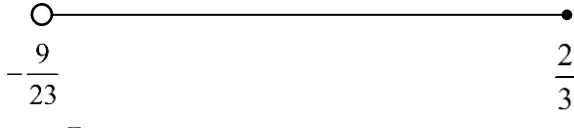


## 2024 Sec 3 E.Math Practice Paper C Solution

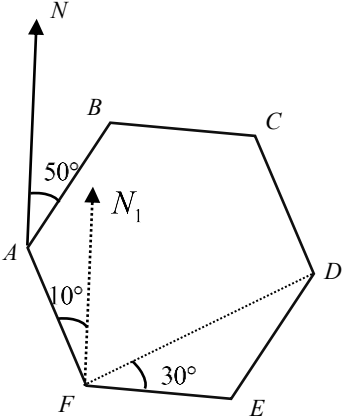
| Qn    | Solutions                                                                                                                                                                                                                                                     |
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| 1(i)  | $y < 0$                                                                                                                                                                                                                                                       |
| 1(ii) | $10a\sqrt{by^2 + 1} = 3y$ $100a^2(by^2 + 1) = 9y^2$ $100a^2 = 9y^2 - 100a^2by^2$ $100a^2 = y^2(9 - 100a^2b)$ $y = -\sqrt{\frac{100a^2}{9 - 100a^2b}} \quad (\text{reject +ve since } y < 0)$ $y = \frac{10a}{\sqrt{9 - 100a^2b}} \quad (\text{since } a < 0)$ |
| 2(a)  | $3(x - 8)(x + 4) = 7(x - 8)$ $3(x - 8)(x + 4) - 7(x - 8) = 0$ $(x - 8)[3(x + 4) - 7] = 0$ $(x - 8)(3x + 5) = 0$ $x = 8 \text{ or } x = -\frac{5}{3} \text{ or } -1\frac{2}{3}$                                                                                |
| 2(b)  | $9^{2x+1} = \frac{27}{81^{4x-3}}$ $3^{2(2x+1)} = \frac{3^3}{3^{4(4x-3)}}$ $3^{4x+2} = 3^{3-16x+12}$ $4x + 2 = 15 - 16x$ $20x = 13$ $x = \frac{13}{20}$                                                                                                        |

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| <b>3</b>    | $-\frac{2}{3}x + 2 < 7x + 5 \leq -2x + 11$ $-\frac{2}{3}x + 2 < 7x + 5 \quad \text{AND} \quad 7x + 5 \leq -2x + 11$ $-3 < 7x + \frac{2}{3}x \quad \text{AND} \quad 9x \leq 6$ $x > -\frac{9}{23} \quad \text{AND} \quad x \leq \frac{2}{3}$ <p>i.e. <math>-\frac{9}{23} &lt; x \leq \frac{2}{3}</math></p>  |
| <b>4(a)</b> | $\frac{u^3 - 2u^2 + u}{u^3 - 1}$ $= \frac{u(u^2 - 2u + 1)}{(u - 1)(u^2 + u + 1)}$ $= \frac{u(u - 1)^2}{(u - 1)(u^2 + u + 1)}$ $= \frac{u(u - 1)}{(u^2 + u + 1)} \text{ or } \frac{u^2 - u}{(u^2 + u + 1)}$                                                                                                                                                                                    |
| <b>4(b)</b> | $4x^2 - 2x - 4x\sqrt{y} + y + \sqrt{y}$ $= (4x^2 - 4x\sqrt{y} + y) + (-2x + \sqrt{y})$ $= (2x - \sqrt{y})^2 - (2x - \sqrt{y})$ $= (2x - \sqrt{y})(2x - \sqrt{y} - 1)$                                                                                                                                                                                                                         |

