CEDAR GIRLS' SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025
SECONDARY FOUR

CANDIDATE NAME	

CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS

4049/02

Paper 2

28 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ \dots\ (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 An object was launched from a machine. Its height, y metres above ground, t seconds after launch, can be modelled by a quadratic function $y = f(t)$. Four seconds after launch, the object reached its maximum height of 81 m and then dropped to a height of 54 m after a further three seconds.

(a) Find the initial height of the object when it was launched. [4]

$$y = a(t-4)^2 + 81$$

Substitute (7, 54)

$$54 = a(7-4)^2 + 81$$

$$a = -3$$

$$y = -3(t-4)^2 + 81$$

At $t = 0$

$y = 33$

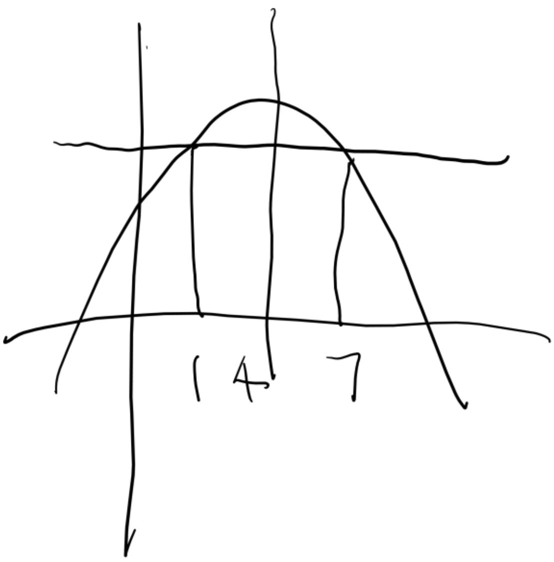
Initial height the object was launched = 33 m

(b) Without solving a quadratic equation, explain and determine how long did the object stay above 54 m? [2]

Since the line of symmetry is $t = 4$, the object was at height 54 at $t = 1$.

Length of time $= 2 \times 3$

$= 6$ seconds



2 Show that the solution of the equation $12^{x+1} = 36 \times 4^{x+2}$ can be expressed as $1 + \dfrac{\lg a}{\lg b}$, where a and b are integers. [5]

$$12^{x+1} = 36 \times 4^{x+2}$$
$$12 \times 12^x = 36 \times 4^x \times 16$$
$$\frac{12^x}{4^x} = 48$$
$$3^x = 48$$
$$\lg 3^x = \lg 48$$
$$x \lg 3 = \lg 48$$
$$x \lg 3 = \lg(16 \times 3)$$
$$= \lg 16 + \lg 3$$
$$x = 1 + \frac{\lg 16}{\lg 3}$$

3 **(a)** Solve the equation $3x^3-11x^2+3x+2=0$, expressing non-integer roots in surd form. [5]

Let f(x) $= 3x^3-11x^2+3x+2$

$f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^3-11\left(\frac{2}{3}\right)^2+3\left(\frac{2}{3}\right)+2$

$= 0$

Therefore $3x-2$ is a factor of f(x)

f(x) $= (3x-2)(x^2+bx-1)$

Comparing coefficient of x^2,

$3b-2 = -11$

$b = -3$

f(x) $= (3x-2)(x^2-3x-1)$

When $3x^3-11x^2+3x+2=0$

$(3x-2)(x^2-3x-1) = 0$

$x=\frac{2}{3}$ or $x=\frac{3\pm\sqrt{3^2-4(1)(-1)}}{2(1)}$

$x=\frac{3\pm\sqrt{13}}{2}$

(b) Hence, solve the equation $x^3+11x^2+9x-18=0$, expressing non-integer roots in surd form. [2]

$x^3+11x^2+9x-18=0$

$-\frac{x^3}{9}-\frac{11x^2}{9}-x+2=0$

$3\left(-\frac{x}{3}\right)^3-11\left(-\frac{x}{3}\right)^2+3\left(-\frac{x}{3}\right)+2=0$

