Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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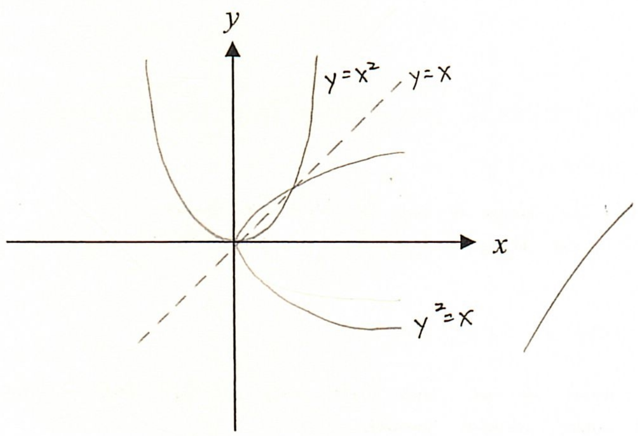
MA302.4.X2 – Power Functions with Rational Indices

The graph of $y = ax^{\frac{1}{n}}$, where n is an integer

Earlier you have sketched the graphs of linear, quadratic and cubic as well as reciprocal functions. Such graphs are part of the family of curves of $y = ax^k$, where k is an integer.

In this section, you will explore the shape of the graphs when k takes the fraction $\frac{1}{n}$, where n is a positive integer.

X1. How are the graphs of $y = x^2$ and $x = y^2$ related? Try sketching them on the **same axes**.



$y^2 = x$
$y = \pm\sqrt{x}$

X2. Make x the subject in the equation $y = x^{\frac{1}{2}}$. What do you notice for the graph of $y = x^{\frac{1}{2}}$?

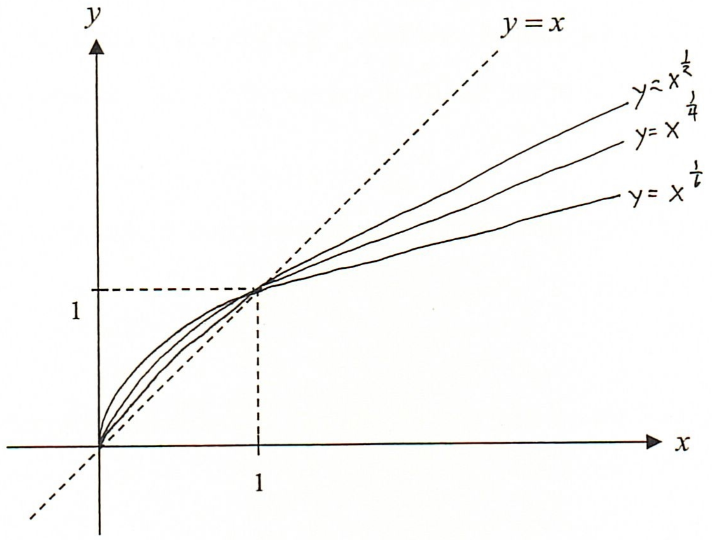
$y = \sqrt{x}$

$y^2 = x$

$x = y^2$

$y = x^{\frac{1}{2}}$ is a inverse function of $y = x^2$.

X3. Sketch the graphs of $y = x^{\frac{1}{6}}$, $y = x^{\frac{1}{4}}$ and $y = x^{\frac{1}{2}}$ on the same axes below.



(a) What similarities do you observe from the above about the three graphs? How do the three graphs differ?

When $x<1$, the bigger the root, the higher the number.
when $x>1$, the bigger the root, the lower the number.

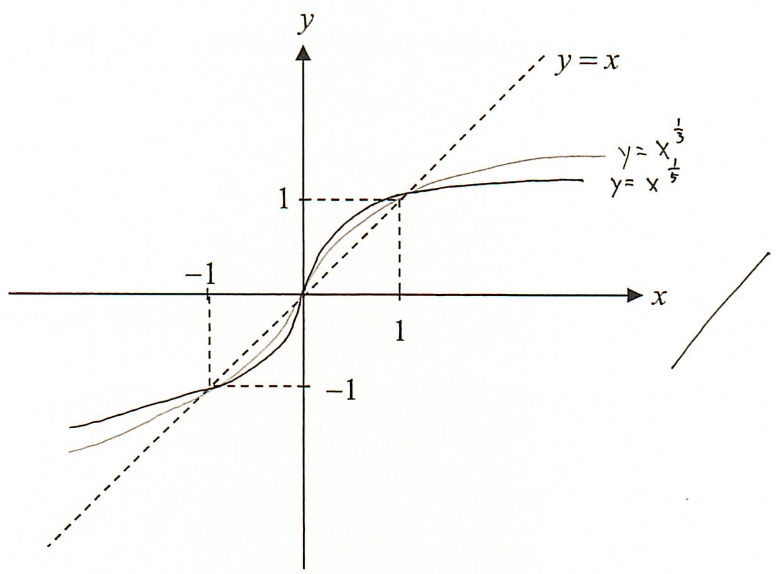
(b) Why cannot x be negative?

