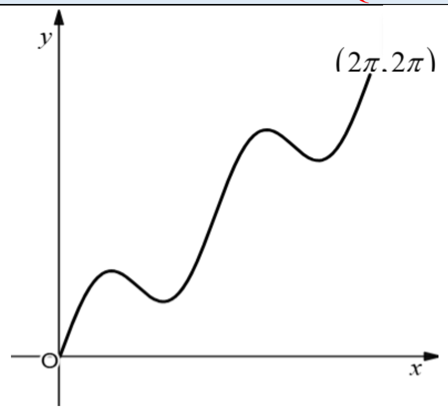
SOLUTIONS:

Qn	Source: VJC Promo 2025 Qn 1	

1



The shell method uses vertical strips, each of which gives rise to a distinct shell. The disc method involves horizontal strips which overlap due to the turning points. As such, turning points need to be determined and the resulting volume is a combination of adding and subtracting volumes formed by different regions under the graph.

$$V=\int_0^{2\pi} 2\pi xy\ \mathrm{d}x$$

$$=2\pi\int_0^{2\pi} x(x+\sin 2x)\ \mathrm{d}x$$

$=499.77...$ which is just under 500 (shown)

Qn	**Suggested Solution**	

1

Perimeter of $C=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\sqrt{\left(4a\cos 2\theta\right)^2+\left(-8a\sin 2\theta\right)^2}\ \mathrm{d}\theta$

$$=4a\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\sqrt{\cos^2 2\theta+4\sin^2 2\theta}\ \mathrm{d}\theta$$

$$=4a\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\sqrt{1+3\sin^2 2\theta}\ \mathrm{d}\theta$$

$=8a\int_0^{\frac{\pi}{4}}\sqrt{1+3\sin^2 2\theta}\ \mathrm{d}\theta$ (as $\sqrt{1+3\sin^2 2\theta}$ is a even fn)

Let $2\theta=\omega$ $\quad=8a\int_0^{\frac{\pi}{2}}\sqrt{1+3\sin^2 w}\ \frac{1}{2}\mathrm{d}w$

$$=4a\int_0^{\frac{\pi}{2}}\sqrt{1+3\sin^2 w}\ \mathrm{d}w$$

Perimeter of $E=\int_{-\pi}^{\pi}\sqrt{(-2a\sin\phi)^2+(a\cos\phi)^2}\ \mathrm{d}\phi$

$$=a\int_{-\pi}^{\pi}\sqrt{3(\sin\phi)^2+1}\ \mathrm{d}\phi$$

	$= 2a\int_0^{\pi} \sqrt{3(\sin\phi)^2 + 1}\ \mathrm{d}\phi$ (as $\sqrt{1+3\sin^2\phi}$ is a even fn) $= 4a\int_0^{\frac{\pi}{2}} \sqrt{3(\sin\phi)^2 + 1}\ \mathrm{d}\phi$ (as $\sin\phi = \sin(\phi + \frac{\pi}{2})$ for $0 < \phi < \frac{\pi}{2}$) Thus perimeter of C = perimeter of E.	

