JURONG PIONEER JUNIOR COLLEGE
9478 H2 PHYSICS

COLLISIONS TUTORIAL SOLUTIONS

Self-Check Questions

S1 Impulse of a force is the product of the force and the time interval for which the force acts on the body. It is mathematically equal to the area under the force-time graph.

S2 The principle of conservation of momentum states that the total momentum of a system remains constant provided no net external force acts on it.

S3 Consider a moving object A, of mass m_A and velocity u_A, collides with an object B, of mass m_B and velocity u_B, moving in the same direction as in Fig. 3.1.

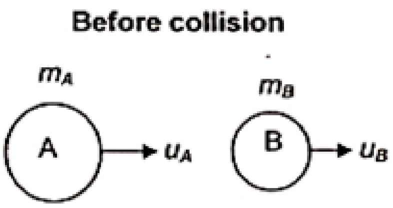


Fig. 3.1

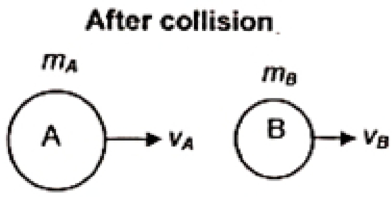


Fig. 3.2

After the collision, they move off with velocities v_A and v_B as shown in Fig. 3.2.

During the brief collision, these objects exert large forces on one another. By Newton's third law, the force F_{BA} exerted by B on A is equal in magnitude and opposite in direction to the force F_{AB} exerted by A on B, as shown in Fig. 3.3.

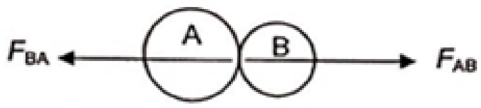


Fig. 3.3

$$F_{BA} = -F_{AB}$$

$$\frac{\Delta p_A}{\Delta t} = -\frac{\Delta p_B}{\Delta t}$$

$$\frac{m_A v_A - m_A u_A}{\Delta t} = -\frac{m_B v_B - m_B u_B}{\Delta t}$$

$$m_A v_A - m_A u_A = -(m_B v_B - m_B u_B)$$

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

Total initial momentum of bodies = Total final momentum of bodies
(provided no net external force act on bodies A and B)

S4 Kinetic energy is conserved in elastic collisions but not in inelastic collisions. Objects stick together in completely inelastic collisions. For all collisions, the total momentum and total energy of the system are conserved.

Self-Practice Questions

Qn	Ans	Solution
P1	**10 N**	Area under *F*-*t* graph = impulse = change in momentum $\Rightarrow \frac{1}{2}(5+2)x = 35$ $\Rightarrow x = 10\text{ N}$
P2	**C**	By conservation of linear momentum, Total initial moment of system = total final momentum of the system Hence, total momentum $=(2\times12)+(4\times0)=24\text{ kg m s}^{-1}$
P3	**A**	Before release, the system of trolleys X and Y has a total momentum of zero. With no external resultant force acting on the system, the total momentum of the system remains at zero. So, after collision, both trolleys must remain stationary in order for the total momentum to be zero. Hence, the speed of Y after collision is zero.
P4	**C**	Principle of conservation of momentum $0.30(1.2)+0.60(-1.8)=0.30v_P+0.60v_Q$ $2v_Q+v_P=-2.4$ Relative speed of approach = Relative speed of separation $1.2-(-1.8)=v_Q-v_P$ $v_Q-v_P=3.0$ Solving $v_P=-2.8\text{ m s}^{-1}$ $v_Q=0.2\text{ m s}^{-1}$

Discussion Questions

1 After collision, let velocity of mass *m* be v_m and velocity of mass *M* be v_M.
By conservation of linear momentum,
$mv_m + Mv_M = mu$ --- (1)

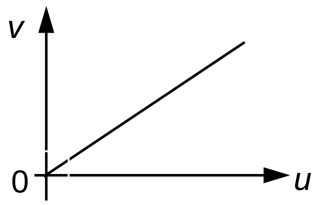
By relative speed of approach = relative speed of separation
$u - 0 = v_M - v_m$
$v_M - v_m = u$ --- (2)

