SERANGOON JUNIOR COLLEGE

2012 JC2 PRELIMINARY EXAMINATION

MATHEMATICS

Higher 2 **9740/2**

Thursday **23 Aug 2012**

Additional materials: Writing paper

List of Formulae (MF15)

TIME : 3 hours

READ THESE INSTRUCTIONS FIRST

Write your name and class on the cover page and on all the work you hand in.
Write in dark or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets **[ ]** at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

Total marks for this paper is 100 marks.

This question paper consists of 7 printed pages (inclusive of this page) and 1 blank page.

[TURN OVER]

Section A: Pure Mathematics [40 marks]

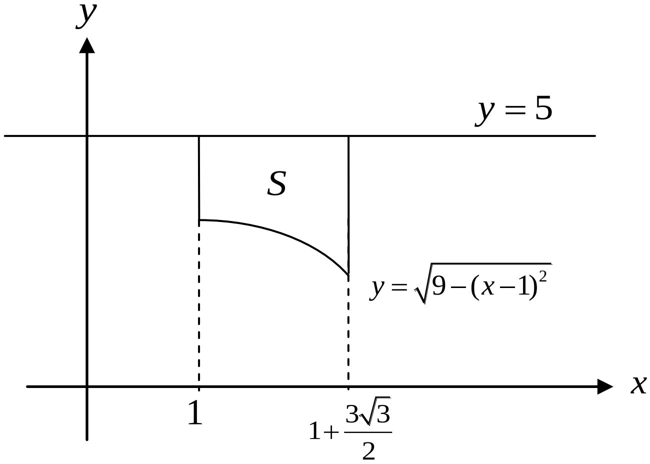
1	A curve is defined by the parametric equations $$x = ku(3-u), \qquad y = 2k(1+u),$$ where k is a positive constant and u is a variable not equal to $\frac{3}{2}$. **(i)** Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of u.	**[2]**
	(ii) Given that x increases at the rate of $\frac{1}{3}$ units per second, find, in terms of k, the rate of change of xy when $y = 4k$.	**[3]**
	Solution	
	(i) $x = ku(3-u)$ $\quad$ $y = 2k(1+u)$ $= 3ku - ku^2$ $\quad$ $= 2k + 2ku$ $\frac{\mathrm{d}x}{\mathrm{d}u} = 3k - 2ku$ $\quad$ $\frac{\mathrm{d}y}{\mathrm{d}u} = 2k$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$ $= 2k \times \frac{1}{3k-2ku}$ $= \frac{2}{3-2u}$	
	(ii) Given $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{3}$, $y = 4k$ $\Rightarrow u = 1, x = 2k$ $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$ $= \frac{2}{3-2(1)} \times (\frac{1}{3})$ $= \frac{2}{3}$ $\frac{\mathrm{d}}{\mathrm{d}t}(xy) = x\frac{\mathrm{d}y}{\mathrm{d}t} + y\frac{\mathrm{d}x}{\mathrm{d}t}$ $= (2k)(\frac{2}{3}) + (4k)(\frac{1}{3})$ $= \frac{8k}{3}$	

2 **(a)** **(i)** Using the substitution $x = 3\cos t + 1$, show that

$$\int_1^{1+\frac{3\sqrt{3}}{2}} \sqrt{9-(x-1)^2}\ \mathrm{d}x = \frac{9}{2}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right).$$

[4]

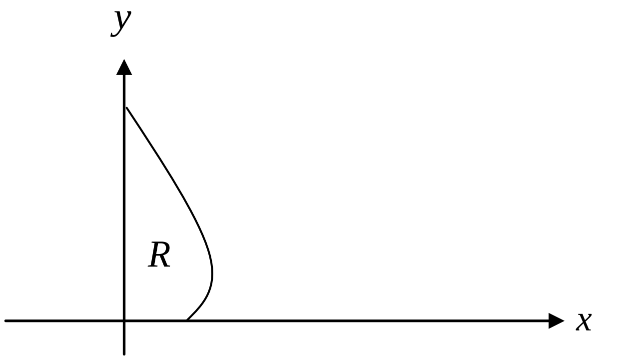
(ii) The curve C has equation $y = \sqrt{9-(x-1)^2}$. The region enclosed by the curve, the horizontal line $y = 5$, and the vertical lines $x = 1$ and $x = 1+\frac{3\sqrt{3}}{2}$ is denoted by S as shown in the diagram below.



By considering the graph of $y = \sqrt{9-(x-1)^2} - 5$ and using the results in **(i)**, find the volume of the solid generated when the region S is rotated through 2π radians about the horizontal line $y = 5$, leaving your answer in the form $\frac{a\sqrt{3}\pi}{8} + b\pi^2$ where a and b are integers to be determined. **[3]**

(b) The curve is defined parametrically by $x = e^t \cos t,\ y = e^t \sin t,\ 0 \le t \le \frac{\pi}{2}$.

Find the area of the region R enclosed by the curve and the axes as shown below.



