Topical Revision Paper 8

Term 2 Unit 8: Matrices

Question 1:

Given that $\begin{pmatrix} 7 & -1 & -1 \\ -3 & m & 0 \\ -3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 3 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, find the value of m. [2]

Answer: $m = 1$

Solution:

$(-3)(1) + m(4) + (0)(3) = 1$

$4m - 3 + 0 = 1$

$4m = 4$

$m = 1$

Question 2:

It is given matrices $\mathbf{M} = \begin{pmatrix} 0 & x \\ y & -1 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 1 & 0 \\ 2y-3 & 1 \end{pmatrix}$.

(i) Find $\mathbf{N} - \mathbf{M}$ and $\mathbf{M}^2$ in terms of x and y. [2]

(ii) If $\mathbf{M} + \mathbf{M}^2 = \mathbf{N}$, find the value of x and of y. [3]

Solution:

(i)

$$\mathbf{N} - \mathbf{M} = \begin{pmatrix} 1 & 0 \\ 2y-3 & 1 \end{pmatrix} - \begin{pmatrix} 0 & x \\ y & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -x \\ y-3 & 2 \end{pmatrix}$$

$$\mathbf{M}^2 = \begin{pmatrix} 0 & x \\ y & -1 \end{pmatrix}\begin{pmatrix} 0 & x \\ y & -1 \end{pmatrix}$$

$$= \begin{pmatrix} xy & -x \\ -y & xy+1 \end{pmatrix}$$

(ii)

$\mathbf{M}+\mathbf{M}^2=\mathbf{N}$

$\mathbf{M}^2=\mathbf{N}-\mathbf{M}$

$$\begin{pmatrix} 1 & -x \\ y-3 & 2 \end{pmatrix}=\begin{pmatrix} xy & -x \\ -y & xy+1 \end{pmatrix}$$

$\Rightarrow y-3=-y$

$xy=1$

$\therefore y=\dfrac{3}{2}, x=\dfrac{2}{3}$

Question 3:

The price of ticket in each category at Cheap Cinema is as follows:
Child - \$3, Adult - \$8 and Senior citizen - \$5.

(i) Write down the column matrix $\mathbf{C}$ represents the above information. [1]

The number of tickets sold on one weekend is given by the matrix $\mathbf{N}$ as follows:

	Children	Adult	Senior Citizen
Saturday	20	45	70
Sunday	50	90	40

Given that $\mathbf{T}=\mathbf{NC}$,

(ii) evaluate $\mathbf{T}$. [3]

(iii) What information can be derived from $\mathbf{T}$? [1]

To improve revenue during weekends, two proposals are examined:

Proposal 1: Double the price on Sunday only.

Proposal 2: Increase the price by 25% on each day.

The matrix $\mathbf{P}$ is such that $\mathbf{PT}$ gives the revenue for the weekend under proposal 1.

The matrix $\mathbf{Q}$ is such that $\mathbf{QT}$ gives the revenue for the weekend under proposal 2.

(iv) Evaluate $\mathbf{PT}$ and $\mathbf{QT}$ and state which proposal would be more profitable? [3]

Solution:

(i)

$$C=\begin{bmatrix} 3 \\ 8 \\ 5 \end{bmatrix}$$

(*ii*)

$$T=\begin{bmatrix}30 & 45 & 68\\ 60 & 95 & 38\end{bmatrix}\begin{bmatrix}3\\ 8\\ 5\end{bmatrix}$$

$$T=\begin{bmatrix}3(30)+45(8)+68(5)\\ 60(3)+95(8)+38(5)\end{bmatrix}$$

$$T=\begin{bmatrix}790\\ 1130\end{bmatrix}$$

(iii)
T represents the amount collected by selling tickets on Saturday and Sunday

(iv)

$$T=\begin{bmatrix}790\\ 1130\end{bmatrix}; P=\begin{bmatrix}1 & 2\end{bmatrix}; \text{Q}=\begin{bmatrix}1.25 & 1.25\end{bmatrix}$$

$$PT=\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}790\\ 1130\end{bmatrix}$$

$$PT=\begin{bmatrix}3050\end{bmatrix}$$

$$QT=\begin{bmatrix}1.25 & 1.25\end{bmatrix}\begin{bmatrix}790\\ 1130\end{bmatrix}$$

$$QT=\begin{bmatrix}2400\end{bmatrix}$$

Proposal 1 will be more profitable

Question 4:

Given that **P** = $\begin{pmatrix}3 & 0\\ 0 & 5\end{pmatrix}$ and **Q** = $\begin{pmatrix}a & b\\ 0 & c\end{pmatrix}$, find the values of *a*, *b* and *c* such that

5**PQ** = 8**P** – 3**Q.** [4]

Solution:

$$5\begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = 8\begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix} - 3\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$$

$$5\begin{pmatrix} 3a & 3b \\ 0 & 5c \end{pmatrix} = \begin{pmatrix} 24-3a & -3b \\ 0 & 40-3c \end{pmatrix}$$

$$15a = 24 - 3a$$

$$a = \frac{4}{3}$$

$$b = 0$$

$$25c = 40 - 3c$$

$$c = \frac{40}{28} = \frac{10}{7}$$

$$a = \frac{4}{3}, \quad b = 0, \quad c = \frac{10}{7}$$

Question 5:

The following table shows the amount of flour, butter and sugar needed in making 12 chocolate cupcakes and 12 lemon cupcakes.

	Flour	Butter	Sugar
Chocolate cupcakes	125 g	125 g	96 g
Lemon cupcakes	225 g	125 g	128 g

The amount of the 3 main ingredients used in making these cupcakes can be represented by the matrix **M** = $\begin{pmatrix} 125 & 125 & 96 \\ 225 & 125 & 128 \end{pmatrix}$.

(i) Given that **N** = $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, evaluate **MN**. [2]

(ii) What do the elements in the product matrix **MN** represent? [1]

The cost of 100 g of flour is \$0.80, 100 g butter is \$2.40 and 100 g of sugar is \$0.20.

(iii) It is decided that the selling price of the cupcakes should include an increase of 450% of the total cost of using these 3 main ingredients. Write down the matrix

S in order to calculate the selling price of 12 chocolate cupcakes and 12 lemon cupcakes respectively. Calculate the respective selling price using MATRIX method. [4]

(iv) A secret recipe of 12 luxurious lemon cupcakes requires the same amount of flour and sugar used as the lemon cupcakes, but with an unknown amount of butter. Fortunately, it is known that the total cost of flour, butter and sugar for making the luxurious lemon cupcakes is 10% more than that of the lemon cupcakes. Find the amount of butter needed for this secret recipe to make 12 luxurious cupcakes, to the nearest gram. [2]

(i)

$$\mathbf{MN} = \begin{pmatrix} 125 & 125 & 96 \\ 225 & 125 & 128 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\mathbf{MN} = \begin{pmatrix} 125+125+96 \\ 225+125+128 \end{pmatrix}$$

$$\mathbf{MN} = \begin{pmatrix} 346 \\ 478 \end{pmatrix}$$

(ii) The total mass of the 3 main ingredients used in 12 chocolate cupcakes and 12 lemon cupcakes.

(iii)

$$\mathbf{S} = 5.5\begin{pmatrix} 125 & 125 & 96 \\ 225 & 125 & 128 \end{pmatrix}$$

$5.5\begin{pmatrix} 125 & 125 & 96 \\ 225 & 125 & 128 \end{pmatrix}\begin{pmatrix} 0.008 \\ 0.024 \\ 0.002 \end{pmatrix}$ $= 5.5\begin{pmatrix} 4.192 \\ 5.056 \end{pmatrix}$ $= \begin{pmatrix} 23.056 \\ 27.808 \end{pmatrix}$ ***The selling price of 12 chocolate cupcakes is $23.06 and*** ***of 12 lemon cupcakes is $27.81.***
(iv)Let the amount of butter be *x* g for 12 luxurious lemon cupcakes. $225 \times 0.008 + x \times 0.024 + 128 \times 0.002 = 5.056 \times 1.1$ $x = 146.1\text{ g} = 146\text{ g}$ (3 s. f.)