Substitute (2) into (1) and solve for v_M.
$$v_M = \frac{2mu}{M+m}$$

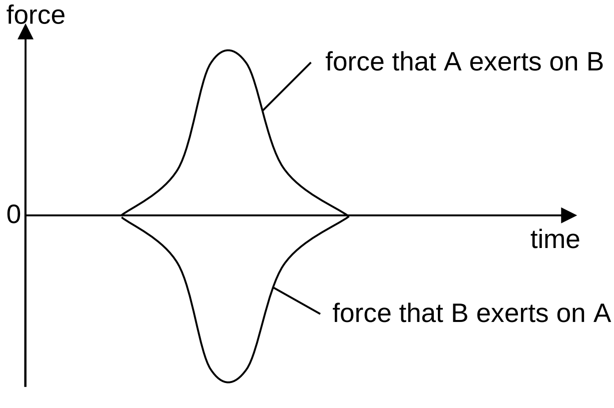
Substitute the above into (2) to solve for v_m.
$$v_m = \frac{2mu}{M+m} - u$$

Solve for *v*:
$$v = v_M - v_m = \frac{2mu}{M+m} - \left(\frac{2mu}{M+m} - u\right) = u$$

Therefore, v is directly proportional to u, hence, sketch a straight-line graph that cuts the origin where gradient is 1.



2 **(a)** **(i)**



(ii) By Newton's third law of motion, the force that A exerts on B is equal in magnitude but opposite in direction to the force B exerts on A.

(iii) The area below the *F*-*t* graph gives the change in momentum experienced by body. The change in momentum of body B is positive while the change in momentum of body A is negative (graph is below *t*-axis), and they have the same magnitude. Hence, the overall change in momentum experienced by the collision system of bodies A and B is zero.

Hence, this is consistent with principle of conservation of momentum where there is no change in momentum of the system as a whole, provided no external resultant force acts on the system.

(b) By Impulse-Momentum Theory,

Change in momentum = Impulse = Ft

Change in velocity of car = $\dfrac{Ft}{m_{car}} = \dfrac{(72000)(0.25)}{1200} = 15\ \text{ms}^{-1}$

Change in velocity of truck = $-\dfrac{Ft}{m_{truck}} = \dfrac{-(72000)(0.25)}{12000} = -1.5\ \text{ms}^{-1}$

Note that the velocity of the truck decreases.

(c) The force experienced by the vehicles is not constant in reality. The force varies as both the truck and car undergo deformation during a collision.

(d) The seat belt and air bag provide a backward force to stop the passenger from continuing to move forward due to his inertia. This backward force is provided over a prolonged period of time. Applying Newton's second law, this also means that the rate of change of momentum of the passenger is lower. The resultant force acting on the passenger is thus reduced, lowering the risk of injury.

(e) Any three factors:
- surface composition of the road
- driving speed
- tyre pressure
- tyre quality and erosion
- tyre maintenance
- road visibility
- road structure (e.g. curves, widths, slopes)

3 **(a)** **(i)** Like charges repel.

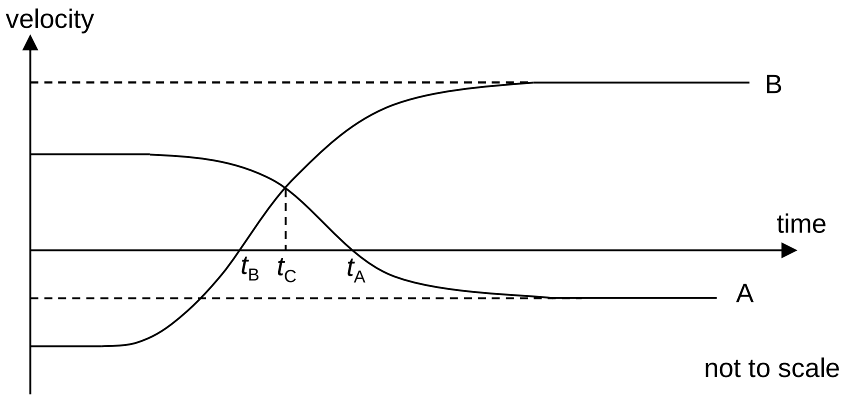
(ii) There are no external forces acting on the system, hence total momentum of the system is conserved. However if both were to stop at the same time, it means the total momentum at that instant is zero, which is impossible since the initial momentum of the system is $3mv - 2mv = mv$ which is not zero.