Qn	Suggested Solution	
2	Let $\omega = \cos\frac{2\pi}{9} + \mathrm{i}\sin\frac{2\pi}{9}$. **(a) (i)** $z^9 - 1 = 0 \Rightarrow z^9 = 1$ $\Rightarrow z = \mathrm{e}^{\mathrm{i}\left(\frac{2k\pi}{9}\right)}, k = 0,1,2,\ldots,8$ $= \omega^k, k = 0,1,2,\ldots,8$ since $\omega = \mathrm{e}^{\mathrm{i}\left(\frac{2\pi}{9}\right)}$ $\Rightarrow z = \omega^0\left(=1\right), \omega, \omega^2, \omega^3, \omega^4, \omega^5, \omega^6, \omega^7, \omega^8$ **(ii)** $1 + \omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6 + \omega^7 + \omega^8 = \frac{1-\omega^9}{1-\omega} = 0$ since $\omega^9 = 1, \omega \neq 1$. **(b) (i)** $w^{-1} = w^8$ $w^{-2} = w^7$ (Since $w^9 = 1$) $w^{-3} = w^6$ $w^{-4} = w^5$ $\omega + \omega^2 + \omega^3 + \omega^4 + \omega^{-4} + \omega^{-3} + \omega^{-2} + \omega^{-1}$ $= \omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6 + \omega^7 + \omega^8$ $= -1$ **(ii)** $z_2 = \omega^{-1} + \omega^{-2} + \omega^{-3} + \omega^{-4} = \omega^* + \left(\omega^2\right)^* + \left(\omega^3\right)^* + \left(\omega^4\right)^* = z_1^*$. From **(b)(i)**, $z_1 + z_2 = -1 \Rightarrow z_1 + z_1^* = -1$ $\Rightarrow 2\,\mathrm{Re}\left(z_1\right) = -1$ $\Rightarrow \mathrm{Re}\left(\omega + \omega^2 + \omega^3 + \omega^4\right) = -\frac{1}{2}$ $\Rightarrow \mathrm{Re}\left(\omega\right) + \mathrm{Re}\left(\omega^2\right) + \mathrm{Re}\left(\omega^3\right) + \mathrm{Re}\left(\omega^4\right) = -\frac{1}{2}$ $\Rightarrow \cos\frac{2\pi}{9} + \cos\frac{4\pi}{9} + \cos\frac{6\pi}{9} + \cos\frac{8\pi}{9} = -\frac{1}{2}$ **(iii)** $z_1 z_2 = \left(\omega + \omega^2 + \omega^3 + \omega^4\right)\left(\omega^{-1} + \omega^{-2} + \omega^{-3} + \omega^{-4}\right)$ $\left(\omega + \omega^2 + \omega^3 + \omega^4\right)\left(\omega^{-1} + \omega^{-2} + \omega^{-3} + \omega^{-4}\right)$ $= 4 + 3\left(\omega + \omega^{-1}\right) + 2\left(\omega^2 + \omega^{-2}\right) + \omega^3 + \omega^{-3}$	

	$=4+3\left(\omega+\omega^*\right)+2\left(\omega^2+\left(\omega^2\right)^*\right)+\omega^3+\left(\omega^3\right)^*$ $=4+3\left(2\operatorname{Re}(\omega)\right)+2\left(2\operatorname{Re}\left(\omega^2\right)\right)+2\operatorname{Re}\left(\omega^3\right)$ $=4+6\cos\frac{2\pi}{9}+4\cos\frac{4\pi}{9}+2\cos\frac{6\pi}{9}$	

Qn	Suggested Solution	
3	$\begin{pmatrix} 0 & -k & 1 & 2 \\ 1 & k & 0 & 0 \\ 2k & k & k & 2k \\ -1 & 0 & -1 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & k & 0 & 0 \\ 0 & k & -1 & -2 \\ 0 & -2k^2+k & k & 2k \\ 0 & k & -1 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & k & 0 & 0 \\ 0 & k & -1 & -2 \\ 0 & 0 & 1-k & 2-2k \\ 0 & 0 & 0 & 0 \end{pmatrix}$ If dim(N) = 1 and dim(R) = 3, $k \neq 0$ and $k \neq 1$. Case of dim(N) = 0 and dim(R) = 4 is not possible as there are at most 3 pivots.	
	(i) $\left(\begin{array}{cccc\|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{array}{c} x_1 = -x_2 \\ x_2 = x_3 + 2x_4 \end{array} \Rightarrow \mathbf{x} = \lambda\begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \mu\begin{pmatrix} -2 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \quad \lambda, \mu \in \mathbb{R}.$ A basis for N is $\left\{\begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 2 \\ 0 \\ 1 \end{pmatrix}\right\}$. A basis for R is $\left\{\begin{pmatrix} 0 \\ 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}\right\}$. Since $\begin{pmatrix} -2 \\ 2 \\ 0 \\ 1 \end{pmatrix} \notin R$. A basis for $N \cap R$ is $\left\{\begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}\right\}$.	
	(ii) Note that $\mathbf{A}(\mathbf{x}) \in$ Range space of T Hence $\mathbf{A}(\mathbf{x}) = \lambda\begin{pmatrix} 0 \\ 1 \\ 2 \\ -1 \end{pmatrix} + \mu\begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ where $\lambda, \mu \in \mathbb{R}$	