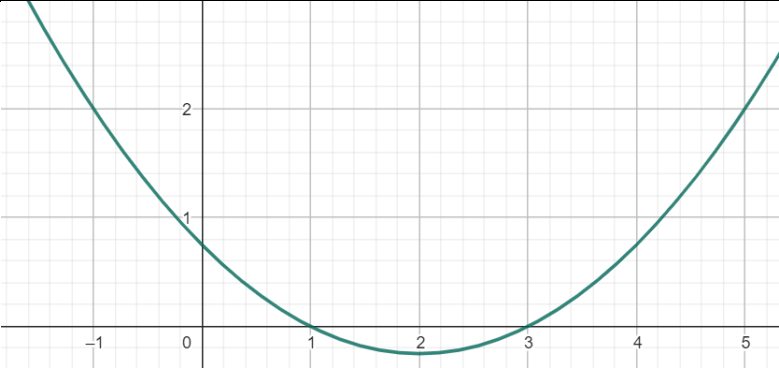
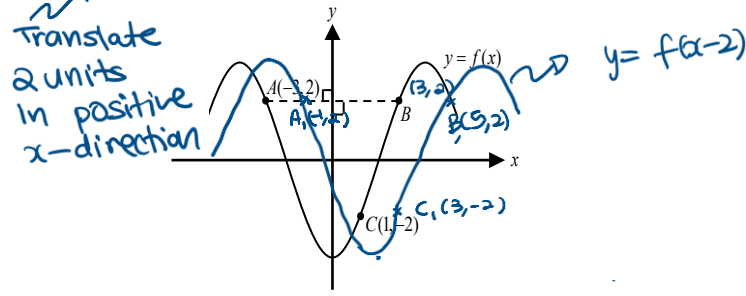
|       |                                                                                                                                                                                                                                                                                                                                                                                                             |
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| 5     | $2x - y = 6$ $x^2 + y^2 = 8$ <p>Sub. <math>y = 2x - 6</math> into <math>x^2 + y^2 = 8</math></p> $x^2 + (2x - 6)^2 = 8$ $x^2 + 4x^2 - 24x + 36 = 8$ $5x^2 - 24x + 28 = 0$ $(5x - 14)(x - 2) = 0$ $x = 2 \text{ or } x = 2.8$ $y = -2 \text{ or } y = -0.4$ <p>Points of intersection: <math>A(2, -2)</math> and <math>B(2.8, -0.4)</math><br/> or : <math>B(2, -2)</math> and <math>A(2.8, -0.4)</math></p> |
| 6(i)  | <p>Centre of circle, <math>C(6, -8)</math></p> <p>Radius <math>r = \sqrt{6^2 + (-8)^2 + 525}</math><br/> <math>= 25</math></p> <p>Alternatively:</p> $x^2 + y^2 - 12x + 16y - 525 = 0$ $(x - 6)^2 - 36 + (y + 8)^2 - 64 - 525 = 0$ $(x - 6)^2 + (y + 8)^2 = 25^2$ <p>Centre of circle, <math>C(6, -8)</math> and<br/> Radius = 25</p>                                                                       |
| 6(ii) | <p>Equation of <math>CQ</math> :</p> $\frac{y + 28}{x - 21} = \frac{-28 + 8}{21 - 6}$ $\frac{y + 28}{x - 21} = \frac{-4}{3}$ $3y = -4x$ $y = -\frac{4}{3}x$ <p>As <math>(0, 0)</math> lies on <math>3y = 4x</math>, <math>CQ</math> passes through <math>O</math>.</p>                                                                                                                                      |

