H2 Mathematics (9758)
Chapter 5 Vectors
Assignment Suggested Solutions

1 **2019/TMJC JC1 Promo/Q5**

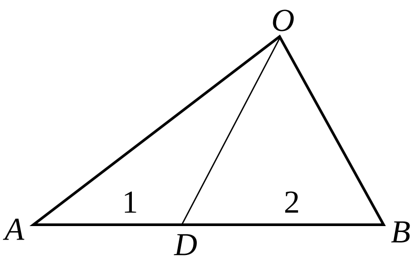
Referred to the origin O, points A, B and C have position vectors $\mathbf{a} = -2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{b} = 3\mathbf{j} - 3\mathbf{k}$ and $\mathbf{c} = \mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$ respectively.

(i) Show that A, B and C are collinear. [2]

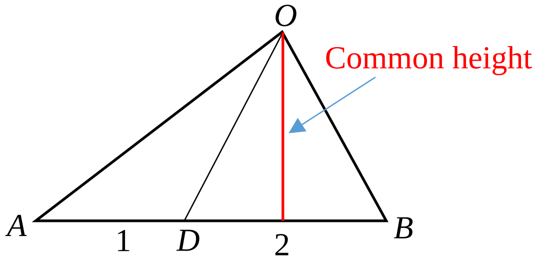
(ii) Point D lies on AB such that $AD : DB = 1 : 2$. Find the position vector of D. [2]

(iii) Find the area of triangle OAB. Hence write down the area of triangle OBD. [4]

(iv) Evaluate $|\mathbf{c} \cdot \hat{\mathbf{a}}|$ and give a geometrical interpretation of $|\mathbf{c} \cdot \hat{\mathbf{a}}|$. [3]

Q1	Solution
(i)	$\overrightarrow{AC} = \begin{pmatrix}1\\4\\-4\end{pmatrix} - \begin{pmatrix}-2\\1\\-1\end{pmatrix} = \begin{pmatrix}3\\3\\-3\end{pmatrix} = 3\begin{pmatrix}1\\1\\-1\end{pmatrix}$ $\overrightarrow{AB} = \begin{pmatrix}0\\3\\-3\end{pmatrix} - \begin{pmatrix}-2\\1\\-1\end{pmatrix} = \begin{pmatrix}2\\2\\-2\end{pmatrix} = 2\begin{pmatrix}1\\1\\-1\end{pmatrix}$ Since $\overrightarrow{AC} = 3\begin{pmatrix}1\\1\\-1\end{pmatrix} = \frac{3}{2}\begin{pmatrix}2\\2\\-2\end{pmatrix} = \frac{3}{2}\overrightarrow{AB}$, hence A, B and C are collinear. Recommend to show that the 2 vectors are parallel like this, where you can see the relationship properly i.e. express $\overrightarrow{AC}$ as $\begin{pmatrix}2\\2\\-2\end{pmatrix}$ and balancing the equation, then replace it with $\overrightarrow{AB}$. This ensures you get the correct relationship between the 2 vectors.
(ii)	Using ratio theorem and ΔOAB, $\overrightarrow{OD} = \dfrac{2\overrightarrow{OA} + \overrightarrow{OB}}{3}$ $= \frac{1}{3}\left[2\begin{pmatrix}-2\\1\\-1\end{pmatrix} + \begin{pmatrix}0\\3\\-3\end{pmatrix}\right]$ $= \frac{1}{3}\begin{pmatrix}-4\\5\\-5\end{pmatrix}$ 

Steps
1) Form any 2 vectors using the 3 points
2) Show that the 2 vectors are parallel
3) Write the conclusion