$-\frac{x}{3}=\frac{2}{3}$ or $-\frac{x}{3}=\frac{3\pm\sqrt{13}}{2}$

$x=-2$ or $x=\frac{-9\pm3\sqrt{13}}{2}$

4 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{12}{(3-2x)^2}$. Given that the curve passes through the points $\left(\frac{1}{2},1\right)$ and $\left(1,\ln(8e^2)\right)$, show that the y-intercept of the curve can be expressed as $a\ln\left(\frac{b}{c}\right)$ where a, b and c are constants. [8]

$$\frac{d^2y}{dx^2}=\frac{12}{(3-2x)^2}$$

$$\frac{d^2y}{dx^2}=12(3-2x)^{-2}$$

$$\frac{dy}{dx}=\frac{12(3-2x)^{-1}}{-2(-1)}+c$$

$$= 6(3-2x)^{-1}+c$$

$$y=\frac{6\ln(3-2x)}{-2}+cx+d$$

$$= -3\ln(3-2x)+cx+d$$

Since $\left(1,\ln(8e^2)\right)$ lies on the curve

$\ln(8e^2) = c + d$

$d = \ln(8e^2) - c$ ----- (1)

Since $\left(\frac{1}{2},1\right)$ lies on the curve

$1 = -3\ln 2 + \frac{c}{2} + d$ ----- (2)

$1 = -3\ln 2 + \frac{c}{2} + \ln(8e^2) - c$

$c = 2$

$d = 3\ln 2$

$y = -3\ln(3-2x)+2x+3\ln 2$

When $x = 0$

$y = -3\ln 3+3\ln 2$

$= 3\ln\left(\frac{2}{3}\right)$

5 A tangent to a circle at the point (4, 3) intersects the x-axis at $x = 7$. The line with the equation $3y = 2x + 6$ is normal to the circle at another point.

(a) Find the equation of the circle. [8]

Gradient of tangent $= \dfrac{3-0}{4-7}$

$= -1$

Gradient of normal $= 1$

Sub (4, 3) into $y = x + c$

$3 = 4 + c$

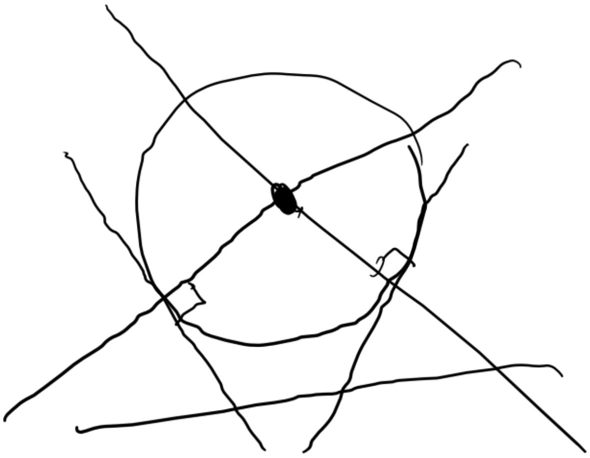
$c = -1$

Equation of normal is $y = x - 1$

$y = x - 1$ -------- (1)

$3y = 2x + 6$ -------- (2)

Sub (1) in (2)



$3(x-1) = 2x + 6$

$3x - 3 = 2x + 6$

$x = 9$

$y = 8$

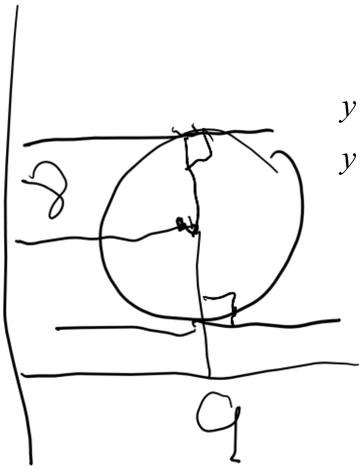
Centre of circle is (9, 8)

Radius $= \sqrt{(9-4)^2 + (8-3)^2}$

$= \sqrt{50}$

Equation of circle is $(x-9)^2 + (y-8)^2 = 50$

(b) Find, in exact form, the equations of the tangents to the circle that are parallel to the x-axis. [2]



$y = 8 + \sqrt{50}$

$y = 8 - \sqrt{50}$

6 A particle, P, starts moving in a straight line with a velocity of 8 m/s at a displacement of 3 m from a fixed point O. The velocity, v m/s of the particle t seconds after moving is given by $v = 5\cos t + 2\sin t + c$.