	$\mathbf{A}(\mathbf{Ax})=\mathbf{A}\left(\lambda\begin{pmatrix}0\\1\\2\\-1\end{pmatrix}+\mu\begin{pmatrix}-1\\1\\1\\0\end{pmatrix}\right)$ $=\lambda\mathbf{A}\begin{pmatrix}0\\1\\2\\-1\end{pmatrix}+\mu\mathbf{A}\begin{pmatrix}-1\\1\\1\\0\end{pmatrix}$ $=\lambda\begin{pmatrix}-1\\1\\1\\0\end{pmatrix}+\mathbf{0}$ $\mathbf{A}^3\mathbf{x}=\mathbf{A}\left(\lambda\begin{pmatrix}-1\\1\\1\\0\end{pmatrix}\right)=\mathbf{0}$. Hence $m=3$.	

	Source 2025 ACJC Promo Qn 8
4(i)	From diagram, $B\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$. $S=OP^2+AP^2+BP^2$ $=r^2+(r\cos\theta-1)^2+(r\sin\theta)^2+\left(r\cos\theta-\frac{1}{2}\right)^2$ $+\left(r\sin\theta-\frac{\sqrt{3}}{2}\right)^2$ $=r^2+r^2-2r\cos\theta+1+r^2-r\cos\theta-r\sqrt{3}\sin\theta+\frac{1}{4}+\frac{3}{4}$ $=3r^2-\left[3\cos\theta+\sqrt{3}\sin\theta\right]r+2$ $\frac{\partial S}{\partial r}=6r-\left[3\cos\theta+\sqrt{3}\sin\theta\right]$ and $\frac{\partial S}{\partial\theta}=-\left[-3\sin\theta+\sqrt{3}\cos\theta\right]r$ For stationary points, $\frac{\partial S}{\partial r}=0$ and $\frac{\partial S}{\partial\theta}=0$. $-\left[-3\sin\theta+\sqrt{3}\cos\theta\right]r=0$

As $r>0$: $3\sin\theta=\sqrt{3}\cos\theta$

$$\tan\theta=\frac{1}{\sqrt{3}}\Rightarrow\theta=\frac{\pi}{6}$$

$$6r-\left[3\cos\theta+\sqrt{3}\sin\theta\right]=0$$

$$6r=3\cos\frac{\pi}{6}+\sqrt{3}\sin\frac{\pi}{6}$$

$$6r=\frac{3\sqrt{3}}{2}+\frac{\sqrt{3}}{2}$$

$$r=\frac{\sqrt{3}}{3}$$

Thus, the centroid $P(x,y)=P\left(\frac{\sqrt{3}}{3}\cos\frac{\pi}{6},\frac{\sqrt{3}}{3}\sin\frac{\pi}{6}\right)=P\left(\frac{1}{2},\frac{\sqrt{3}}{6}\right)$.

To prove that this gives a minimum point at the centroid:

$$\frac{\partial^2 S}{\partial r^2}=6$$

$$\begin{aligned}\frac{\partial^2 S}{\partial\theta\partial r}&=3\sin\theta-\sqrt{3}\cos\theta\\&=3\sin\frac{\pi}{6}-\sqrt{3}\cos\frac{\pi}{6}\\&=0\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 S}{\partial\theta^2}&=\left[3\cos\theta+\sqrt{3}\sin\theta\right]r\\&=\left[3\cos\frac{\pi}{6}+\sqrt{3}\sin\frac{\pi}{6}\right]\left(\frac{\sqrt{3}}{3}\right)\\&=\left[2\sqrt{3}\right]\left(\frac{\sqrt{3}}{3}\right)\\&=2\end{aligned}$$

As $\frac{\partial^2 S}{\partial r^2}\frac{\partial^2 S}{\partial\theta^2}-\left(\frac{\partial^2 S}{\partial\theta\partial r}\right)^2=12-0=12>0$ and $\frac{\partial^2 S}{\partial r^2}=6>0$ (or equivalently, $\frac{\partial^2 S}{\partial\theta^2}=2>0$), then this is a minimum point by the second derivative test.

Alternatively,
Provided that S is first expressed in terms of r and θ as asked, this problem can be solved by recasting S in Cartesian form and finding $S_x,S_y,S_{xx},S_{xy},S_{yy}$ respectively. This is left as an exercise to the reader.