(iii)	Area of triangle $OAB = \frac{1}{2}\lvert \mathbf{a}\times\mathbf{b}\rvert$ $= \frac{1}{2}\left\lvert \begin{pmatrix}-2\\1\\-1\end{pmatrix}\times\begin{pmatrix}0\\3\\-3\end{pmatrix}\right\rvert$ $= \frac{1}{2}\left\lvert \begin{pmatrix}0\\-6\\-6\end{pmatrix}\right\rvert$ $= 3\left\lvert \begin{pmatrix}0\\-1\\-1\end{pmatrix}\right\rvert = 3\sqrt{2}$  **Note:** Triangle *OAB* and triangle *OBD* share common height Area of triangle $OBD = 2\sqrt{2}$
(iv)	$\lvert \mathbf{c}\cdot\hat{\mathbf{a}}\rvert = \left\lvert \begin{pmatrix}1\\4\\-4\end{pmatrix}\bullet\frac{1}{\sqrt{6}}\begin{pmatrix}-2\\1\\-1\end{pmatrix}\right\rvert$ $=\frac{1}{\sqrt{6}}\lvert -2+4+4\rvert$ $=\sqrt{6}$ $\lvert \mathbf{c}\cdot\hat{\mathbf{a}}\rvert$ is the length of projection of **c** onto **a**.

2 **2019/ASRJC JC1 Promo/Q7 (Modified)**

Referred to the origin *O*, **a**, **b** and **c** are non-zero and non-parallel vectors denoting the position vectors of the points *A*, *B* and *C* respectively.

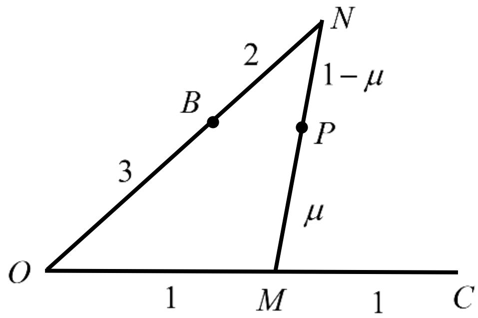
(i) Given that $\mathbf{a}\times\mathbf{b} = 3\mathbf{a}\times\mathbf{c}$ and $\mathbf{b}\neq 3\mathbf{c}$, show that $\mathbf{b}-3\mathbf{c}=\lambda\mathbf{a}$ where λ is a scalar. [2]

The point *M* is the mid-point of *OC* and the point *N* lies on *OB* produced such that $3ON = 5OB$. The point *P* lies on *MN* such that $MP : MN = \mu : 1$. It is given that the position vector of *P* is $\frac{10}{9}\mathbf{b}+\frac{1}{6}\mathbf{c}$.

(ii) Find the value of μ. [3]

It is given that **b** is a unit vector, $\lvert\mathbf{c}\rvert=\sqrt{2}$, $\lvert\mathbf{b}-3\mathbf{c}\rvert=\sqrt{13}$ and the angle between **b** and **c** is $45°$.

(iii) Find the exact length of projection of $\overrightarrow{OP}$ on $\overrightarrow{OA}$. [4]