[3]

Solution

(i) $x = 3\cos t + 1$

$\frac{\mathrm{d}x}{\mathrm{d}t} = -3\sin t$

When $x = 1$, $\quad 1 = 3\cos t + 1$

$\cos t = 0$

$$t = \frac{\pi}{2}$$

When $x = 1+\frac{3\sqrt{3}}{2}$, $\quad 1+\frac{3\sqrt{3}}{2} = 3\cos t + 1$

$$\cos t = \frac{\sqrt{3}}{2}$$

$$t = \frac{\pi}{6}$$

$$\int_{1}^{1+\frac{3\sqrt{3}}{2}} \sqrt{9-(x-1)^2}\ \mathrm{d}x = \int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \sqrt{9-\left[(3\cos t+1)-1\right]^2}\ (-3\sin t)\ \mathrm{d}t$$

$$= \int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \sqrt{9-9\cos^2 t}\ (-3\sin t)\ \mathrm{d}t$$

$$= \int_{\frac{\pi}{2}}^{\frac{\pi}{6}} 3\sqrt{1-\cos^2 t}\ (-3\sin t)\ \mathrm{d}t$$

$$= -9\int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \sin^2 t\ \mathrm{d}t$$

$$= 9\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1-\cos 2t}{2}\ \mathrm{d}t$$

$$= \frac{9}{2}\left[t-\frac{\sin 2t}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \frac{9}{2}\left[\left(\frac{\pi}{2}-\frac{\sin \pi}{2}\right)-\left(\frac{\pi}{6}-\frac{\sin 2\left(\frac{\pi}{6}\right)}{2}\right)\right]$$

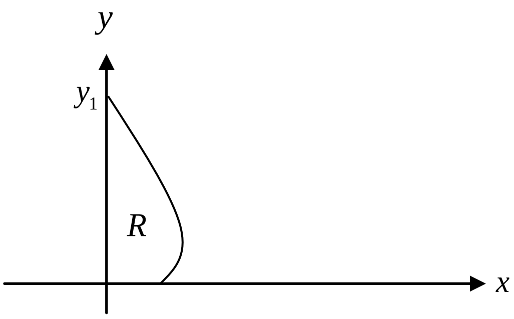
$$= \frac{9}{2}\left[\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right] \text{ (Shown)}$$

(ii) Require volume $\quad = \pi\int_{1}^{1+\frac{3\sqrt{3}}{2}} \left(\sqrt{9-(x-1)^2}-5\right)^2\ \mathrm{d}x$

$$= \pi\int_{1}^{1+\frac{3\sqrt{3}}{2}} 9-(x-1)^2+25-10\sqrt{9-(x-1)^2}\ \mathrm{d}x$$

$$= \pi\int_{1}^{1+\frac{3\sqrt{3}}{2}} 34-(x-1)^2\ \mathrm{d}x - 10\pi\int_{1}^{1+\frac{3\sqrt{3}}{2}} \sqrt{9-(x-1)^2}\ \mathrm{d}x$$

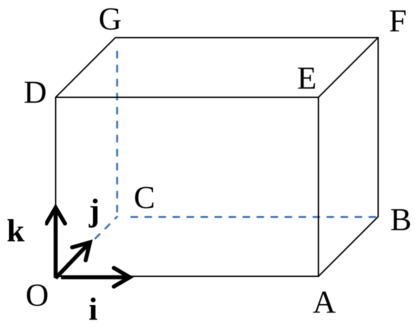
$$= \pi\left[34x-\frac{(x-1)^3}{3}\right]_{1}^{1+\frac{3\sqrt{3}}{2}} - 10\pi\left[\frac{9}{2}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right)\right]$$

	$$=\pi\left[51\sqrt{3}-\frac{27\sqrt{3}}{8}\right]-15\pi^2-\frac{45\sqrt{3}\pi}{4}$$ $$=\frac{291\sqrt{3}\pi}{8}-15\pi^2 \text{ units}^3$$ $\therefore a=291$ and $b=-15$	
	(b)  When y = 0, $0=e^t\sin t$ $\sin t = 0$ ($\because e^t>0$) $t = 0$ When x = 0, $0=e^t\cos t$ $\cos t = 0$ ($\because e^t>0$) $t=\frac{\pi}{2}$ $y=e^t\sin t$ $\frac{\mathrm{d}y}{\mathrm{d}t}=e^t\sin t+e^t\cos t$ Required area = $\int_0^{y_1} x\ \mathrm{d}y$ $=\int_0^{\frac{\pi}{2}}(e^t\cos t)\ (e^t\sin t+e^t\cos t)\ \mathrm{d}t$ = 5.54 units^2	
3	**(a)** Relative to the origin O, the position vectors of A,B and C are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively where $\mathbf{c}=3\mathbf{a}-2\mathbf{b}$.	
	Given that $\mathbf{a}$ is a unit vector, $\lvert\mathbf{b}\rvert=3$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{2\pi}{3}$, **(i)** find the exact value of λ such that $\mathbf{a}$ is perpendicular to $3\mathbf{b}-4\mathbf{a}+\lambda\mathbf{c}$,	**[3]**
	(ii) show that the exact area of triangle OBC is $\frac{9\sqrt{3}}{4}$ units^2.	**[2]**
	(b) The diagram shows a rectangular box with unit vectors **i**, **j**, **k** as shown. The lengths of OA, OC and OD are 20 units, 12 units and 15 units respectively. The point P lies on BD and divides BD in the ratio $3:2$.	