Negative numbers do not have real roots as the roots in the equation are even, they will not accept negative numbers.

(c) What can you infer about the graphs of $y = x^{\frac{1}{n}}$ where n is even?

The x and y values are always non-negative

S1. Sketch the graphs of $y = x^{\frac{1}{5}}$ and $y = x^{\frac{1}{3}}$ on the same axes below.



(a) What similarities do you observe from the above about the graphs? How do the two graphs differ?

S: Both graph pass through (−1, 1), (0, 0), (1, 1)

D: when $y < 0$, the gradient of $y=x^{\frac{1}{3}}$ is steeper.

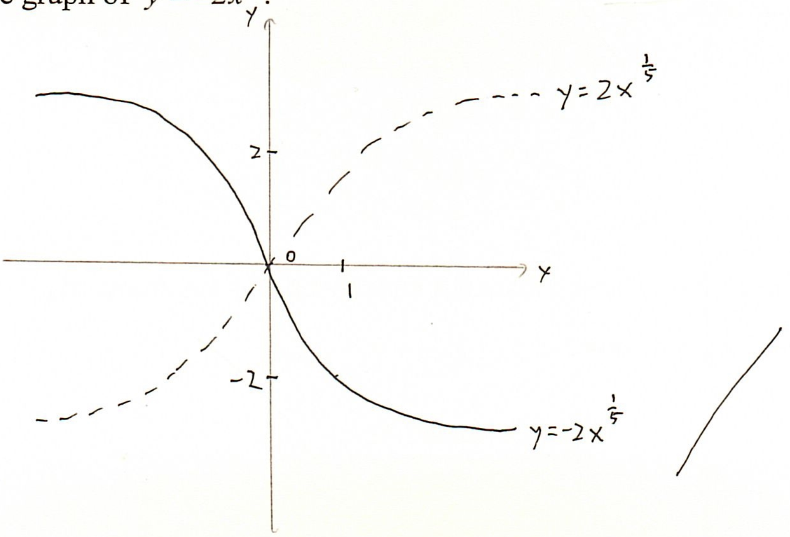
when $y > 0$, the gradient of $y=x^{\frac{1}{5}}$ is steeper.

(b) What can you infer about the graph of $y = x^{\frac{1}{n}}$ where n is odd?

The x and y values can be negative as well as non-negative.

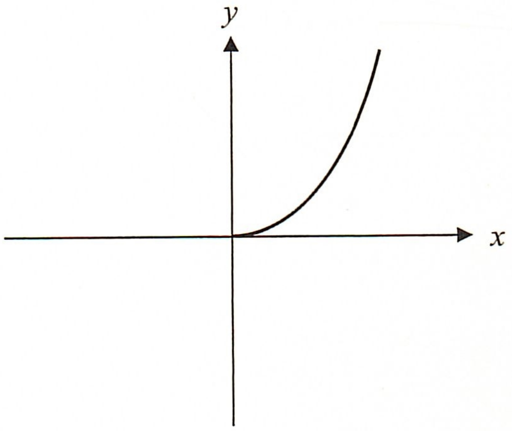
Order 2 rotational symmetry

S2. Sketch the graph of $y = -2x^{\frac{1}{5}}$.



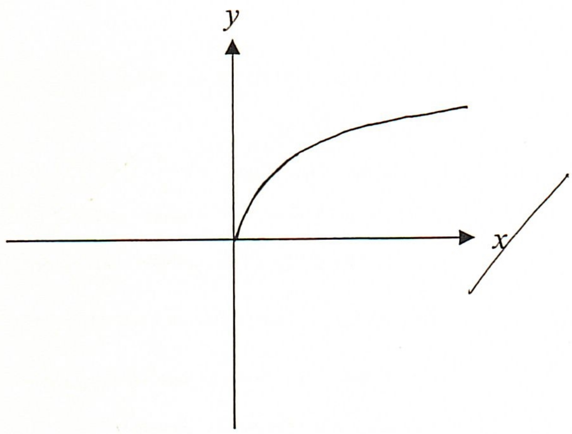
The graph of $y = ax^n$

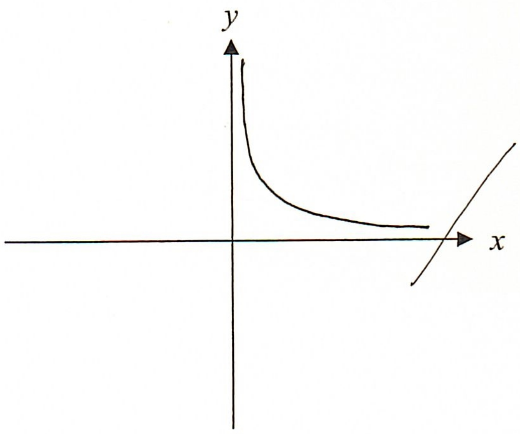
In Standard Graphs, the graphs of $y = x^n$ have been discussed when n takes integer values. What about values of n which are rational numbers? Broadly, they fall into three types.