(a) Find the value of c. [2]

(a) $v = 5\cos t + 2\sin t + c$
When $t = 0$
$8 = 5 + c$
$c = 3$

(b) Find the initial acceleration of the particle. [2]

(b) $a = -5\sin t + 2\cos t$
Initial acceleration $= 2$ m/s^2

(c) By explaining with relevant working, show the particle changed its direction of motion during the 3rd second. [3]

(c) $v = 5\cos t + 2\sin t + 3$

At $t = 2$,

$v = 5\cos 2 + 2\sin 2 + 3$

$= 2.7379$

At $t = 3$,

$v = 5\cos 3 + 2\sin 3 + 3$

$= -1.6677$

Since the velocity changed sign from positive to negative during $t = 2$ to $t = 3$, the particle changed its direction during the 3rd second.

(d) Find displacement of P from O at $t = 6$. [3]

Displacement $= 5\sin t - 2\cos t + 3t + d$

When $t = 0$, $s = 3$

$-2 + 0 + d = 3$

$d = 5$

Therefore, $s = 5\sin t - 2\cos t + 3t + 5$

When $t = 6$,

$s = 5\sin 6 - 2\cos 6 + 3(6) + 5$

$= 19.7$ m

Alternative

When $t = 0$, $s = 0$

$-2 + 0 + d = 0$

$d = 2$

Therefore, $s = 5\sin t - 2\cos t + 3t + 2$

When $t = 6$,

$s = 5\sin 6 - 2\cos 6 + 3(6) + 2$

$= 16.7$ m

7 It is given that there is a term in x^2 in the expansion of $\left(\sqrt{x}-\dfrac{2}{x}\right)^n$, where n is a positive integer.

(a) Find the smallest possible value of n, explaining your answer. [4]

$$
\begin{aligned}
T_{r+1} &= \binom{n}{r}\left(x^{\frac{1}{2}}\right)^{n-r}\left(-\frac{2}{x}\right)^{r} \\
&= \binom{n}{r}x^{\frac{1}{2}n-\frac{1}{2}r}(-2)^{r}x^{-r} \\
&= \binom{n}{r}x^{\frac{1}{2}n-\frac{3}{2}r}(-2)^{r}
\end{aligned}
$$

$$\frac{1}{2}n-\frac{3}{2}r = 2$$

$$n = 3r+4$$

Smallest possible value of $r = 0$.
Therefore, the smallest possible value of $n = 4$.

(b) In the case where $n = 7$, show that there is no constant term in the expansion of $\left(\frac{48}{x^2} - x^4\right)\left(\sqrt{x} - \frac{2}{x}\right)^n$. [5]

$T_{r+1} = \binom{7}{r} x^{\frac{7}{2}-\frac{3}{2}r} (-2)^r$

$\frac{7}{2} - \frac{3}{2}r = 2$

$r = 1$

coefficient of x^2 $= \binom{7}{1}(-2)^1$

$= -14$

$\frac{7}{2} - \frac{3}{2}r = -4$

$r = 5$

coefficient of x^{-4} $= \binom{7}{5}(-2)^5$

$= -672$

Contant term $= 48(-14) - (1)(-672)$

$= 0$

Therefore there is no constant term. (shown)

8 The temperature, $\theta°C$, of a cup of hot chocolate milk, as it cools to room temperature, can be modelled by the formula $\theta = 27 + qb^{-t}$, where q and b are constants and t is the time of cooling in minutes. Measurements of t and θ are shown in the table below.

t	2	4	6	8	10	12
θ	71	59	48	43	38	35

(a) Explain how a straight line graph can be drawn to represent the formula and draw it for the given data. [4]