A line ℓ has Cartesian equation $\dfrac{x+1}{-3}=\dfrac{y-6}{2}=z+3$.

(i) Find the vector equation of line *PE*. **[3]**

(ii) The line *PE* intersects ℓ at *Q*, find the position vector of *Q* .



[3]

Solution

(ai)

Since angle between **a** and **b** is $\dfrac{2\pi}{3}$,

$$\cos\left(\frac{2\pi}{3}\right)=\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

$$-\frac{1}{2}=\frac{\mathbf{a}\cdot\mathbf{b}}{(1)|\mathbf{b}|}$$

$$\mathbf{a}\cdot\mathbf{b}=-\frac{3}{2}$$

$$\mathbf{a}\cdot\left(3\mathbf{b}-4\mathbf{a}+3\lambda\,\mathbf{a}-2\lambda\,\mathbf{b}\right)=0$$

$$3\mathbf{a}\cdot\mathbf{b}-4|\mathbf{a}|^2+3\lambda|\mathbf{a}|^2-2\lambda(\mathbf{a}\cdot\mathbf{b})=0$$

$$-\frac{9}{2}-4+3\lambda+3\lambda=0$$

$$\lambda=\frac{17}{12}$$

(ii)

$$|\mathbf{b}\times\mathbf{a}|=|\mathbf{b}||\mathbf{a}|\sin(\frac{2\pi}{3})=3\left(\frac{\sqrt{3}}{2}\right)=\frac{3\sqrt{3}}{2}$$

$$\text{Area}=\frac{1}{2}\left|\mathbf{b}\times\left(3\mathbf{a}-2\mathbf{b}\right)\right|$$

$$=\frac{1}{2}\left|3\mathbf{b}\times\mathbf{a}-2\mathbf{b}\times\mathbf{b}\right|$$

$$=\frac{1}{2}(3)|\mathbf{b}\times\mathbf{a}|$$

$$=\frac{9\sqrt{3}}{4}$$

(bi) $\overrightarrow{OA}=\begin{pmatrix}20\\0\\0\end{pmatrix}$, $\overrightarrow{OC}=\begin{pmatrix}0\\12\\0\end{pmatrix}$, $\overrightarrow{OD}=\begin{pmatrix}0\\0\\15\end{pmatrix}$

Line ℓ : $\dfrac{x+1}{-3}=\dfrac{y-6}{2}=z+3 \Rightarrow \mathbf{r}=\begin{pmatrix}-1\\6\\-3\end{pmatrix}+\mu\begin{pmatrix}-3\\2\\1\end{pmatrix}, \mu\in\mathbb{R}$

$$\overrightarrow{OP}=\frac{1}{5}(2\overrightarrow{OB}+3\overrightarrow{OD})=\frac{1}{5}\left[2\begin{pmatrix}20\\12\\0\end{pmatrix}+3\begin{pmatrix}0\\0\\15\end{pmatrix}\right]=\begin{pmatrix}8\\4.8\\9\end{pmatrix}=\frac{1}{5}\begin{pmatrix}40\\24\\45\end{pmatrix}$$

$$\overrightarrow{PE}=\overrightarrow{OE}-\overrightarrow{OP}=\begin{pmatrix}20\\0\\15\end{pmatrix}-\begin{pmatrix}8\\4.8\\9\end{pmatrix}=\begin{pmatrix}12\\-4.8\\6\end{pmatrix}=\frac{6}{5}\begin{pmatrix}10\\-4\\5\end{pmatrix}$$

Therefore line PE is $\mathbf{r}=\begin{pmatrix}20\\0\\15\end{pmatrix}+\lambda\begin{pmatrix}10\\-4\\5\end{pmatrix}, \lambda\in\mathbb{R}$.

(ii) Since the line PE intersects ℓ,

$$\begin{pmatrix}20\\0\\15\end{pmatrix}+\lambda\begin{pmatrix}10\\-4\\5\end{pmatrix}=\begin{pmatrix}-1\\6\\-3\end{pmatrix}+\mu\begin{pmatrix}-3\\2\\1\end{pmatrix}$$

