Topic 10: Area & Volume of Similar Figures and Solids

Singapore Secondary 3/4 E Math EOY Intensive Masterclass

Core Scale Factor Rules for Similar Shapes & Solids

1. **Length Scale Factor (k):**
$$k = \frac{l_1}{l_2} = \frac{h_1}{h_2} = \frac{r_1}{r_2}$$
2. **Area Ratio (k^2):**
$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 = k^2$$
3. **Volume Ratio (k^3):**
$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3 = k^3$$
4. **Mass & Density Relationship:** If two similar solids are made of the **same material** (same density $\rho = \frac{m}{V}$):
$$\frac{M_1}{M_2} = \frac{V_1}{V_2} = k^3$$
5. **Frustum / Truncated Solid Strategy:** Volume of Frustum = Volume of Original Large Solid − Volume of Removed Small Solid.

Part I: Exam-Level Practice Questions

Q1: Two similar triangles ABC and PQR have corresponding side lengths $AB = 6$ cm and $PQ = 9$ cm. Given that the area of triangle ABC is 24 cm^2, calculate the area of triangle PQR.

Q2: Two similar solid cylindrical bottles have heights 12 cm and 18 cm.

(a) Given that the smaller bottle has a volume of 320 cm^3, calculate the volume of the larger bottle.

(b) Given that the total surface area of the larger bottle is 450 cm^2, calculate the surface area of the smaller bottle.

Q3: Two mathematically similar bronze statues have masses of 1.6 kg and 5.4 kg. Given that the height of the smaller statue is 20 cm, calculate the height of the larger statue.

Q4: A solid cone of height 30 cm and base radius 12 cm is cut parallel to its base at a height of 20 cm from the base to remove a smaller top cone.

(a) Find the height and base radius of the small cone that was removed.

(b) Find the ratio of the volume of the small top cone to the volume of the original large cone.

(c) Hence, find the ratio of the volume of the small top cone to the volume of the remaining frustum.

Q5: The total surface area of two mathematically similar spheres are in the ratio 16 : 25.

(a) Find the ratio of their radii.

(b) Given that the volume of the smaller sphere is 128 cm^3, calculate the volume of the larger sphere.

Part II: Fully Worked Step-by-Step Solutions

Solution to Q1

Length scale factor $k = \frac{PQ}{AB} = \frac{9}{6} = \frac{3}{2}$.

Area ratio: $\frac{\text{Area}_{PQR}}{\text{Area}_{ABC}} = k^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$.

$\text{Area}_{PQR} = 24 \times \frac{9}{4} = \mathbf{54}$ cm$^\mathbf{2}$.

Solution to Q2

Length scale factor $k = \frac{h_{\text{large}}}{h_{\text{small}}} = \frac{18}{12} = \frac{3}{2} = 1.5$.

(a) Volume ratio $\frac{V_{\text{large}}}{V_{\text{small}}} = k^3 = (1.5)^3 = 3.375$.
$V_{\text{large}} = 320 \times 3.375 = \mathbf{1080}$ cm$^\mathbf{3}$.

(b) Area ratio $\frac{A_{\text{large}}}{A_{\text{small}}} = k^2 = (1.5)^2 = 2.25$.
$A_{\text{small}} = \frac{A_{\text{large}}}{2.25} = \frac{450}{2.25} = \mathbf{200}$ cm$^\mathbf{2}$.

Solution to Q3

Since both statues are made of bronze (same density), Mass ratio = Volume ratio:
$\frac{V_2}{V_1} = \frac{M_2}{M_1} = \frac{5.4}{1.6} = \frac{54}{16} = \frac{27}{8}$.

Length scale factor $k = \frac{h_2}{h_1} = \sqrt[3]{\frac{V_2}{V_1}} = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$.

$h_2 = 20 \times \frac{3}{2} = \mathbf{30}$ cm.

Solution to Q4

(a) Original cone height $H = 30$ cm. Cut is at 20 cm from base.
Height of top small cone $h = 30 - 20 = \mathbf{10}$ cm.
Length ratio $k = \frac{h}{H} = \frac{10}{30} = \frac{1}{3}$.
Base radius of small cone $r = k \times R = \frac{1}{3} \times 12 = \mathbf{4}$ cm.

(b) Volume ratio $\frac{V_{\text{small}}}{V_{\text{large}}} = k^3 = \left(\frac{1}{3}\right)^3 = \mathbf{\frac{1}{27}}$ (or $1:27$).

(c) $V_{\text{frustum}} = V_{\text{large}} - V_{\text{small}} = 27$ parts $- 1$ part $= 26$ parts.
Ratio of $V_{\text{small}} : V_{\text{frustum}} = \mathbf{1 : 26}$.

Solution to Q5

(a) Surface area ratio $\frac{A_1}{A_2} = \frac{16}{25} = k^2$.
Ratio of radii $k = \sqrt{\frac{16}{25}} = \mathbf{\frac{4}{5}}$ (or $4:5$).

(b) Volume ratio $\frac{V_1}{V_2} = k^3 = \left(\frac{4}{5}\right)^3 = \frac{64}{125}$.
$V_{\text{large}} = 128 \times \frac{125}{64} = 2 \times 125 = \mathbf{250}$ cm$^\mathbf{3}$.