(ii)

$$G\left(\frac{0.5+1}{2},\frac{\sqrt{3}}{4}\right)=G\left(\frac{3}{4},\frac{\sqrt{3}}{4}\right)$$

	$\Rightarrow m_{OG}=\dfrac{\sqrt{3}}{3}$ $M\left(\dfrac{1}{2},\dfrac{\sqrt{3}}{6}\right)=M\left(\dfrac{3}{6},\dfrac{\sqrt{3}}{6}\right)$ $\Rightarrow m_{OM}=\dfrac{\sqrt{3}}{3}$ As both lines have the same gradient with O as a common point, O, M and G are collinear. **Alternatively,** A "vectors" argument can be used to show that $\overrightarrow{OM}=\dfrac{2}{3}\overrightarrow{OG}$ with O as a common point, or use a "polar coordinates" argument to say that the polar angle of M and G is $\dfrac{\pi}{6}$.

Qn	Suggested Solution	
5	Let $\mathbf{A}$ be a Markov matrix, then the sum of entries of the rows in $\mathbf{A}^T$ equal to 1. $\therefore \mathbf{A}^T\begin{pmatrix}1\\1\\\vdots\\1\end{pmatrix}=\begin{pmatrix}1\\1\\\vdots\\1\end{pmatrix}\Rightarrow \mathbf{A}^T$ has eigenvalue 1 and corresponding eigenvector $\begin{pmatrix}1\\1\\\vdots\\1\end{pmatrix}$. $\mathbf{A}$ and $\mathbf{A}^T$ have same eigenvalues since $\lvert\mathbf{A}-\lambda\mathbf{I}\rvert=\lvert\mathbf{A}^T-\lambda\mathbf{I}\rvert$. Thus, $\mathbf{A}$ has eigenvalue 1. (i) The matrix representing the migration, $\mathbf{M}=\begin{pmatrix}0.95&0.02&0.05\\0.03&0.90&0.05\\0.02&0.08&0.90\end{pmatrix}$. (ii) The characteristic polynomial: $\det(\mathbf{M}-\lambda\mathbf{I})=\det\begin{pmatrix}0.95-\lambda&0.02&0.05\\0.03&0.90-\lambda&0.05\\0.02&0.08&0.90-\lambda\end{pmatrix}$. $=(0.95-\lambda)\begin{vmatrix}0.90-\lambda&0.05\\0.08&0.90-\lambda\end{vmatrix}-0.03\begin{vmatrix}0.02&0.05\\0.08&0.90-\lambda\end{vmatrix}+0.02\begin{vmatrix}0.02&0.05\\0.90-\lambda&0.05\end{vmatrix}$ $=(0.95-\lambda)(0.806-1.8\lambda+\lambda^2)-0.03(0.014-0.02\lambda)+0.02(-0.044+0.05\lambda)$ $=0.7644-2.5144\lambda+2.75\lambda^2-\lambda^3$ (iii) After 1 day, the distribution	

$$= \mathbf{M}\begin{pmatrix}0.35\\0.35\\0.3\end{pmatrix} = \begin{pmatrix}0.95 & 0.02 & 0.05\\0.03 & 0.90 & 0.05\\0.02 & 0.08 & 0.90\end{pmatrix}\begin{pmatrix}0.35\\0.35\\0.3\end{pmatrix}$$

$$= \begin{pmatrix}0.355\\0.341\\0.305\end{pmatrix}$$

The distribution of caravans after a day is 0.355, 0.341 and 0.305 for the airport, Dandeton and Morningong respectively.

Using GC, eigenvalues are 0.84, 0.91 and 1.

When $\lambda = 0.84$, $\mathbf{M} - \lambda\mathbf{I} = \begin{pmatrix}0.11 & 0.02 & 0.05\\0.03 & 0.06 & 0.05\\0.02 & 0.08 & 0.06\end{pmatrix}$

Using GC, an associated eigenvector is $\begin{pmatrix}1\\2\\-3\end{pmatrix}$.

When $\lambda = 0.91$, $\mathbf{M} - \lambda\mathbf{I} = \begin{pmatrix}0.04 & 0.02 & 0.05\\0.03 & -0.01 & 0.05\\0.02 & 0.08 & -0.01\end{pmatrix}$

Using GC, an associated eigenvector is $\begin{pmatrix}-3\\1\\2\end{pmatrix}$.