$10\lambda+3\mu=-21$

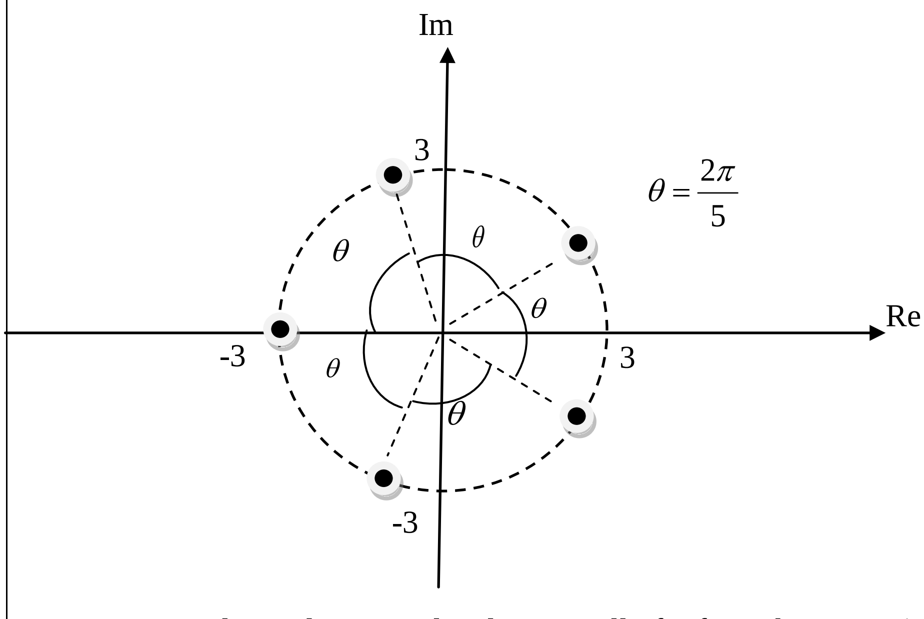
$-4\lambda-2\mu=6$

$5\lambda-\mu=-18$

Using GC, $\lambda=-3,\quad \mu=3$

$\therefore \overrightarrow{OQ}=\begin{pmatrix}-10\\12\\0\end{pmatrix}$

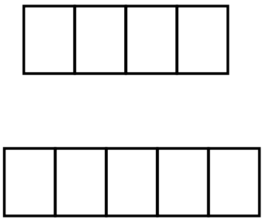
4	**(i)** The polynomial P(z) has real coefficients. The equation P(z) = 0 has a root $re^{i\theta}$, where r > 0 and $0<\theta<\pi$. Show that a quadratic factor of P(z) is $z^2-2rz\cos\theta+r^2$.	**[3]**
	(ii) Solve the equation $z^5=-243$, expressing the solutions in the form $re^{i\theta}$, where r > 0 and $-\pi<\theta\le\pi$.	**[2]**
	(iii) Hence express z^5+243 as the product of one linear factor and two quadratic factors with real coefficients (in trigonometrical form).	**[2]**

	(iv) Show all the roots found in **(ii)** on an Argand diagram. Given that these roots form the vertices of a pentagon, calculate the area of this pentagon.	**[4]**
	(v) If one of the roots in **(ii)** is z_1 where $0 < \arg(z_1) < \dfrac{\pi}{2}$, find the values of n such that $(z_1)^n$ is purely imaginary.	**[3]**
	Solution	
	(i) Since P(z) has real coefficients and $re^{i\theta}$ is a root of P(z) = 0, $re^{-i\theta}$ is also a root of P(z) = 0.	
	A quadratic factor is a product of two linear factors $= (z - re^{i\theta})(z - re^{-i\theta})$	
	$= z^2 - zr\,(e^{i\theta} + e^{-i\theta}) + r^2$	
	$= z^2 - zr\,(2\cos\theta) + r^2$	
	$= z^2 - 2rz\cos\theta + r^2$	
	(ii) $z^5 = -243$	
	$z^5 = 243e^{\pi i}$, $k = 0, \pm1, \pm2$	
	$z = 3e^{\left(\frac{\pi+2k\pi}{5}\right)i}$, $k = 0, \pm1, \pm2$	
	$z = -3,\ 3e^{\pm\frac{\pi}{5}i},\ 3e^{\pm\frac{3\pi}{5}i}$	
	(iii) $z^5 + 243$	
	$= (z+3)(z - 3e^{\frac{\pi}{5}i})(z - 3e^{-\frac{\pi}{5}i})(z - 3e^{\frac{3\pi}{5}i})(z - 3e^{-\frac{3\pi}{5}i})$	
	$= (z+3)\left(z^2 - 6z\cos\left(\dfrac{\pi}{5}\right) + 9\right)\left(z^2 - 6z\cos\left(\dfrac{3\pi}{5}\right) + 9\right)$	
	 G1 – Roots are located corrected and are equally far from the origin (3 units) G1 – Adjacent roots are equally spaced out	
	Area of the pentagon $= 5\left(\dfrac{1}{2}\right)(3)^2\left(\sin\dfrac{2\pi}{5}\right)$	

	$= 21.4$ square units	
	(v) $z_1 = 3e^{i(\frac{\pi}{5})} = 3[\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}]$	
	$(z_1)^n = 3^n[\cos\frac{n\pi}{5} + i\sin\frac{n\pi}{5}]$	
	For $(z_1)^n$ to be purely imaginary, $\cos\frac{n\pi}{5} = 0$ $\Rightarrow \frac{n\pi}{5} = \frac{(2k-1)\pi}{2}$ or $\frac{(2k+1)\pi}{2}$ where $k \in \mathbb{Z}$	
	$\therefore n = \frac{5(2k-1)}{2}$ or $\frac{5(2k+1)}{2}$ where $k \in \mathbb{Z}$	

