative motion between solid and fluid i.e. the solid is moving relative to the fluid. • Frictional forces between solids are independent of velocity, whereas viscous forces change with velocity.

3 CENTRE OF GRAVITY

(f) show an understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity.

The **centre of gravity** of a body is the single point at which the entire weight of the body can be considered to act.

- If a body is symmetrical and of uniform density, then the centre of gravity is at the geometrical centre of the body.
- An object will always balance if you support it at its centre of gravity.

- For an irregularly shaped object, or one of non-uniform density, the centre of gravity can be found by using a plumbline.

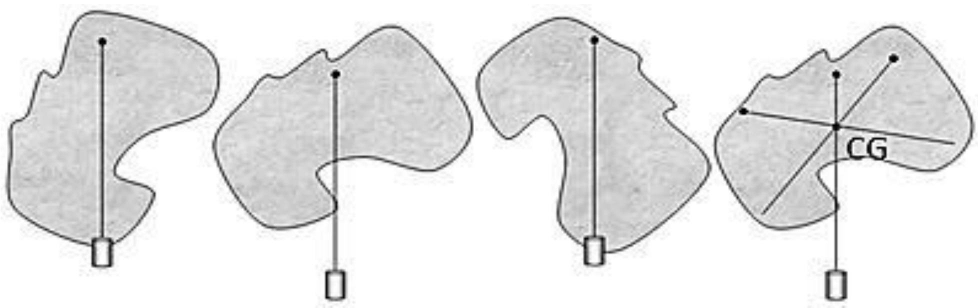


Fig. 11 Determining the centre of gravity of an irregularly shaped object

4 MOMENT AND TORQUE

(d) define and apply the moment of a force and the torque of a couple.

4.1 Moment of a force

Besides causing a change in the translational motion of a body, a force can also produce a turning effect or rotation of the body about a pivot. The turning effect of a force is called its ***moment***.

> ***The moment of a force about a pivot is the product of the force and the perpendicular distance from the pivot to the line of action of the force.***

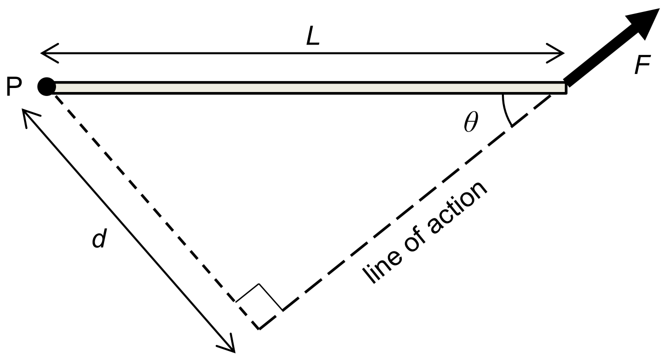


Fig. 12 A force *F* acting on a light rod of length *L*, at a perpendicular distance of *d* from the pivot P.

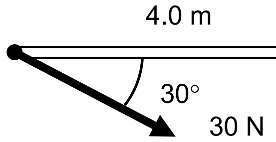
The line of action of the force refers to the line along which the force *F* is acting.

Length *d* refers to the perpendicular distance between the pivot P and the line of action of the force. In Fig. 12, $d = L\sin\theta$.

Thus, the anti-clockwise moment of force *F* about P $= F \times d = FL\sin\theta$

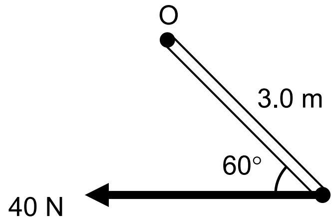
EXAMPLE 3
Calculate the moment of each of the following forces acting on a rod about point O.

(i)



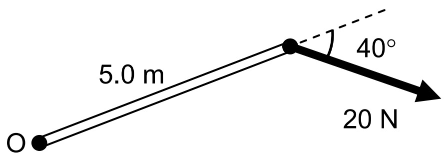
Moment of force about O $= 30 \times 4.0 \sin 30^\circ$
$= 60$ N m
(anti-clockwise)

(ii)



Moment of force about O $= 40 \times 3.0 \sin 60^\circ$
$= 100$ N m
(clockwise)

(iii)



Moment of force about O $= 20 \times 5.0 \sin 40^\circ$
$= 64$ N m
(clockwise)

4.2 Couple

(e) show an understanding that a couple is a pair of forces which tends to produce rotation only.