When $\lambda = 1$, $\mathbf{M} - \lambda\mathbf{I} = \begin{pmatrix}-0.05 & 0.02 & 0.05\\0.03 & -0.10 & 0.05\\0.02 & 0.08 & -0.10\end{pmatrix}$

Using GC, an associated eigenvector is $\begin{pmatrix}15\\10\\11\end{pmatrix}$.

Diagonalising, $\mathbf{M} = \mathbf{QDQ}^{-1}$ where $\mathbf{Q} = \begin{pmatrix}1 & -3 & 15\\2 & 1 & 10\\-3 & 2 & 11\end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix}0.84 & 0 & 0\\0 & 0.91 & 0\\0 & 0 & 1\end{pmatrix}$.

$$\mathbf{M}^n\begin{pmatrix}0.35\\0.35\\0.3\end{pmatrix}=\mathbf{Q}\begin{pmatrix}0.84^n&0&0\\0&0.91^n&0\\0&0&1^n\end{pmatrix}\mathbf{Q}^{-1}\begin{pmatrix}0.35\\0.35\\0.3\end{pmatrix}$$

$$=\mathbf{Q}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}\mathbf{Q}^{-1}\begin{pmatrix}0.35\\0.35\\0.3\end{pmatrix}\quad \text{as } n\to\infty$$

$$=\begin{pmatrix}0.417\\0.278\\0.306\end{pmatrix}$$

The distribution of caravans on the long run is 0.417, 0.278 and 0.306 for the Airport, Dandeton and Morningong respectively. This will pose a problem to the space constraints in the Airport.

Qn	Suggested Solution	
6	(i) $X\sim U(0,m)\quad \mathrm{f}(x)=\dfrac{1}{m}$ For $w\in(0,m)$ $\mathrm{P}(W<w)=\mathrm{P}\left(\max\{X_1,\ X_2,\ \dots,\ X_n\}<w\right)$ $=\mathrm{P}\left(X_1<w, X_2<w, \dots, X_n<w\right)$ $=\left[\mathrm{P}\left(X<w\right)\right]^n$ $=\left[\dfrac{w}{m}\right]^n$ (ii) For $w\in(0,m)$ $\mathrm{f}(w)=\dfrac{\mathrm{d}}{\mathrm{d}w}\mathrm{P}(W<w)=n\left[\dfrac{w}{m}\right]^{n-1}\dfrac{1}{m}=\dfrac{nw^{n-1}}{m^n}$ (iii)	

$$\mathrm{E}(W)=\int_0^m w\cdot\frac{nw^{n-1}}{m^n}\ \mathrm{d}w$$
$$=\frac{n}{m^n}\int_0^m w^n\ \mathrm{d}w$$
$$=\frac{n}{m^n}\left[\frac{w^{n+1}}{n+1}\right]_0^m$$
$$=\frac{n}{m^n}\frac{m^{n+1}}{n+1}$$
$$=\frac{mn}{n+1}$$

Qn	Suggested Solution
7	

H_0: The performances of students in the lecture test on integration is independent of the total number of conceptual and interpretational errors made.

H_1: The performances of students in the lecture test on integration is not independent of the total number of conceptual and interpretational errors made.

At 1% significance level.

Expected frequencies

Grades	**Total number of Conceptual and Interpretation Errors made**			**Total**
	≤ 4	**5 – 6**	**≥ 7**	
A	6 (2.1053)	0 (1.8947)	0 (2)	6
B	5 (2.1053)	1 (1.8947)	0 (2)	6
C	3 (1.4035)	1 (1.2632)	0 (1.3333)	4
D	4 (4.2105)	6 (3.7895)	2 (4)	12
E	0 (1.4035)	3 (1.2632)	1 (1.3333)	4
S	1 (1.0526)	1 (0.9474)	1 (1)	3
U	1 (7.7193)	6 (6.9474)	15 (7.3333)	22
Total	20	18	19	57

The expected frequencies are in parenthesis.