Section B: Statistics [60 marks]

5	In 2011, a study suggested that approximately 92 per cent of the adult population are right-handed.	
	(i) Sixty samples, each consisting of 20 adults are chosen from different parts of a cosmopolitan city. Find the probability that the total number of right-handed adults is less than 1100.	**[2]**
	(ii) Another sample consisting of n adults is taken where $50 < n < 60$. Using a suitable approximation, find the largest value of n so that the probability that the number of right-handed adults is at most 48 is more than 0.4.	**[3]**
	Solution	
	(i) Let X be the random variable no. of right-handed adults out of 20 adults. $X \sim B(20, 0.92)$ Let X_k denote the number of right-handed adults in k^{th} sample of 20 adults.	
	Since n =60 is large, by C.L.T, $T = X_1 + X_2 + \ldots\ldots X_{60} \sim N(18.4 \times 60, 1.472 \times 60)$ approx $= N(1104, 88.32)$ approx	
	$P(T < 1100) = 0.335$	
	(ii) Let Y be the random variable no of adults who are not right-handed out of n adults. $Y \sim B(n, 0.08)$	
	Since n is large, $n(0.08) < 4.8 < 5$ (as $50 < n < 60$) $Y \sim Po(0.08n)$ approx	
	$P(Y \geq n - 48) > 0.4$ $P(Y \leq n - 49) < 0.6$	
	From G.C, When $n = 52$, $P(Y \leq n - 49) = 0.40285$ When $n = 53$, $P(Y \leq n - 49) = 0.58206$ When $n = 54$, $P(Y \leq n - 49) = 0.73333$	
	From GC largest n = 53	

6	**(a)** Find the number of ways in which 6 different pens can be distributed among 5 students such that each student can receive any number of pens.	**[1]**
	(b) A fleet of 9 different cars are to be parked at a reserved bay of 9 identical lots of which 4 lots are on one side and 5 lots are on the other. 	
	Find the number of possible parking arrangements of these 9 cars	
	(i) if two particular cars cannot be parked side by side.	**[3]**
	(ii) if two particular cars can only be parked at corner lots.	**[2]**
	Solution	
	(a) Number of ways $= 5^6$ $= 15625$	
	(b)(i) Number of parking arrangements $= 9! - (\text{cases where the two cars are taking the side with 4 lots})$ $- (\text{cases where the two cars are taking the side with 5 lots})$ $= 9! - {}^7C_2 3!2!5! - {}^7C_3 4!2!4!$ $= 292320$	
	(ii) Number of parking arrangements $= {}^4C_2(2!)(7!)$ $= 60480$	
7	**(a)** Given that the two events A and B are such that $\mathrm{P}(A\mid B) = \frac{2}{3}$, $\mathrm{P}(A \cap B') = \frac{1}{4}$ and $\mathrm{P}(A \cap B) = \frac{5}{12}$,	
	(i) determine if A and B are independent,	**[1]**
	(ii) find $\mathrm{P}(A \cup B)$.	**[2]**
	(b) Ali and Bob take turns to shoot an arrow in an Archery training session. The probability that Ali hits the bull's eye is 0.3 and for Bob, it is 0.4. Ali shoots first.	
	Find the probability that Ali hits the bull's eye first.	**[3]**
	(c) Calvin, his parents and 7 other people are seated in a round table of 10. Find the probability that his parents are seated together but Calvin is not seated beside either of his parents.	**[2]**
	Solution	
	(ai) $P(A) = P(A \cap B) + P(A \cap B') = \frac{1}{4} + \frac{5}{12} = \frac{2}{3}$	

Since $P(A)=\frac{2}{3}=P(A|B)$, therefore A and B are independent

(ii) $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{2}{3}$

$P(A\cap B)=\frac{2}{3}P(B)$

$\frac{5}{12}=\frac{2}{3}P(B)$

$P(B)=\frac{5}{8}$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{2}{3}+\frac{5}{8}-\frac{5}{12}=\frac{7}{8}$

(b) $0.3+0.7\times0.6\times0.3+0.7\times0.6\times0.7\times0.6\times0.3+\ldots\ldots$

$=0.3\times[1+0.7\times0.6+(0.7\times0.6)^2+\ldots\ldots]$

$=0.3\left[\frac{1}{1-0.42}\right]$

$=\frac{15}{29}$

(c) Required probability = $\frac{2!(7-1)!\,{}^7C_2\,(2!)}{(10-1)!}=\frac{1}{6}$

8 The table gives the values of the GDP per capita, x (in USD$1000) and the total fertility rate (y) for eleven countries.