A **couple** is a pair of *equal* and *opposite* forces *acting on a body*, and the *lines of action* of these forces *do not coincide*, as shown in Fig. 13.

- As the forces are equal and opposite, the resultant force is zero and so there is ***no linear acceleration***. In other words, ***a couple produces only rotation***.

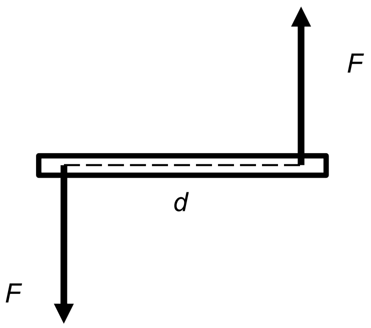


Fig. 13 A couple acting on a rod

4.3 Torque

Torque is the turning effect of a couple.

Turning effect of a couple or Torque   = **one of the forces × the perpendicular distance between the two forces**

If we take moments about the centre of a rod (like the one in Fig. 13),

$$\text{torque, } \tau = F\left(\frac{d}{2}\right) + F\left(\frac{d}{2}\right)$$

$$\tau = Fd \quad \text{(anti-clockwise moment)}$$

Taking moments about any arbitrary point, e.g. A in Fig. 14,

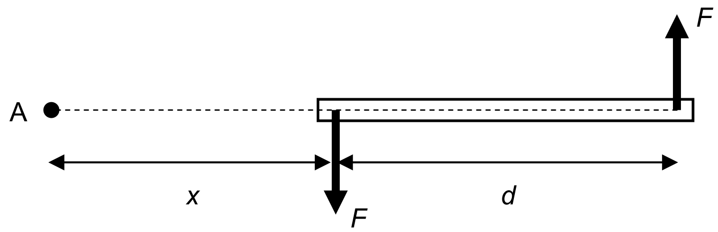


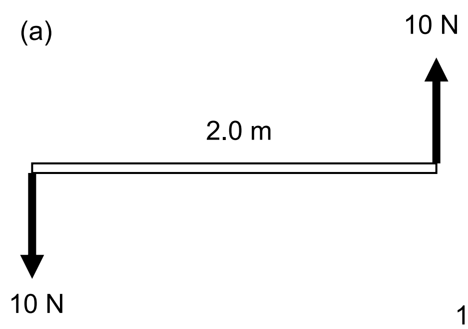
Fig. 14 Taking moments of the forces *F* about an arbitrary point A

Total anti-clockwise moment = (F)(d + x) – (F)(x) = Fd

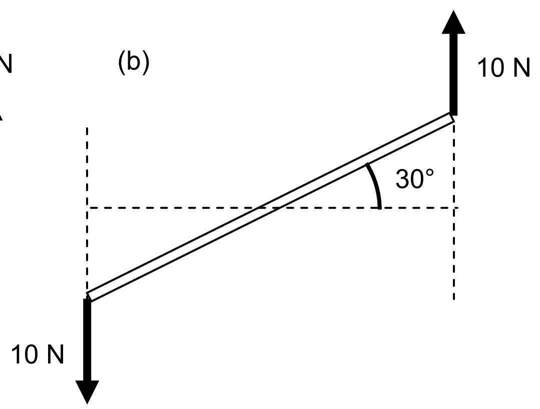
EXAMPLE 4

Calculate the torque acting on the rod of length 2.0 m.

(a)



(b)



Solutions

(a) Torque = $F\,d$ = 10 × 2.0 = 20 N m (anti-clockwise)

(b) Perpendicular distance between 10 N forces = 2.0 cos 30°

Torque = 10 × 2.0 cos 30° = 17 N m (anti-clockwise)

