Tutorial 18 Alternating Currents Suggested Solutions

D1 **[C]** In order to melt identical fuse, power dissipated in the fuse by the d.c. source must be equal to the mean power dissipated in the fuse by the a.c. source:

$$\begin{aligned} P_{dc} &= \langle P_{ac} \rangle \\ I_{dc}^{2} R &= \langle I_{ac}^{2} \rangle R \\ I_{dc} &= \sqrt{\langle I_{ac}^{2} \rangle} \\ &= I_{rms} \end{aligned}$$

Hence, if a fuse melts when alternating current just exceeds 13 A r.m.s., it will also melt when direct current just exceeds 13 A.

D2

In Fig. 2a, mean power $P = \dfrac{V_{\text{r.m.s.1}}^2}{R} = \dfrac{\left(V_0/\sqrt{2}\right)^2}{R} = \dfrac{V_0^{\,2}}{2R}$

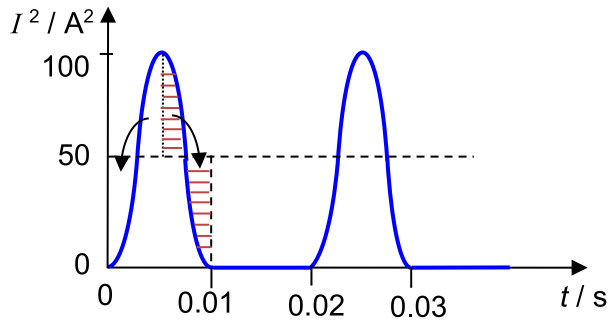
In Fig. 2b, mean power $P' = \dfrac{V_{\text{r.m.s.2}}^2}{R} = \dfrac{V_0^{\,2}}{R} = 2P$

D3 **(a)** Case (i) and (ii):

$I_{\text{r.m.s.}} = \dfrac{10}{\sqrt{2}} = 7.07 \text{ A}$

(b) The r.m.s. current of sinusoidal waveforms is independent of frequency or period.

(c) Case (iii): Using graphical method,



1. Square the instantaneous current I

2. Mean square current
= (total area in one period) / (time for one period)
$= \dfrac{(50 \times 0.01) + 0}{0.02} = 25 \text{ A}^2$

3. Take square root of the mean square current $= 5$ A

Therefore $I_{\text{r.m.s.}} = 5$ A

These steps prove that, for half-wave rectified current with peak value I_0, the r.m.s. current is given by

$I_{\text{r.m.s.}} = \dfrac{I_0}{2}$

Alternatively: For full sinusoidal waveform, mean power delivered is

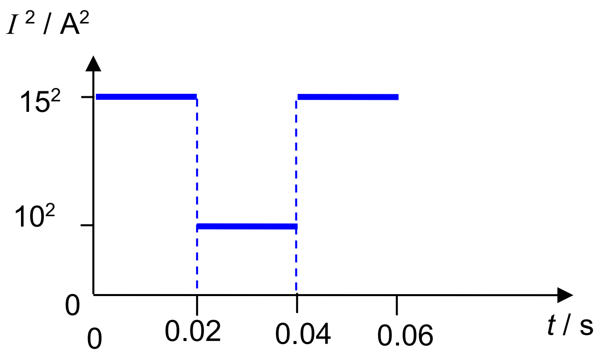
$\langle P \rangle \propto (I_{\text{r.m.s.}})^2$ ----- (1)

For half-wave rectified waveform, half the mean power is "lost",

$\dfrac{\langle P \rangle}{2} \propto (I'_{\text{r.m.s.}})^2$ ----- (2)