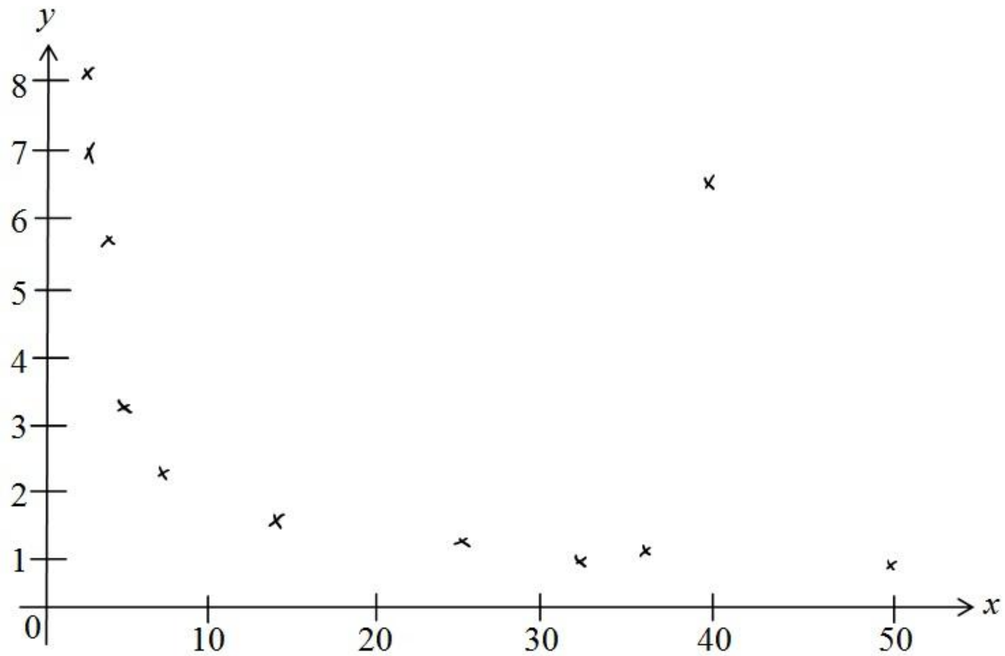
Country	A	B	C	D	E	F	G	H	I	J	K
x (USD$1000)	50	40	37	33	25	14	7	5	4	3	3
y	1.0	6.6	1.1	1.0	1.2	k	2.2	3.2	5.7	7.0	8.1

(i) A statistician thought that a linear model of the form $y=a+bx$ fits the set of data and found that the equation of the regression line of y on x is given by $y=5.0739-0.077885x$. Show that the value of k is 1.5 correct to 1 decimal place. **[2]**

(ii) Sketch a scatter diagram for the set of data and identify the outlier. **[2]**

(iii) Remove the outlier and calculate the value of the product moment correlation coefficient. Hence explain why the statistician's linear model may not be the best model. **[2]**

[TURN OVER]

	(iv) Using the model $y = a + \dfrac{b}{x-2}$, estimate the total fertility rate for a particular country Z when its GDP per capita is USD$1 000, giving your answer to 1 decimal place and comment on the reliability of the estimate.	**[3]**
	Solution:	
	(i) Using GC, $\bar{x} = \dfrac{221}{11}$ $\bar{y} = \dfrac{37.1+k}{11}$	
	Sub $\bar{x} = \dfrac{221}{11}$, $\bar{y} = \dfrac{37.1+k}{11}$ into the equation of the regression line,	
	$\dfrac{37.1+k}{11} = 5.0739 - 0.077885\left(\dfrac{221}{11}\right)$ $\Rightarrow k \approx 1.5$	
	(ii) 	
	Outlier is (40, 6.6)	
	(iii) $r \approx -0.731$	
	Even though the r value calculated in (c) indicate a strong negative linear correlation between y and x but the scatter diagram shows a trend where the y-values decrease at a decreasing rate and so a linear model may not be the best model.	
	(iv) Eqn of regression line of y on $\dfrac{1}{x-2}$: $y = 0.9747 + \dfrac{6.864}{x-2}$	
	$y = 0.9747 + \dfrac{6.864}{1-2} \approx -5.9$	

	Since the value of x =1 substituted is outside the data range, the relationship between the two variables y and $\dfrac{1}{x-2}$ may not be linear and so the estimate obtained for y may not be reliable. OR Since we cannot have a negative fertility rate, the estimate obtained for y is not reliable.	
9	The distance travelled by students on bus A to attend Euler College is modelled by a normal distribution with mean 5 km and standard deviation ® km.	
	(i) If 10% of students travel more than 8 km, show that the value of ® = 2.34.	**[2]**
	(ii) Five different students are randomly selected and are asked one at a time by a teacher on the distance they need to travel on bus A to the College. Find the probability that the fifth student will be the third student travelling less than 8 km on bus A to the College.	**[2]**
	At Gauss College, the distance travelled by students on bus B to attend their school is modelled by a normal distribution with mean 6 km and standard deviation 1.5 km.	
	(iii) Seven students are randomly selected, three from Euler College and four from Gauss College. Find the probability that the average distance travelled by the seven students exceeds 5.9 km.	**[3]**
	(iv) The cost to travel by bus A and bus B is \$0.10/km and \$0.07/km respectively. Find the probability that thrice the cost paid by a student travelling on bus B exceeds the total cost paid by two students travelling on bus A by not more than \$0.20.	**[3]**
	Solution	
	(i) Let X be the r.v. "distance travelled by a randomly chosen student from Euler College". $X \sim N\left(5, \sigma^2\right)$	
	$\mathrm{P}\left(X > 8\right) = 0.1$ $\mathrm{P}\left(X \le 8\right) = 0.9$ $\mathrm{P}\left(Z \le \dfrac{3}{\sigma}\right) = 0.9$	
	$\dfrac{3}{\sigma} = 1.2816$	
	® = 2.34 (shown)	
	(ii) Required probability = $\left[\mathrm{P}\left(X > 8\right)\right]^2 \times \left[\mathrm{P}\left(X < 8\right)\right]^3 \times \dfrac{4!}{2!2!}$ $= \left(0.1\right)^2 \times \left(0.9\right)^3 \times \dfrac{4!}{2!2!}$	