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| <p><b>6 (iii)</b></p> | <p>Centre of new circle = Midpoint of <math>OQ</math><br/> <math>= (10.5, -14)</math></p> <p>Radius of circle = Half of diameter <math>OQ</math><br/> <math>= \frac{1}{2} \times \sqrt{21^2 + 28^2}</math><br/> <math>= 17.5</math></p> <p>Hence, equation of the circle:<br/> <math>(x - 10.5)^2 + (y + 14)^2 = 17.5^2</math><br/> <math>x^2 - 21x + 110.25 + y^2 + 28y + 196 = 360.25</math><br/> <math>x^2 + y^2 - 21x + 28y = 0</math><br/> <math>k = -21</math><br/> <math>q = 28</math></p> |
| <p><b>7(i)</b></p>    | <p>Since <math>\angle BCO = 90^\circ</math>, therefore<br/> <math>\cos(0.9273) = \frac{x}{x+6}</math><br/> <math>x \cos(0.9273) + 6 \cos(0.9273) = x</math><br/> <math>x = \frac{6 \cos(0.9273)}{1 - \cos(0.9273)}</math><br/> <math>x = 9.00</math> (shown)</p>                                                                                                                                                                                                                                  |
| <p><b>7(ii)</b></p>   | <p>Area of Sector <math>OBD = \frac{1}{2}(8.999 + 6)^2(0.9273)</math><br/> <math>= 104.307 \text{ cm}^2</math></p> <p>Area of <math>\triangle OBC = \frac{1}{2}(9)(14.999)\sin(0.9273)</math><br/> <math>= 53.997 \text{ cm}^2</math></p> <p>Shaded Area <math>= 104.307 \text{ cm}^2 - 53.997 \text{ cm}^2</math><br/> <math>= 50.3 \text{ cm}^2</math> (3s.f)</p>                                                                                                                               |
| <p><b>8(i)</b></p>    | <p><math>\mathbf{B} = 16 \begin{pmatrix} 7 &amp; 6 \\ 4 &amp; 3 \end{pmatrix}</math><br/> <math>= \begin{pmatrix} 112 &amp; 96 \\ 64 &amp; 48 \end{pmatrix}</math></p>                                                                                                                                                                                                                                                                                                                            |

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| <b>8(ii)</b>  | $\mathbf{C} = \begin{pmatrix} 45 \\ 65 \end{pmatrix}$                                                                                                                                                                                                                |
| <b>8(iii)</b> | $\begin{aligned} \mathbf{D} &= \begin{pmatrix} 112 & 96 \\ 64 & 48 \end{pmatrix} \begin{pmatrix} 45 \\ 65 \end{pmatrix} \\ &= \begin{pmatrix} 11280 \\ 6000 \end{pmatrix} \end{aligned}$                                                                             |
| <b>8(iv)</b>  | The elements represent the total amount Peter earns from the 16-week block of sessions on weekdays and on weekends respectively.                                                                                                                                     |
| <b>8(v)</b>   | $\begin{pmatrix} 36 \\ 52 \end{pmatrix}$                                                                                                                                                                                                                             |
| <b>8(vi)</b>  | $\begin{aligned} \mathbf{T} &= 16(1 \quad 1) \begin{pmatrix} 10 & 8 \\ 7 & 4 \end{pmatrix} \begin{pmatrix} 36 \\ 52 \end{pmatrix} \\ &= (19776) \end{aligned}$                                                                                                       |
| <b>9(i)</b>   | <p>Equation of line <math>Y = 2.5X + c</math> and <math>(4, 8)</math> lies on the line,</p> $8 = 10 + c$ $c = -2$ <p>Hence, the equation of the line:</p> $xy = 2.5y - 2$ $xy - 2.5y = -2$ $y(x - 2.5) = -2$ $y = \frac{-2}{x - 2.5}$ $a = -2 \text{ and } b = -2.5$ |
| <b>9(ii)</b>  | $\tan \theta = 2.5$ $\theta = 1.19 \text{ (3 s.f)}$                                                                                                                                                                                                                  |

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| <b>10(i)</b>   | $\text{Interior } \angle AFE = \frac{180^\circ(6-2)}{6}$ $= 120^\circ$                                                                                                                                                                                                                                                              |
| <b>10(ii)</b>  | <p>Bearing of <math>D</math> from <math>F</math> :</p> $= \angle AFE - \angle AFN_1 - \angle DFE$ $= 120^\circ - (180^\circ - 170^\circ) - \left( \frac{180^\circ - 120^\circ}{2} \right)$ $= 120^\circ - (10^\circ) - (30^\circ)$ $= 080^\circ$  |
| <b>10(iii)</b> | $AE = \sqrt{5^2 + 5^2 - 2(5)(5)\cos 120^\circ}$ $= 5\sqrt{3}$ <p>OR</p> $AE = 2 \times 5 \cos 30^\circ$ $= 5\sqrt{3}$ $\tan \theta = \frac{2}{AE}, \text{ where } \theta = \angle \text{ of elevation}$ $\tan \theta = \frac{2}{5\sqrt{3}}$ $\theta = 13.0^\circ \text{ (1d.p)}$                                                    |