When $n > 1$, the graph increases at an increasing rate. For example, $y = x^{\frac{4}{3}}$.

When $0 < n < 1$, the graph increases at a decreasing rate.





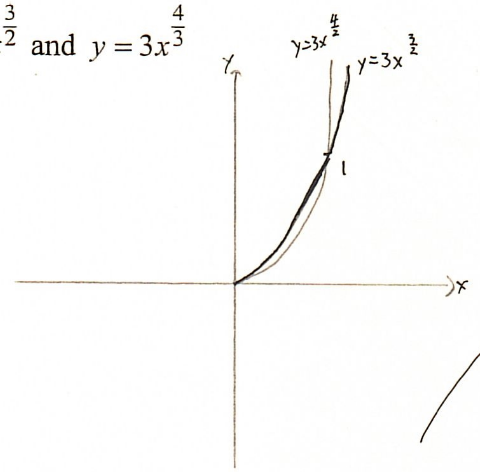
When $n < 0$, the graph follows that of a reciprocal shape, decreasing at a decreasing rate towards the x-axis.

In all these graphs, negative values of x are not considered. Do you know why?

Only even roots have negative values.

S3. Sketch the following graphs on the same pair of axes, for $x > 0$, showing clearly the points of intersection.

(a) $y = 3x^{\frac{3}{2}}$ and $y = 3x^{\frac{4}{3}}$

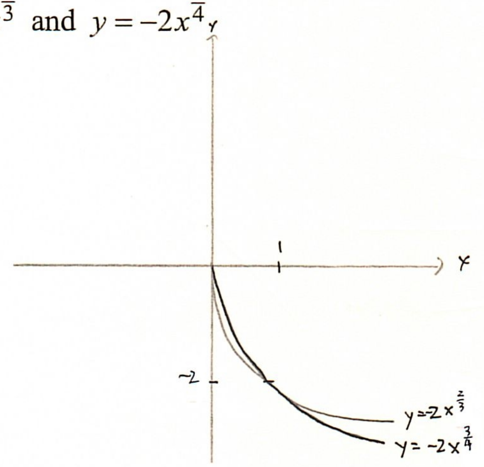


if $x>1$, $x^{\frac{5}{2}} > x^{\frac{2}{5}}$

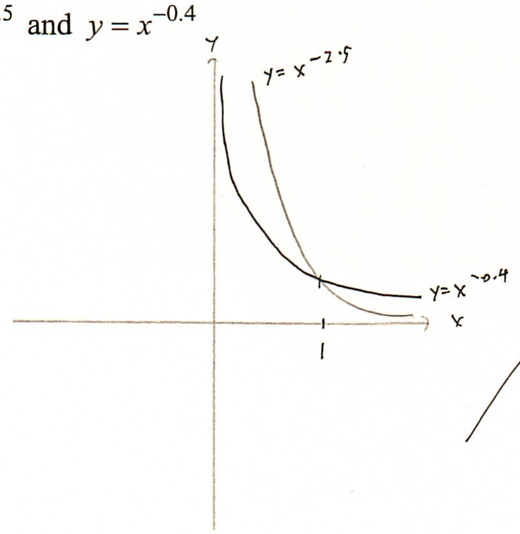
$\frac{1}{x^{\frac{5}{2}}} > \frac{1}{x^{\frac{2}{5}}}$

$\therefore x^{-\frac{5}{2}} < x^{-\frac{2}{5}}$

(b) $y = -2x^{\frac{2}{3}}$ and $y = -2x^{\frac{3}{4}}$



(c) $y = x^{-2.5}$ and $y = x^{-0.4}$



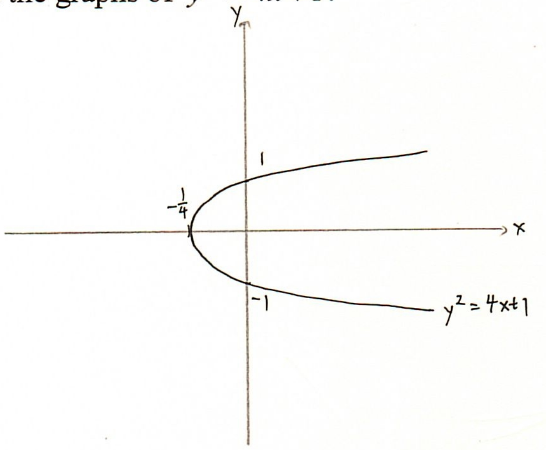
The graph of $y^2 = kx$

X4. What is the difference between $y = \sqrt{x}$ and $y^2 = x$, if the latter is just the square of the former?

For $y = \sqrt{x}$, y values that are negative are rejected.

For $y^2 = x$, y values that are negative can be accepted.

X5. Sketch the graphs of $y^2 = 4x + 1$.



S4. Sketch the graph of $y^2 + 9x = 0$.