As some of the expected frequencies are less than 5, we will need to combine some rows (sensible combination of rows expected):

Grades	Total number of Conceptual and Interpretation Errors made			Total
	≤ 4	5 – 6	≥ 7	
A – C	14 (5.614)	2 (5.0526)	0 (5.3333)	16
D – E	4 (5.614)	9 (5.02526)	3 (5.3333)	16
S – U	2 (8.7719)	7 (7.8947)	16 (8.3333)	25
Total	20	18	19	57

The expected frequencies are in parenthesis.

Degree freedom = $(3-1)\times(3-1)=4$

Under H_0, $X^2=\sum_{i=1}^{2}\sum_{j=1}^{3}\frac{(O_{ij}-E_{ij})^2}{E_{ij}}\sim\chi^2(4)$.

Reject H_0 if p-value ≤ 0.01

Using GC, p-value = 2.1209×10^{-7}

Since p-value < 0.01, we reject H_0. Thus, at 1% level of significance, there is sufficient evidence to conclude that the performances of students in the lecture test on integration is not independent of the total number of conceptual and interpretational errors made.

Qn	Suggested Solution	
8	(i) Using GC, $\int_0^2 \frac{3}{22}\left(4-x^2\right)\,dx=0.727$ $P(X\le a)=0.9>0.727$, so $2<a\le 4$ $0.72727+\int_2^a \frac{3}{22}\,dx=0.9$ $a=\frac{49}{15}$ or 3.27 (ii) $E\left(X^2\right)=\int_0^2\frac{3x^2}{22}\left(4-x^2\right)\,dx+\int_2^4\frac{3}{22}x^2dx=\frac{172}{55}$ (using GC) $\mathrm{Var}\left(X\right)=\frac{172}{55}-\left(\frac{15}{11}\right)^2=\frac{767}{605}$ (iii) Using Central Limit Theorem, since n is large $\sum X_i\sim N\left(75,\frac{767}{11}\right)$ approximately. Probability required $=P\left(56<\sum X<78\right)\approx 0.629$	

Qn	Suggested Solution	
9	(i) Probability that Mitch wins the game $= \text{P}(\text{HH, THH, HTHH, THTHH, HTHTHH, THTHTHH...})$ $= p^2 + qp^2 + pqp^2 + qpqp^2 + pqpqp^2 + qpqpqp^2$ $= p^2\left(1+q+pq+pq^2+p^2q^2+p^2q^3...\right)$ $= p^2\left(1+q+pq(1+q)+(pq)^2(1+q)+...\right)$ $= \dfrac{p^2(1+q)}{1-pq}$	
	(ii) $\text{P}(X=2k) = \text{P}(\underbrace{\text{HT...HT}}_{k-1 \text{ pairs of HT}}\ \text{HH or } \underbrace{\text{TH...TH}}_{k-1 \text{ pairs of TH}}\ \text{TT},)$ $= (pq)^{k-1}p^2 + (pq)^{k-1}q^2$ $= (pq)^{k-1}\left(p^2+q^2\right)$ $\text{P}(X=2k+1) = \text{P}(\text{T}\ \underbrace{\text{HT...HT}}_{k-1 \text{ pairs of HT}}\ \text{HH or H}\ \underbrace{\text{TH...TH}}_{k-1 \text{ pairs of TH}}\ \text{TT},)$ $= q(pq)^{k-1}p^2 + p(pq)^{k-1}q^2$ $= (pq)^k(p+q)$ $= (pq)^k$	
	(iii) $\text{E}(X) = \sum_{k=1}^{\infty} 2k\text{P}(X=2k) + \sum_{k=1}^{\infty}(2k+1)\text{P}(X=2k+1)$ $= \sum_{k=1}^{\infty} 2k(pq)^{k-1}\left(p^2+q^2\right) + \sum_{k=1}^{\infty}(2k+1)(pq)^k$ $= 2\left(p^2+q^2\right)\sum_{k=1}^{\infty} k(pq)^{k-1} + 2pq\sum_{k=1}^{\infty} k(pq)^{k-1} + \sum_{k=1}^{\infty}(pq)^k$ $= 2\left(p^2+q^2\right)\dfrac{1}{(1-pq)^2} + 2pq\dfrac{1}{(1-pq)^2} + \dfrac{pq}{(1-pq)}$ When $p=0.4$, $\text{E}(X) = 2.95$	