$\theta = 27 + qb^{-t}$

$\theta - 27 = qb^{-t}$

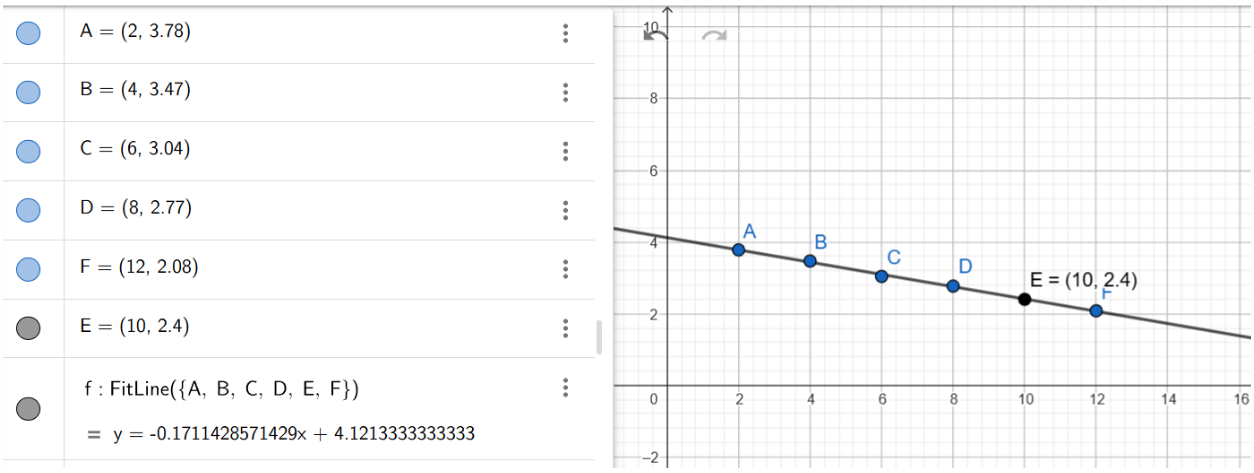
$\ln(\theta - 27) = \ln(qb^{-t})$

$\ln(\theta - 27) = \ln q - t \ln b$

$\ln(\theta - 27) = (-\ln b)t + \ln q$

Plot $\ln(\theta - 27)$ against t to get a straight line

t	2	4	6	8	10	12
$\theta - 27$	44	32	21	16	11	8
$\ln(\theta - 27)$	3.78	3.47	3.04	2.77	2.40	2.08



(b) Estimate the value of q and of b. [3]

Vertical intercept $= 4.1$ (allow ± 0.1)

$\ln q = 4.1$

$q = e^{4.1}$

$= 60.340$

$= 60.3$ (allow 54.6 to 66.7)

Gradient $= -\ln b = \dfrac{4.1 - 2.2}{0 - 11.2}$

$\ln b = 0.16964$ (0.16 to 0.18)

$b = e^{0.16964}$

$= 1.1849$

$= 1.18$ (allow 1.17 to 1.20)

(c) Use your graph to estimate the temperature of the cup of tea at $t = 7$. [2]

At $t = 7$,

$\ln(\theta - 27) = 2.925$ (allow 2.8 to 3.0) B1

$\theta = 27 + e^{2.925}$

$= 45.634$

$= 45.6°C$ (allow 43.4 to 47.1) B1

(d) What is the room temperature? Explain how you obtain your answer. [2]

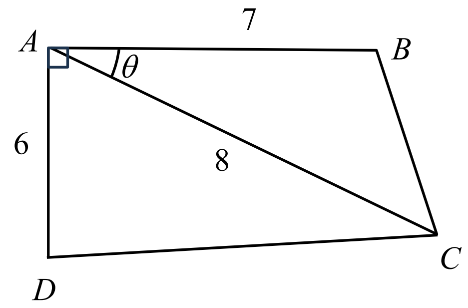
Since $\theta = 27 + 60.3(1.18)^{-t}$

As t increases, 1.18^{-t} approaches 0.

Therefore $60.3(1.18)^{-t}$ approaches 0.

Thus θ approaches 27 which is the room temperature.

9



A farmer owns two triangular plots of land ABC and ADC as shown in the diagram above. It is given that $AB = 7$ m, $AC = 8$ m and $AD = 6$ m, angle BAD is a right angle and angle $BAC = \theta$ radians.

(a) **(i)** Show the total area of the two plots of land, A m^2, can be expressed in the form $a\sin\theta + b\cos\theta$, where a and b are integers. [3]

(a) (i) Total area $= \frac{1}{2}(7)(8)\sin\theta + \frac{1}{2}(6)(8)\sin\left(\frac{\pi}{2} - \theta\right)$

$= \frac{1}{2}(7)(8)\sin\theta + \frac{1}{2}(6)(8)\cos\theta$

$= 28\sin\theta + 24\cos\theta$

(ii) Express A in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

$\tan\alpha = \frac{24}{28} = \frac{6}{7}$

$\alpha = \tan^{-1}\frac{6}{7}$

$= 0.70863$

$A = \sqrt{1360}\sin(\theta + 0.70863)$

$= 4\sqrt{85}\sin(\theta + 0.70863)$

(iii) Find the greatest possible value of A and the value of θ at which this occurs. [3]