(b) By principle of conservation of momentum and taking right +ve:

$$3mv + 2m(-v) = 3mv_f + 2mv_f$$

$$v_f = 0.2v$$

(c) **(i)**



(ii) Let final velocity of A be v_A and that of B be v_B.
Taking right as positive,
By principle of conservation of momentum:

$3mv + 2m(-v) = 3mv_A + 2mv_B$ --- (1)

Since interaction is elastic, thus
relative speed of approach = relative speed of separation

$2v = v_B - v_A$ --- (2)

Solving both equations yields $v_A = -0.6v$ and $v_B = 1.4v$
Thus speed of A = $0.6v$ and speed of B = $1.4v$

Note: The negative sign of v_A shows that A actually changes direction. Similarly, the fact that v_B is positive shows that B has also changed direction. A and B repel each other and move away in opposite directions.

4 **(a)** **(i)** Knowing that their collision is elastic tells us that the total kinetic energy of the molecules is conserved in this collision. Moreover the relative speed of approach of the molecules equals their relative speed of separation.

(ii) A head-on collision is one where the molecules remain moving along the same straight line joining their centres of mass before and after collision.

(b) **(i)** Since collision is elastic: (taking to the right as positive)

relative velocity of separation of the two molecules = relative velocity of approach

relative velocity of approach = $u_1 - u_2 = 1.88 \times 10^3 - (-405) = 2285 \text{ m s}^{-1}$

Hence relative velocity of separation of the two molecules,
$v_2 - v_1 = 2285 \text{ m s}^{-1}$ ---- (1)

(ii) let the final velocity of the hydrogen molecule be v_1 and that of the oxygen molecule be v_2, taking right to be positive:

$2.00u\ (1.88 \times 10^3) + 32.0u\ (-405) = 2.00u\ (v_1) + 32.0u\ (v_2)$ ---- (2)

Solving equations (1) and (2) yields $v_1 = -2420 \text{ m s}^{-1}$, $v_2 = -136 \text{ m s}^{-1}$

Hydrogen molecule will move to the left with speed 2420 m s^{-1}.
Oxygen molecule will move to the left with speed 136 m s^{-1}.

5(a)(i) Both trolleys could not be stationary at the same time as this would imply that the total momentum of the system is zero at that instant. Since the total momentum of the system before collision was (1.5)(0.90) + (1.2)(-2.2) = -1.29 kg m s^{-1}, with no external resultant force acting on the system, the total momentum should remain constant at this value throughout, according to the Principle of Conservation of Momentum.

5(a)(ii) According to Newton's third law, the forces on trolleys A and B form an action-reaction pair. Hence, the average force between the trolleys may be determined by calculating the average force on trolley A.

According to Newton's second law of motion,
the average force on A = (change in momentum of A) / time
= [(1.5)(-0.70) – (1.5)(0.90)]/0.30
= -8.0 N

5(b)(i) Let the speed of trolley B after the collision be *v*.

By Newton's third law, the average force on trolley B is equal in magnitude but opposite in direction to the average force on trolley A.
Hence, the average force on trolley B = 8.0 N.
=> (change in momentum of B) / time = 8.0 N
=> [(1.2)(v) – (1.2)(-2.2)]/0.30 = 8.0
=> v = -0.2 m s^{-1}

Therefore, final speed of trolley B after collision is 0.2 m s^{-1}.

5(b)(ii) Since the rightward direction was assigned as the positive direction, the negative sign in (b)(i) implies that trolley B was still moving towards the left after the collision, i.e. still in the same direction as that of its initial velocity.

5(c) Relative speed of approach = 0.90 - (-2.2) = 3.1 m s^{-1}
Relative speed of separation = -0.2 - (-0.70) = 0.50 m s^{-1}

Since the relative speed of approach is not equal to the relative speed of separation, the collision is inelastic.