Qn	Suggested Solution	
10	(i) A paired-sample *t*-test is more appropriate as we are interested to compare the difference in scores of the **same** golfer.	
	(ii) Let X and Y be the scores obtained in Day 1 and Day 2 respectively. Let $D = X - Y$. Let μ_1 and μ_2 be the population mean scores of the golfer in Day 1 and Day 2 respectively. Let	

	To test $\begin{matrix} H_0 : \mu_1 - \mu_2 = 0 \\ H_1 : \mu_1 - \mu_2 < 0 \end{matrix}$ at 10% level of significance. Assume that the difference in scores follows a normal distribution. Under H_0, since n is small and population variance is unknown, $T = \dfrac{\bar{D} - 0}{\sqrt{\dfrac{S_D{}^2}{12}}} \sim t(12-1)$ Reject H_0 if p-value ≤ 0.10 $\bar{d} = -1.1667$, $s_D{}^2 = (2.6912)^2$ $t_{calc} = -1.5017$ p-value $= 0.0807$ Since p-value $= 0.0807 < 0.10$, we reject H_0 and conclude that there is sufficient evidence from the sample data at 10% significance level that the scores on Day 2 is more than that of Day 1.	
	Now, to test $\begin{matrix} H_0 : \mu_1 - \mu_2 = 0 \\ H_1 : \mu_1 - \mu_2 \neq 0 \end{matrix}$ at 10% level of significance. For comparable results, do not reject H_0 $\lvert t_{calc} \rvert = \left\lvert \dfrac{-1.5017 + k - 0}{\sqrt{\dfrac{(2.6912)^2}{12}}} \right\rvert < 1.7959$ $0.10650 < k < 2.8969$ Possible k is 1 or 2.	

Qn	Suggested Solution	
11	(i) Let A be the random variable denoting the number of people arriving at the queue in a k-minute interval. $A \sim \text{Po}(0.1k)$. $\text{P}(A \leq 1) < 0.2$ $\text{P}(A = 0) + \text{P}(A = 1) < 0.2$ $\dfrac{\text{e}^{-0.1k}(0.1k)^0}{0!} + \dfrac{\text{e}^{-0.1k}(0.1k)^1}{1!} < 0.2$ $\text{e}^{-0.1k}(1 + 0.1k) < 0.2$ From GC, $k > 29.943 \Rightarrow k \geq 30.0$	
	(ii) Let X be the random variable denoting the number of people arriving at the lift lobby in a 20-minute interval. $X \sim \text{Po}(2)$	

	Required probability = $\text{P}(X \le 6) = 0.995$	
	(iii) Let Y_1 and Y_2 be the random variables denoting the number of people arriving at the lift lobby in the first and second 10-min interval respectively. Then $Y_1 \sim \text{Po}(1), Y_2 \sim \text{Po}(1)$ Required probability $= \text{P}(Y_1 \le 3, Y_2 \le 3) + \text{P}(Y_1 = 4, Y_2 \le 2) + \text{P}(Y_1 = 5, Y_2 \le 1) + \text{P}(Y_1 = 6, Y_2 \le 0)$ $= 0.979$	
	(iv) $W \sim \text{Exp}\left(\frac{1}{5}\right)$. P(at least a visitor leaves the queue within first t minutes) $= 1 -$ P (no visitors leave within first t minutes) $= 1 - [\text{P}(W > t)]^n$ $= 1 - \text{e}^{-\frac{nt}{5}}$ $\text{P}(T \le t) = \text{P}(\text{first visitor leaves in the first } t \text{ mins})$ $= 1 - \text{e}^{-\frac{nt}{5}}$ Hence $T \sim \text{Exp}\left(\frac{n}{5}\right)$. $\text{E}(T) = \frac{5}{n}$. (v) $\text{E}(T) = \frac{5}{n}$ when there are n visitors in the queue. By memoryless property of exponential distribution, after first visitor is leaves, there are n-1 visitors left and the expected time between 1st and 2nd visitor leaving is $\frac{5}{n-1}$ and so on. (vi) Hence, expected time $= \frac{5}{n} + \frac{5}{n-1} + \frac{5}{n-2} + \ldots + \frac{5}{1} = \sum_{r=1}^{n} \frac{5}{r} \le 10$ Using GC, largest n is 3.	