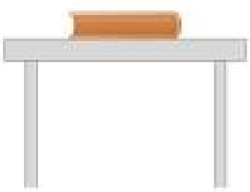
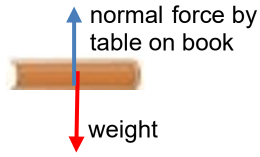
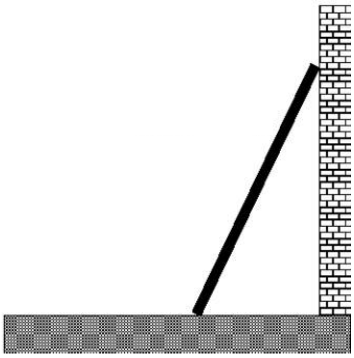
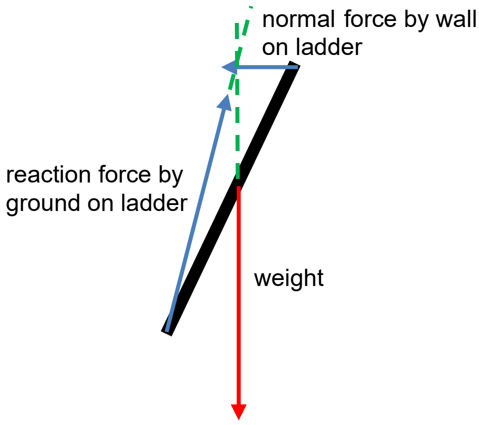
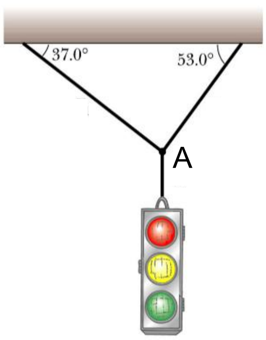
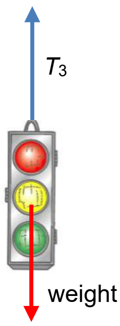
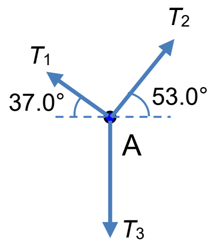
5 FREE-BODY DIAGRAMS

Free-body diagrams (FBD) of a body are diagrams used to show the points of action and the direction of all external forces acting on it in a given situation.

The direction of the arrow reveals the direction in which the force is acting. The arrow is usually drawn such that it is directed away from the body, with its tail starting from the point of action on the body.

These forces are usually labelled as "*(type of)* force (acting on the *body)* by the *external agent*". For example, the (gravitational) force (acting on the body) by the Earth is also known as the weight of the body.

For each of the following cases, draw the FBD of the following object(s):

1) An egg is free-falling from a nest in a tree. Neglect air resistance.	
2) A book resting on a table-top. 	 *Note: The normal force should be exactly on the line of action of weight, but is shown slightly separated to enable viewing.*
3) A uniform ladder leaning against a smooth vertical wall and a rough horizontal ground. 	
4) A suspended traffic light 	traffic light  point A 

(g) apply the principle of moments to new situations or to solve related problems.
(h) show an understanding that, when there is no resultant force and no resultant torque, a system is in equilibrium.
(i) use free-body diagrams and vector triangles to represent forces on bodies that are in rotational and translational equilibrium.

6.1 Conditions for equilibrium

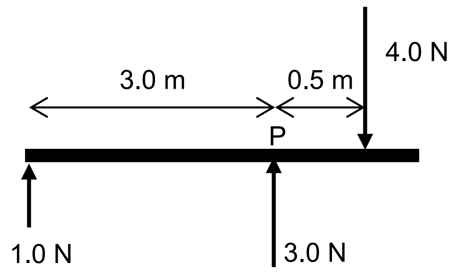
Forces cause objects to move in a linear direction and to rotate about an axis.
For any object to be in equilibrium, the following two conditions must be fulfilled:

1. There must be **no resultant force** acting on the object, i.e. $\sum F = 0$.
   - The vector sum of all the forces acting on the object must be zero.
   - The object is in **translational equilibrium** – there is no linear acceleration.

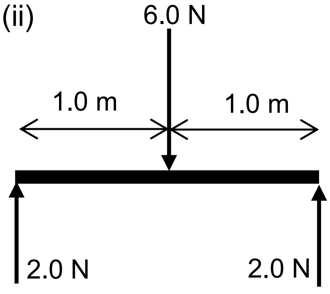
2. There must be **no resultant torque** on the object about any point, i.e. $\sum \tau = 0$.
   - The vector sum of all torques on the object about any point must be zero.
   - **Principle of moments**: the sum of clockwise moments about any point equals the sum of the anticlockwise moments about the same point.
   - The object is in **rotational equilibrium**.

EXAMPLE 5
(a) Determine if the following uniform rods are in equilibrium.