(iii) Greatest possible value is $\sqrt{1360} = 4\sqrt{85}$

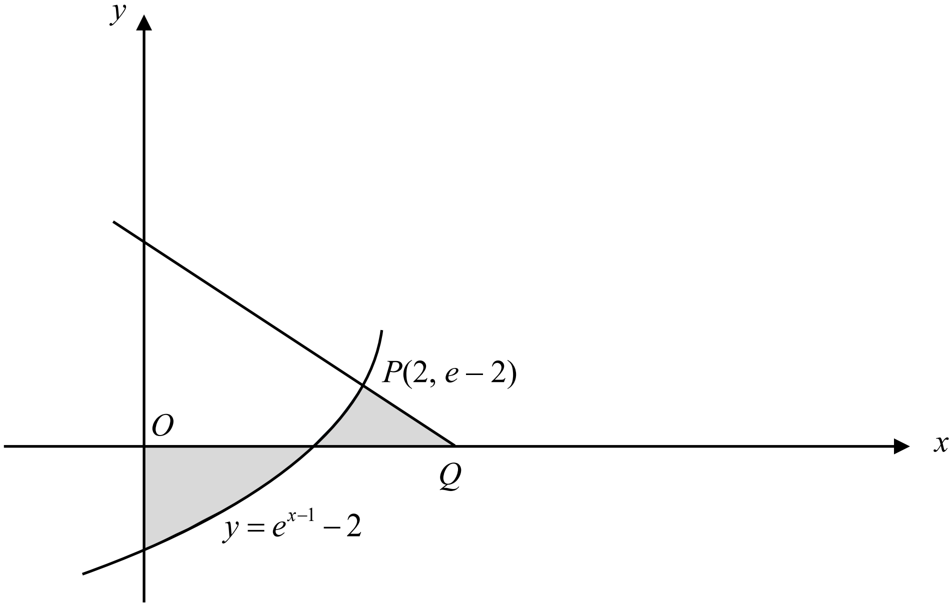
$$\theta + 0.70863 = \frac{\pi}{2}$$
$$= 0.86217$$
$$= 0.862$$

(b) The farmer would like to grow roses on plot *ABC* and hibiscus on plot *ADC*. Given that he wants the area of plot *ABC* to be twice the area of plot *ADC*, find the value of θ. [3]

(b) Area of triangle *ABC* = 2 × Area of triangle *ADC*

$$\frac{1}{2}(7)(8)\sin\theta = 2 \times \frac{1}{2}(6)(8)\cos\theta$$
$$\tan\theta = \frac{12}{7}$$
$$\theta = 1.0427$$
$$= 1.04$$

10



The diagram shows part of the curve $y = e^{x-1} - 2$. The point $P(2,\ e-2)$ lies on the curve and the normal to the curve at P meets the x-axis at Q.

(a) Find the x-coordinate of Q in terms of e. [5]

$y = e^{x-1} - 2$

$\frac{dy}{dx} = e^{x-1}$

Gradient of tangent at P $= e$

Gradient of normal at P $= -\frac{1}{e}$

$\frac{0-(e-2)}{q-2} = -\frac{1}{e}$

$q - 2 = e(e-2)$

$q = e(e-2) + 2$

(b) Find the total area of the shaded region. [7]

$$\begin{aligned} e^{x-1}-2 &= 0 \\ e^{x-1} &= 2 \\ x &= 1+\ln 2 \\ &= 1.6931 \end{aligned}$$

Shaded area under the x-axis $= \int_{1.6931}^{0} e^{x-1}-2dx$

$= \left[e^{x-1}-2x\right]_{1.6931}^{0}$

$= e^{-1} - (e^{0.6931}-3.3862)$

$= 3.3862 + 0.36788 - 1.9999$

$= 1.7542$

Shaded area above the x-axis $= \int_{1.6931}^{2} e^{x-1}-2dx + \frac{1}{2}\times(e-2)(e^2-2e)$

$= \left[e^{x-1}-2x\right]_{1.6931}^{2} + \frac{1}{2}\times(e-2)^2(e)$

$= e - 4 - (e^{0.6931}-3.3862) + 0.70122$

$= 0.10458 + 0.70122$

$= 0.8058$

Total area $= 1.7542 + 0.8058$

$= 2.56$ units^2

End of Paper