	$= 0.0437$	
	Alternatively,	
	Let M be the number of students who travels less than 8 km to school, out of four students. $M \sim B(4, 0.9)$ $\mathrm{P}(M=2)\times 0.9 = 0.0437$	
	(iii) Let Y be the r.v. . "distance travelled by a randomly chosen student from Gauss College". $Y \sim N\left(6, 1.5^2\right)$	
	$A = \dfrac{X_1 + X_2 + X_3 + Y_1 + Y_2 + Y_3 + Y_4}{7} \sim \mathrm{N}(5.5714, 0.51891)$	
	$\mathrm{P}(A > 5.9) = 0.32414 = 0.324$	
	Alternatively,	
	$T = X_1 + X_2 + X_3 + Y_1 + Y_2 + Y_3 + Y_4 \sim \mathrm{N}(39, 25.4268)$	
	$\mathrm{P}(T > 41.3) = 0.32415 = 0.324$	
	(iv) $3(0.07)Y - 0.1(X_1 + X_2) \sim \mathrm{N}\left(3(0.07)(6) - 0.1(2)(5), 3^2(0.07)^2(1.5)^2 + 0.1^2(2)(2.34)^2\right)$ $3(0.07)Y - 0.1(X_1 + X_2) \sim \mathrm{N}(0.26, 0.208737)$	
	$\mathrm{P}\left(3(0.07)Y - 0.1(X_1 + X_2) \le 0.2\right) = 0.448$	
10	A randomly chosen financial consultant in general sells, on average, 0.75 insurance policies per week. The sales of the insurance policies sold by the financial consultants are taken to be independent of each other.	
	(i) State, in the context of this question, an assumption needed for it to be modelled by a Poisson distribution.	**[1]**
	(ii) Find the probability that in a given month (4 weeks), a randomly chosen financial consultant sells more than one insurance policy per week for at least 3 weeks.	**[2]**
	(iii) Financial consultant A wished to be the top 1% financial consultant, in terms of the number of insurance policies sold, for the year 2012. Taking a year to consist of 52 weeks, estimate the minimum number of policies A needs to sell in 2012 in order for him to meet his target.	**[4]**
	(iv) A manager will check on the performance of two financial consultants B and C once every two weeks. If the total number of policies sold by them in two weeks is at most 3, find the probability that C sold at least 2 policies.	**[3]**
	Solution	
	(i) Assume that the average number of insurance policies sold by a randomly chosen financial consultant per week is a constant every week. OR The expected number of insurance policies sold by a randomly chosen financial consultant in a given interval of time is proportional to the size of the interval of	

	time. OR The sales of the insurance policies occur uniformly.	
	(ii) Let X be the number of insurance policies sold by the financial consultant per week. $X \sim \text{Po}(0.75)$ $\text{P}(X>1) = 1-\text{P}(X\le 1) = 0.173$	
	Let Y be the number of weeks when he sells more than one insurance policy per week $Y \sim \text{B}(4, 0.173)$ $\text{P}(Y\ge 3) = 1-\text{P}(Y\le 2) = 0.0181$	
	(iii) Let W be the number of insurance policies sold by a financial consultant in one year.	
	$W \sim \text{Po}(0.75\times 52)$ $W \sim \text{Po}(39)$ Since $\lambda = 39 (>10)$, $W \sim \text{N}(39, 39)$ approximately.	
	$\text{P}(W\ge n)\le 0.01 \xrightarrow[C.C]{} \text{P}(W>n-0.5)\le 0.01$	
	Method 1	
	$\text{P}\left(Z > \dfrac{n-0.5-39}{\sqrt{39}}\right)\le 0.01$	
	$\text{P}\left(Z \le \dfrac{n-0.5-39}{\sqrt{39}}\right)\ge 0.99$	
	$\dfrac{n-39.5}{\sqrt{39}} \ge 2.3263$	
	$n \ge 54.03$	
	Hence he needs to sell at least 55 insurance policies in 2012 to hit his target.	
	Method 2	
	From G.C,	
	When n = 54, $\text{P}(W>n-0.5) = 0.01012$	
	When n = 55, $\text{P}(W>n-0.5) = 0.00653$	
	When n = 56, $\text{P}(W>n-0.5) = 0.00412$	
	Hence he needs to sell at least 55 insurance policies in 2012 to hit his target.	
	(iv) Let X be the number of insurance policies sold by a financial consultant in 2 weeks. $X \sim \text{Po}(1.5)$	
	$\text{P}\left(X_C \ge 2 \mid X_B + X_C \le 3\right)$	
	$= \dfrac{\text{P}(X_C=2)\text{P}(X_B=0)+\text{P}(X_C=2)\text{P}(X_B=1)+\text{P}(X_C=3)\text{P}(X_B=0)}{\text{P}\left(X_B+X_C\le 3\right)}$	
	$= \dfrac{0.16803}{0.64723} = 0.260$	

