RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 6

Term 2 Week 10 Math Focus
Topic: Vectors 1

1 The vectors **u** and **v** are given by $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = a\mathbf{i} + b\mathbf{k}$, where a and b are constants.

(i) Find $(\mathbf{u} + \mathbf{v}) \times (\mathbf{u} - \mathbf{v})$ in terms of a and b. [2]

(ii) Given that the **i**- and **k**-components of the answer to part **(i)** are equal, express $(\mathbf{u} + \mathbf{v}) \times (\mathbf{u} - \mathbf{v})$ in terms of a only. Hence find, in an exact form, the possible values of a for which $(\mathbf{u} + \mathbf{v}) \times (\mathbf{u} - \mathbf{v})$ is a unit vector. [4]

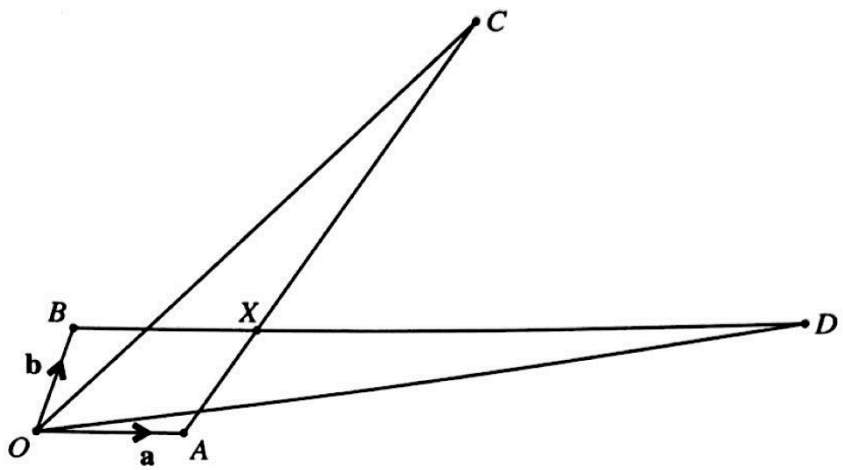
(iii) Given instead that $(\mathbf{u} + \mathbf{v}).(\mathbf{u} - \mathbf{v}) = 0$, find the numerical value of $|\mathbf{v}|$. [2]

2 Vectors **a**, **b** and **c** are such that $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{a} \times 3\mathbf{b} = 2\mathbf{a} \times \mathbf{c}$.

(i) Show that $3\mathbf{b} - 2\mathbf{c} = \lambda\mathbf{a}$, where λ is a constant. [2]

(ii) It is now given that **a** and **c** are unit vectors, that the modulus of **b** is 4 and that the angle between **b** and **c** is 60°. Using a suitable scalar product, find exactly the two possible values of λ. [5]

3



With reference to the origin O. the points A, B, C and D are such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = 2\mathbf{a} + 4\mathbf{b}$ and $\overrightarrow{OD} = \mathbf{b} + 5\mathbf{a}$. The lines BD and AC cross at X (see diagram).

(i) Express $\overrightarrow{OX}$ in terms of **a** and **b**. [4]

The point Y lies on CD and is such that the points O. X and Y are collinear.

(ii) Express $\overrightarrow{OY}$ in terms of **a** and **b** and find the ratio $OX : OY$. [6]