Q2	Solution
(i)	$\mathbf{a}\times\mathbf{b}=3\mathbf{a}\times\mathbf{c}$ $(\mathbf{a}\times\mathbf{b})-(3\mathbf{a}\times\mathbf{c})=\mathbf{0}$ $\mathbf{a}\times(\mathbf{b}-3\mathbf{c})=\mathbf{0}$ $\mathbf{a}$ is parallel to $\mathbf{b}-3\mathbf{c}$, hence $\mathbf{b}-3\mathbf{c}=\lambda\mathbf{a}$. **Note:** this is the zero vector, and not zero as cross product results in a vector. Make sure you write as $\underset{\sim}{0}$ to symbolise that it is a vector
(ii)	By ratio theorem, $\overrightarrow{OP}=\dfrac{\mu\overrightarrow{ON}+(1-\mu)\overrightarrow{OM}}{\mu+1-\mu}$ $\dfrac{10}{9}\mathbf{b}+\dfrac{1}{6}\mathbf{c}=\mu\left(\dfrac{5}{3}\mathbf{b}\right)+(1-\mu)\dfrac{1}{2}\mathbf{c}$  Since $\mathbf{b}$ and $\mathbf{c}$ are non-zero and non-parallel vectors, $\dfrac{10}{9}=\dfrac{5}{3}\mu$ and $\dfrac{1}{6}=\dfrac{1}{2}(1-\mu)$ $\mu=\dfrac{2}{3}$
(iii)	length of projection $=\left\lvert\overrightarrow{OP}\cdot\dfrac{\overrightarrow{OA}}{\lvert\overrightarrow{OA}\rvert}\right\rvert=\left\lvert\dfrac{(20\mathbf{b}+3\mathbf{c})}{18}\bullet\dfrac{\mathbf{a}}{\lvert\mathbf{a}\rvert}\right\rvert$ $=\dfrac{1}{18}\left\lvert\dfrac{(20\mathbf{b}+3\mathbf{c})\cdot\frac{1}{\lambda}(\mathbf{b}-3\mathbf{c})}{\left\lvert\frac{1}{\lambda}\right\rvert\lvert\mathbf{b}-3\mathbf{c}\rvert}\right\rvert$ $\left(\text{From } \textbf{(i)}, \mathbf{a}=\dfrac{1}{\lambda}(\mathbf{b}-3\mathbf{c})\right)$ $=\dfrac{1}{18}\left\lvert\dfrac{20\mathbf{b}\cdot\mathbf{b}-57\mathbf{c}\cdot\mathbf{b}-9\mathbf{c}\cdot\mathbf{c}}{\lvert\mathbf{b}-3\mathbf{c}\rvert}\right\rvert$ $=\dfrac{1}{18}\left\lvert\dfrac{20\lvert\mathbf{b}\rvert^2-57\mathbf{c}\cdot\mathbf{b}-9\lvert\mathbf{c}\rvert^2}{\lvert\mathbf{b}-3\mathbf{c}\rvert}\right\rvert$ $=\dfrac{1}{18}\left\lvert\dfrac{20-18-57\lvert\mathbf{c}\rvert\lvert\mathbf{b}\rvert\cos 45^\circ}{\sqrt{13}}\right\rvert$ $=\dfrac{1}{18}\left\lvert\dfrac{-55}{\sqrt{13}}\right\rvert=\dfrac{55\sqrt{13}}{234}$

3 2019/ RIJC Promo/5

(i) Given **a**, **b** and **c** are unit vectors such that **a**·**b** = 2**a**·**c**, what is the relationship between **a** and **b** – 2**c**? [2]

(ii) It is further given that **b** is perpendicular to **b** – 2**c**. Find the angle between **b** and **c**. [2]

(iii) Comment on the relationship between **a** × **b** and **b** – 2**c**. [1]

Q3	**Solution**
(i)	$\mathbf{a}\cdot\mathbf{b} = 2\mathbf{a}\cdot\mathbf{c}$ $\mathbf{a}\cdot\mathbf{b} - 2\mathbf{a}\cdot\mathbf{c} = 0$ $\mathbf{a}\cdot(\mathbf{b} - 2\mathbf{c}) = 0$ Therefore **a** is perpendicular to $\mathbf{b} - 2\mathbf{c}$ since $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{b} \neq 2\mathbf{c}$. • $\mathbf{a} \neq \mathbf{0}$ as **a** is a unit vector (i.e. $\lvert\mathbf{a}\rvert = 1$) • $\mathbf{b} \neq 2\mathbf{c}$ as both **b** and **c** are unit vectors $\Rightarrow \lvert\mathbf{b}\rvert = 1$ and $\lvert\mathbf{c}\rvert = 1 \Rightarrow \lvert 2\mathbf{c}\rvert = 2$ Since $\lvert\mathbf{b}\rvert \neq \lvert 2\mathbf{c}\rvert \Rightarrow \mathbf{b} \neq 2\mathbf{c}$.
(ii)	Let θ be the angle between **b** and **c**. Given **b** is perpendicular to $\mathbf{b} - 2\mathbf{c}$, $\mathbf{b}\cdot(\mathbf{b} - 2\mathbf{c}) = 0$ $\mathbf{b}\cdot\mathbf{b} = 2\mathbf{b}\cdot\mathbf{c}$ $\lvert\mathbf{b}\rvert^2 = 2\lvert\mathbf{b}\rvert\lvert\mathbf{c}\rvert\cos\theta$ $\cos\theta = \frac{1}{2}$ (since **b** and **c** are unit vectors) $\theta = 60^\circ$
(iii)	From **(i)** and **(ii)**, **b** – 2**c** is perpendicular to **a** and **b**. $\mathbf{a}\times\mathbf{b}$ is perpendicular to **a** and **b**. Therefore, $\mathbf{a}\times\mathbf{b}$ is parallel to **b** – 2**c**.