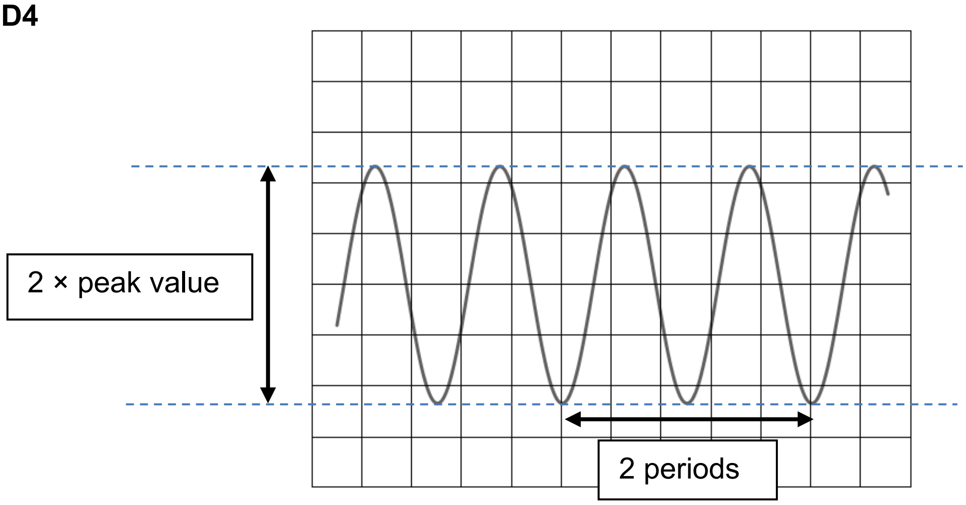
$$\frac{(2)}{(1)} \Rightarrow \frac{1}{2} = \frac{(I'_{\text{r.m.s.}})^2}{(I_{\text{r.m.s.}})^2} \Rightarrow I'_{\text{r.m.s.}} = \frac{1}{\sqrt{2}} I_{\text{r.m.s.}} = \frac{1}{\sqrt{2}}\left(\frac{I_0}{\sqrt{2}}\right) = \frac{I_0}{2} = \frac{10\text{ A}}{2} = 5\text{ A}$$

Case (iv):



$$I_{\text{r.m.s.}} = \sqrt{\frac{(15^2 \times 0.02) + (10^2 \times 0.02)}{0.04}} = 12.7\text{ A}$$

D4



(a) Total time in 2 periods = 5 cm × 0.5 ms cm^{-1} = 2.5 ms
Period = 1.25 ms

(b) Frequency = 1/(1.25 ms) = 800 Hz

(c) Peak value of potential difference = (4 ⅔ cm × 2 V cm^{-1}) / 2 = 4.7 V

(d) Root-mean-square value of potential difference = $\dfrac{4.7}{\sqrt{2}} = 3.3$ V

D5 **[D]**

$$\frac{V_\mathrm{s}}{V_\mathrm{p}} = \frac{N_\mathrm{s}}{N_\mathrm{p}}$$

$$V_\mathrm{s} = \left(\frac{N_\mathrm{s}}{N_\mathrm{p}}\right) V_\mathrm{p} = \frac{3200}{200} \times 15 = 240 \text{ V r.m.s.}$$

$$I_s = \frac{V_\mathrm{s}}{R} = \frac{240}{120} = 2.0 \text{ A}$$

$$P = 240 \times 2.0 = 480 \text{ W}$$

D6 **(a)** **(i)** $\frac{V_\mathrm{s}}{V_\mathrm{p}} = \frac{N_\mathrm{s}}{N_\mathrm{p}} \Rightarrow V_\mathrm{s} = \frac{N_\mathrm{s}}{N_\mathrm{p}} V_\mathrm{p} = \frac{40}{1000} \times 230 = 9.2 \text{ V r.m.s.}$

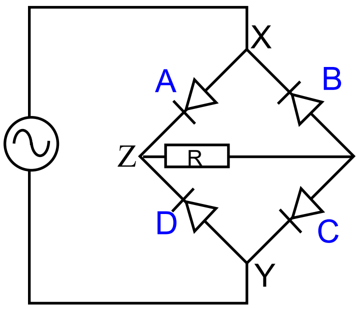
(ii) $V_\mathrm{peak} = 9.2\sqrt{2} = 13.0 \text{ V}$

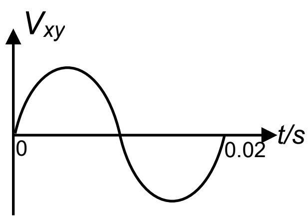
(b) **(i)** To provide half-wave rectification such that current only flows in a direction (clockwise) that charges the battery. If the current flows in the opposite direction, it will discharge the battery.