11	**(i)** Slimmer, a slimming company, advertised that one will lose his or her mass by at least 5 kg after undergoing their specialized treatment for 3 months. Jane thinks that the company has overstated the figure and decided to verify her belief by asking 15 people who had tried out the treatment. She found out that the mean mass loss of the sample is 4.7 kg with a standard deviation of 0.8 kg. Carry out an appropriate test at 9% level of significance. State an assumption needed for the calculation.	**[5]**
	(ii) Natasha, who also expressed interest in checking the claim, obtained another sample of 25 other people and the amount of mass loss, *y*, are recorded and summarised as follows:	
	$\sum y = 129.5$, $\sum y^2 = 689.05$.	
	By combining the two samples, show that the unbiased estimates of the population mean and variance of the mass loss by the people who have undergone the treatment are 5 and $\dfrac{10}{13}$ respectively. If the null hypothesis $\mu = \mu_0$ is being tested against the alternate hypothesis $\mu < \mu_0$, find the range of values of μ_0 such that the conclusion favours the alternate hypothesis $\mu < \mu_0$ at 5% level of significance.	**[7]**
	Solution	
	(i) Assume that the mass loss of a randomly chosen person who undergoes the treatment follows a normal distribution.	
	Let X be the mass loss of a randomly chosen person who has undergone the treatment by the slimming company.	
	$s^2 = \dfrac{15}{14}(0.8^2) = 0.685714$	
	To test $\text{H}_0 : \mu = 5$ against $\text{H}_1 : \mu < 5$ Left-tailed test at 4% level of significance	
	Under H_0, $T \sim t(14)$ OR Under H_0, test statistic: $T = \dfrac{\overline{X} - 5}{\frac{0.82808}{\sqrt{15}}} \sim t(14)$	
	$\mu = 5$, $\bar{x} = 4.7$, $s = 0.82808$, $n = 15$	
	Using G.C, p-value = 0.0912	
	Since **p-value > 0.09**, we **do not reject** H_0 and conclude that there is insufficient evidence to conclude that the mean mass loss is less than 5kg at 9% level of significance	
	(ii) $\dfrac{\sum_{i=1}^{15} x_i}{15} = 4.7$ => $\sum_{i=1}^{15} x_i = 70.5$	

	Also, $\frac{1}{14}\left(\sum_{i=1}^{15} x_i^2 - \frac{\left(\sum_{i=1}^{15} x_i\right)^2}{15}\right) = s^2 = \frac{15}{14}(0.8^2)$	
	Thus $\sum_{i=1}^{15} x_i^2 = 15\left(0.8^2\right) + \frac{\left(\sum_{i=1}^{15} x_i\right)^2}{15} = 15\left(0.8^2\right) + \frac{70.5^2}{15} = 340.95$	
	Let W denote the mass loss of a randomly chosen person from the sample of 40.	
	$\sum_{i=1}^{40} w_i = \sum_{i=1}^{15} x_i + \sum_{i=1}^{25} y_i = 70.5 + 129.5 = 200$	
	$\sum_{i=1}^{40} w_i^2 = \sum_{i=1}^{15} x_i^2 + \sum_{i=1}^{25} y_i^2 = 340.95 + 689.05 = 1030$	
	So $s_w^2 = \frac{1}{n-1}\left[\sum w^2 - \frac{\left(\sum w\right)^2}{n}\right] = \frac{1}{39}\left[1030 - \frac{(200)^2}{40}\right] = \frac{10}{13}$	
	$\bar{w} = \frac{\sum_{i=1}^{40} w_i}{40} = \frac{200}{40} = 5$	
	To test $H_0 : \mu = \mu_0$ against $H_1 : \mu < \mu_0$ Left-tailed test at 5% level of significance	
	Since n = 40 is large, by C.L.T, $\overline{W} \sim N\left(\mu_0, \frac{\left(\frac{10}{13}\right)}{40}\right)$ approx	
	For H_0 to be rejected,	
	$\frac{5-\mu_0}{\sqrt{\left(\frac{10}{13}\right)/40}} < -1.64485$	
	So $5.23 < \mu_0$	

END OF PAPER