(i)



(ii)

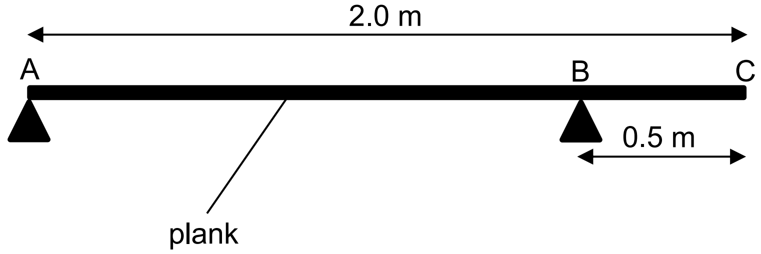


Solutions

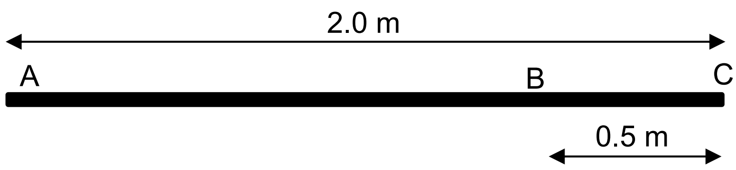
(a)
(i) The vector sum of forces is equal to zero. The rod is in translational equilibrium.
Taking moments about P,
total clockwise moment = (4.0)(0.5) + (1.0)(3.0)
= 5.0 N m
total anti-clockwise moment = 0
The rod will rotate in a clockwise direction. It is not in rotational equilibrium.

(ii) Resultant force = 6.0 – 2.0 – 2.0 = 2.0 N (downward)
It is not in translational equilibrium.
Taking moments about its centre of gravity,
total clockwise moment = (2.0)(1.0) = 2.0 N m
total anti-clockwise moment = (2.0)(1.0) = 2.0 N m
Resultant torque = 0
The rod will not rotate. It is in rotational equilibrium.

(b) A uniform plank of mass 40 kg and length 2.0 m is held horizontally by two identical supports at points A and B. Point B is 0.5 m away from the end of the beam at point C as shown in the figure below.



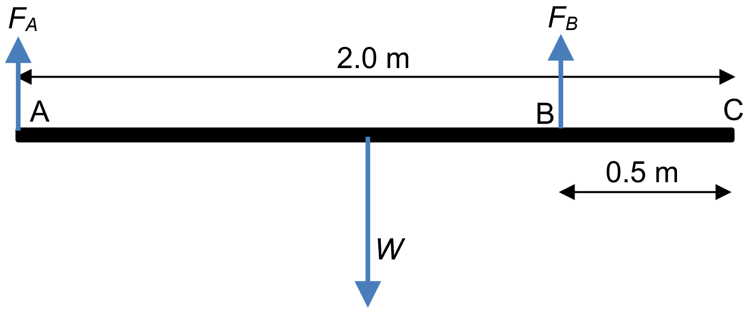
(i) On the figure below, draw the forces acting on the plank.



(ii) Calculate the force provided by the support at points A and B.

Solutions

(b)
(i)



(ii) There is no resultant torque on the object about any point.

Taking moments about A,
clockwise moment = anti-clockwise moment

$W \times 1.0 = F_B \times 1.5$

$(40 \times 9.81) \times 1.0 = F_B \times 1.5$

$$F_B = \frac{(40 \times 9.81) \times 1.0}{1.5} = 261.6 \approx 262 \text{ N}$$

There is no resultant force on the object.

$\sum F = 0 \rightarrow F_A + F_B = W$

$F_A = W - F_B$

$F_A = (40 \times 9.81) - 261.6 = 130.8 \approx 131 \text{ N}$

6.2 Vector diagrams of forces

Fig. 15 shows a free body diagram of a point mass, and how the four forces can be represented in a vector diagram. Since these forces form a closed polygon, it means that there is no net force acting on the point mass, and thus the point mass is in equilibrium.

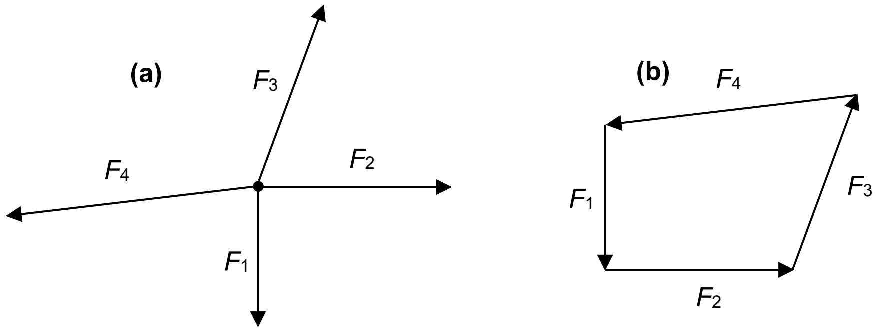


Fig. 15 (a) A free body diagram of a point mass, and
(b) a vector diagram showing a closed polygon

Fig. 16 shows the same point mass with one of the forces removed. The vector diagram shows that there is a net force due to the remaining three forces, thus the point mass is not in equilibrium.