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| <b>11(i)</b>   | $y\text{-coordinate of } P = \frac{1+2}{2} = 1.5$<br>$3x - 4 \times 1.5 = 27$<br>$3x = 33$<br>$x = 11$<br>$x\text{-coordinate of } P = 11$<br>hence, $k + 5 = 11 \times 2$<br>$k = 17$                                                                                              |
| <b>11(ii)</b>  | $\text{Gradient of line } AB = \text{Gradient of line } CD = \frac{4-2}{1-5} = -\frac{1}{2}$<br>Since $(17, 1)$ lies on line $CD$ ,<br>$y = -\frac{1}{2}x + c$<br>$(1) = -\frac{1}{2}(17) + c$<br>$c = 9.5$<br>$y = -\frac{1}{2}x + 9.5$                                            |
| <b>11(iii)</b> | Equation of $PQ$ is $3x - 4y = 27$ (Given)<br>Equation of $CD$ is $y = -\frac{1}{2}x + 9.5$<br>Solving simultaneously:<br>$3x - 4(-\frac{1}{2}x + 9.5) = 27$<br>$3x + 2x - 38 = 27$<br>$x = 13$<br>$y = 3$<br>$Q(13, 3)$                                                            |
| <b>11(iv)</b>  | $P$ has coordinates $(11, 1.5)$<br>$= \frac{1}{2} \begin{vmatrix} 1 & 5 & 11 & 13 & 1 \\ 4 & 2 & 1.5 & 3 & 4 \end{vmatrix}$ $\text{Area of the quadrilateral } ABPQ = \frac{1}{2} [(2 + 7.5 + 33 + 52) - (20 + 22 + 19.5 + 3)]$ $= \frac{1}{2} [94.5 - 64.5]$ $= 15 \text{ unit}^2$ |

|           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
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| 12(a)     | $f(x) = 1 - 2x + 3x^2 - x^3 \quad h(x) = 7 - 4x + 6x^2 - 2x^3$ <p>Let <math>y = 1 - 2x + 3x^2 - x^3</math></p> $h(x) = 7 - 4x + 6x^2 - 2x^3$ <p>I: <math>y = 1 - 2x + 3x^2 - x^3 \longrightarrow y_1 = 2(1 - 2x + 3x^2 - x^3)</math></p> $\therefore y_1 = 2 - 4x + 6x^2 - 2x^3$ <p>Scale with scale factor 2 parallel to <math>y</math>-axis.</p> <p>II: <math>y_1 = 2 - 4x + 6x^2 - 2x^3 \longrightarrow y_2 = 7 - 4x + 6x^2 - 2x^3</math></p> <p>Translate 5 units in the positive <math>y</math>-direction.</p> |
| 12(b)(i)  | $\text{gradient} = \frac{4-1}{13-1}$ $m = \frac{1}{4}$ $y + x - 1 = \frac{1}{4}(x^2 - 1)$ $y = \frac{1}{4}x^2 - x + \frac{3}{4}$                                                                                                                                                                                                                                                                                                                                                                                    |
| 12(b)(ii) |                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| 13(a)     | $C_{\text{new}} : (1-4)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| 13(b)(i)  | <p><math>y_1 = f(x-2)</math></p> <p>Translate 2 units in positive <math>x</math>-direction</p>                                                                                                                                                                                                                                                                                                                                  |

13(b)(ii)

Translate 1 unit in positive y-direction

$$y_2 = 1 - f(x)$$

Reflect about x-axis

$$y = 1 - f(x)$$