(ii) To increase overall resistance of circuit in order to reduce charging current
→ restricts the initial charging current which may be large if the battery's initial e.m.f. is low as compared to the p.d. across the secondary coil of the transformer (9.2 V r.m.s.).
→ less heat is generated within the battery once it is fully charged, maintaining the battery temperature within its safe limits

D7 **[B]** Period is the same. The statements for option A, C and D are correct.

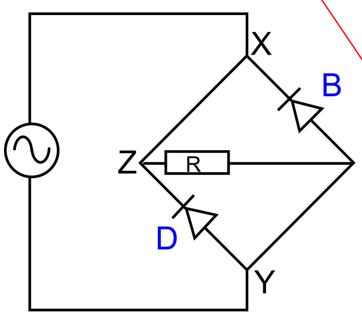
D8 **[A]**

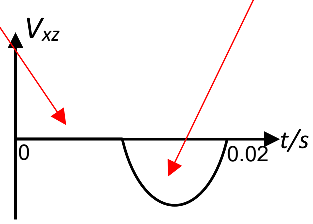


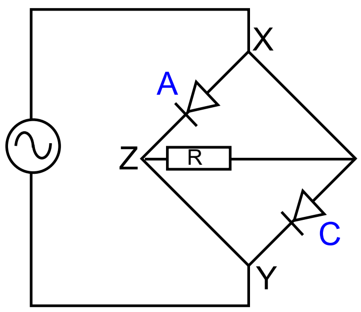


When $V_x > V_y$, diodes A and C are forward biased and conducts, so $V_{xz} = 0$

When $V_x < V_y$, diodes B and D are forward biased and conducts so $V_{xz} = V_{xy}$







Note: Potential difference $V_{XY} = V_X - V_Y$ = potential of X with respect to Y.

C1 In mutual induction, changes of current in one coil (the primary) can cause changes in the flux linkages with another coil (the secondary) and an induced e.m.f. is produced. The flux linkage of the secondary $N_s\Phi_s$ depends on the primary current I_p which is creating the flux.

$$N_s\Phi_s = MI_p$$

where M is the mutual inductance between the two coils, which may or may not be symmetric. M can be shown to be proportional to the product N_pN_s of their numbers of turns, and dependent only on the geometry of the two coils, not on the current.
From this equation,

$$N_s\frac{d\Phi_s}{dt} = M\frac{dI_p}{dt}$$

Or

$$\varepsilon_s = -M\frac{dI_p}{dt}$$

The negative sign is a reflection of Lenz's law.

Similarly, for the opposite case in which a changing current I_s in the secondary causing a changing flux linkage $N_p\Phi_p$ and an e.m.f. ε_p in the primary, we have

$$\varepsilon_p = -M\frac{dI_s}{dt}$$

From Fig. 23.1, $\varepsilon_Q = -M\frac{dI_P}{dt} \Rightarrow 2\text{ mV} = -M\left(5\text{ A s}^{-1}\right)$

From Fig. 23.2, $\varepsilon_P = -M\frac{dI_Q}{dt} \Rightarrow \varepsilon_P = -M\left(2\text{ A s}^{-1}\right)$

Dividing, $\frac{2\text{ mV}}{\varepsilon_P} = \frac{5}{2} \Rightarrow \varepsilon_P = \frac{2}{5}(2\text{ mV}) = 0.80\text{ mV}$

C2 $I = 1500t - 15$

$$I_{\text{r.m.s.}} = \sqrt{\frac{\int_0^T I^2 dt}{T}} = \sqrt{\frac{\int_0^T (1500t-15)^2\,dt}{0.02}}$$

$$= \sqrt{\frac{\int_0^{0.02}\left(2250000t^2 - 45000t + 225\right)dt}{0.02}}$$

$$= \sqrt{\frac{\left[\frac{2250000t^3}{3} - \frac{45000t^2}{2} + 225t\right]_0^{0.02}}{0.02}}$$

$$= \sqrt{75}$$

$$= 8.66\text{ A}$$