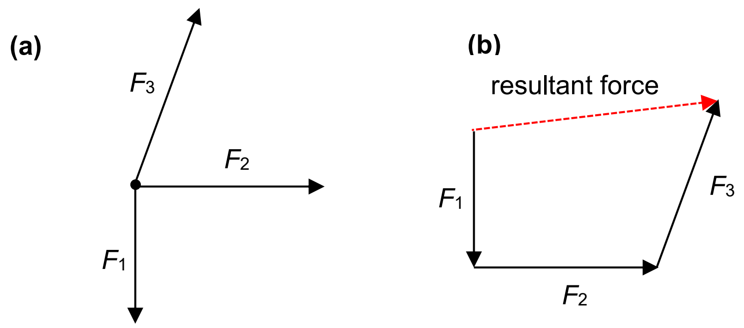


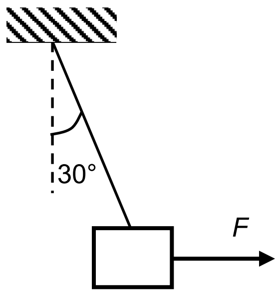
Fig. 16 (a) The same point mass with three forces acting on it, and
(b) a vector diagram showing a net force due to the three forces.

6.3 General Guidelines for solving problems that involve an object in equilibrium

1. Draw a force diagram (free-body diagram) of the body concerned.
   - Indicate **all external forces** acting **on** the body.
2. Resolve all the forces along two suitable perpendicular directions.
3. Apply the condition of **translational equilibrium** for each of the perpendicular directions such that net force is zero.
   - $\sum F_x = 0$, $\sum F_y = 0$.
4. Apply the condition of **rotational equilibrium** (i.e. $\sum \tau = 0$) about any convenient pivot point.
   - A point at which an unknown force is acting is chosen as a pivot. This will help simplify calculations as that particular unknown force is ignored.
5. Solve for unknown quantities.

EXAMPLE 6

A 30 N block is suspended by a rope of negligible mass. A horizontal force *F* is applied until the rope makes an angle of 30° with the vertical.



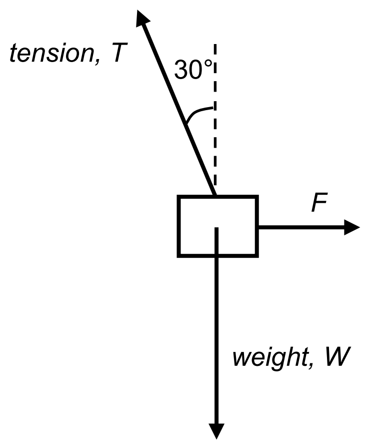
Calculate

(i) the tension in the rope,

(ii) the magnitude of *F* required to hold the block in equilibrium in that position.

<u>Solutions</u>

Method 1: Resolving forces



Vertically,

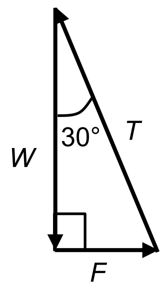
$$W = T\cos 30° \Rightarrow T = \frac{30}{\cos 30°} = 35 \text{ N}$$

Horizontally,

$$F = T\sin 30° = \frac{30}{\cos 30°} \times \sin 30° = 17 \text{ N}$$

Method 2: Drawing a vector diagram

- For 3 forces, a vector triangle is drawn.



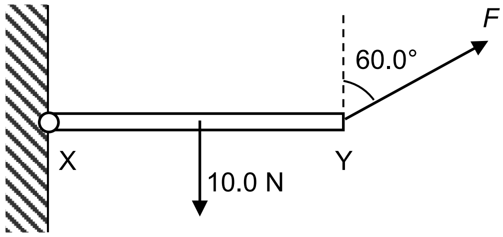
$$\cos 30° = \frac{W}{T} \Rightarrow T = \frac{30}{\cos 30°} = 35 \text{ N}$$

$$\tan 30° = \frac{F}{W}$$

$$\Rightarrow F = W\tan 30° = 30 \times \tan 30° = 17 \text{ N}$$

EXAMPLE 7

A uniform rod XY of weight 10.0 N is freely hinged to a wall at X. It is held horizontal by a force *F* acting from Y at an angle of 60.0° to the vertical as shown in the diagram below.



Determine
(a) the magnitude of *F*.
(b) the magnitude and direction of the reaction force by the hinge on the rod at X.

Solutions

Let the length of the rod XY be *L*.
Let the reaction force exerted by the hinge on the rod be *R*.

(a) Taking moments about X,
Total anti-clockwise moments = total clockwise moments

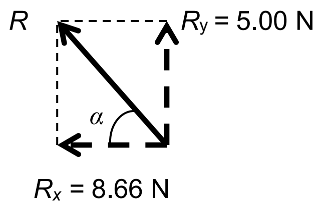
$$(F\cos 60.0°)(L) = (10.0)\left(\frac{L}{2}\right)$$

F = 10.0 N

(b) Let *R* be the reaction force by the hinge on the rod.
Let R_x be the horizontal component and R_y be the vertical component.
The rod is in translational equilibrium.
The weight does not have a horizontal component.
Force *F* has a rightward horizontal component.
Therefore the direction of R_x must be leftward.

Horizontally: $\sum F_x = 0$
$R_x = F \sin 60.0°$
$R_x = 8.66$ N
Vertically: $\sum F_y = 0$
$R_y + F\cos 60.0° = 10.0$
$R_y = 5.00$ N

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{8.66^2 + 5.00^2}$$
$$= 10.0 \text{ N}$$
$$\tan\alpha = \frac{R_y}{R_x}$$
$$\alpha = 30.0° \text{ above the horizontal}$$



Therefore, <u>*R* has a magnitude of 10.0 N, directed at 30.0° above the horizontal</u>, as shown in the diagram above.

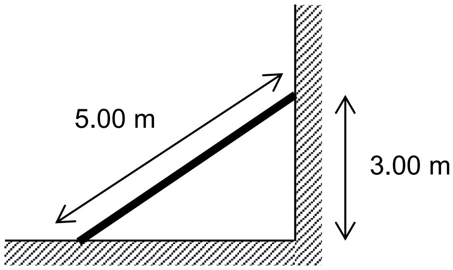
6.4 Special Case: 3 non-parallel forces in equilibrium

When there are only three non-parallel forces acting on a body in equilibrium, the line of actions of these forces must intersect at a single point spatially.

- This can be proven by considering moments about the point of intersection; the perpendicular distance between any force and the pivot is zero.
- Recall that these 3 forces must form a vector triangle.

EXAMPLE 8

A uniform ladder of length 5.00 m and mass 2.00 kg leans against a smooth wall and is supported by a rough ground. Its upper end is at a height of 3.00 m above the ground.



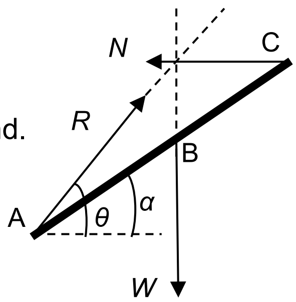
Calculate the force exerted by the smooth wall on the ladder and the force exerted by the rough ground on the ladder.

<u>Solutions</u>

Let θ be the angle between force R and the rough ground.

At equilibrium: $\left.\begin{aligned} W &= R\sin\theta \\ N &= R\cos\theta \end{aligned}\right\}$ translational equilibrium

Let α be the angle between the ladder and the rough ground.



Taking moments about A, at equilibrium,

$\left.\begin{aligned} &N \times 3.00 = 2.00 \times 9.81 \times 2.50 \cos\alpha \\ &N = 13.08\text{ N} \approx 13.1\text{ N} \end{aligned}\right\}$ rotational equilibrium

Consider forces acting vertically,

$R\sin\theta = W = 19.62$ N

Consider forces acting horizontally,

$R\cos\theta = N = 13.08$ N

Therefore $\tan\theta = \dfrac{19.62}{13.08}$

$\theta = 56.3°$

Magnitude of $R = \sqrt{(R\cos\theta)^2 + (R\sin\theta)^2} = \sqrt{13.08^2 + 19.62^2}$

$= 23.6$ N

Force exerted by smooth wall on the ladder, N is 13.1 N to the left.

<u>Force exerted by rough ground on the ladder, R is 23.6 N, 56.3° above the horizontal</u>, as shown in the diagram.