1 Differentiate $\frac{\left(e^{x}-e^{-x}\right)^{2}}{e^{x}}$ with respect to x, leaving your answer as a single fraction. [2]

2 Differentiate each of the following with respect to x, simplifying your answers.

(i) $2e^{x^2}\left(\sqrt[3]{9x-5}\right)$, [3]

2 **(ii)** $\dfrac{4-x^2}{\sqrt{3x-7}}$. [3]

3 Find

(a) $\int \frac{8}{3\sqrt{(5-2x)^7}}\,\mathrm{d}x$, [2]

3 **(b)** $\int_1^3 \frac{6x-1}{4-9x}\,\mathrm{d}x$, leaving your answer to 3 significant figures. [3]

4 The height of a ball above ground, h metres, released by a machine can be modelled by the equation $h = -0.4x^2 + 6x + 3$, where x is the horizontal distance travelled by the ball in metres.

(i) State the height above ground at which the ball is being released. [1]

(ii) Express $h = -0.4x^2 + 6x + 3$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [2]

4 **(iii)** Using your answers from **(i)** and/or **(ii)**,

(a) explain if the machine is safe for use in an indoor sports arena with a ceiling height of 25 metres, [1]

(b) find how far horizontally will the ball land from the point it is released. [2]

5 The equation of a curve is $y = \ln\left[(x-2)\sqrt{2x+1}\right]$ for $x > 2$.

(i) Find $\frac{dy}{dx}$, leaving your answer as a single fraction. [2]

5 **(ii)** Find the coordinates of the point on the curve where the tangent is perpendicular to the line $11y+18x=6$, leaving your answer in the form $(a,\ \ln b)$, where a and b are constants. [3]

6 The equation $\log_3 x + 4\log_3 5 = \log_{27} x$ has the solution $x = 5^k$.

Find the value of k. [4]

7 A is a point (1, 2), $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

(i) Find the exact value of $\left|\overrightarrow{BC}\right|$. [1]

(ii) Express $\overrightarrow{AC}$ as a column vector. [1]

7 **(iii)** Given that $\overrightarrow{CD} = \begin{pmatrix} -2 \\ p \end{pmatrix}$, find the two possible values of p such that $ABCD$ is a trapezium. [2]

7 **(iv)** A student claims that $E(-8,-2)$ lies on CA produced.

Use **vectors** to determine whether his claim is true or not. [3]

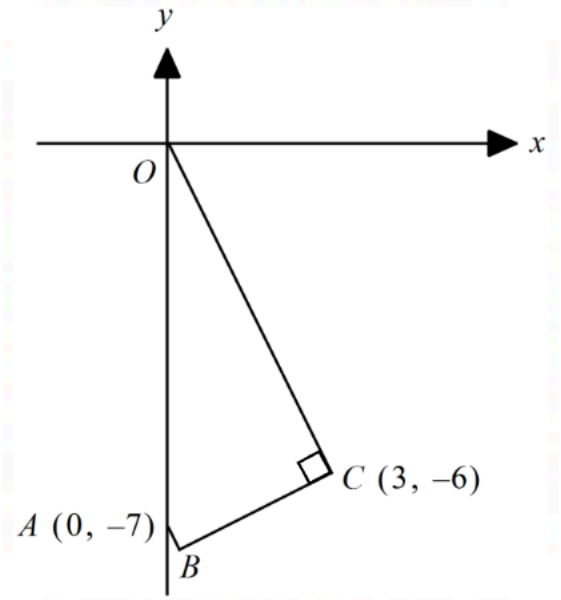
8 The gradient function of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{\sqrt{x^3}} - 2x + 1$ and the gradient to the curve at the point $(4, -6)$ is –9. Find

(i) the value of k, [1]

(ii) the equation of the curve, [2]

8 **(iii)** the range of values of x for which $\frac{dy}{dx} < -2x - 15$. [2]

9 **Solution to this question by accurate drawing will not be accepted.**



In trapezium $OABC$ (not drawn to scale), point A has coordinates $(0,-7)$ and point C has coordinates $(3,-6)$.

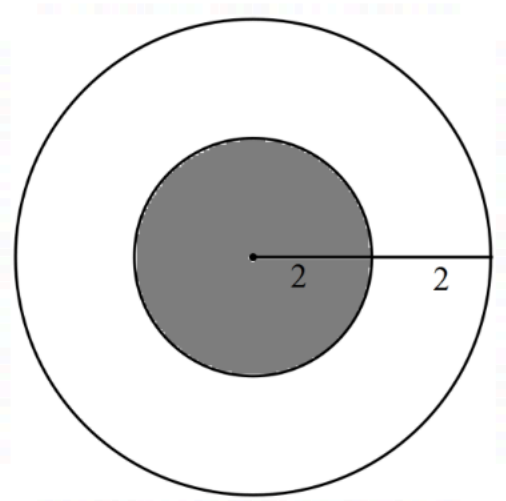
(i) Find the coordinates of B. [4]

9 **(ii)** E is a point which lies on the perpendicular bisector of OC such that the area of $OACE$ is 25.5 units^2. Find the coordinates of E. [4]

10 **(i)** Given that $4x^3 - 19x^2 + 6x + 45 \equiv (x - A)^2(Bx + 5)$, find the values of A and B, where A and B are positive integers. [2]

10 **(ii)** Hence, express $\dfrac{3x^2-27x+88}{4x^3-19x^2+6x+45}$ as partial fractions. [5]

11



The figure above shows a circular target board formed by 2 concentric circles.
The radius of the entire target board is 4 cm.
The radius of the shaded circular region is 2 cm.

(i) A dart is thrown randomly inside the target board. Find the probability that the dart hits the shaded region. [1]

Darts are thrown randomly inside the target board until a dart hits the shaded region.

(ii) Find the probability that exactly 3 throws are required. [2]

11 **(iii)** Find the probability that no more than 3 throws are required. [2]

(iv) How many darts would need to be thrown so that the probability that at least one dart hits the shaded region exceeds 0.99? [3]

12 Food companies Raffles and Gryphon sell the same type of chicken satays, fish ball sticks and sausages. The following table shows the selling price, in dollars, of a box of chicken satays, a box of fish ball sticks and a box of sausages in Raffles and Gryphon.

	Selling price in Raffles ($)	Selling price in Gryphon ($)
A box of chicken satays	a	7.00
A box of fish ball sticks	5.80	6.50
A box of sausages	7.50	b

Kayden and Leo intend to organise a class party. Kayden decides to buy 12 boxes of chicken satays, 15 boxes of fish ball sticks and 8 boxes of sausages. Leo decides to buy 14 boxes of chicken satays, 10 boxes of fish ball sticks and 12 boxes of sausages.

(i) Using relevant information from above, write down two matrices such that the product would give the total amount spent by Kayden and by Leo at each company. Find this product. [3]

12 The next table shows the total amount, in dollars, that Kayden and Leo will spend if they buy the above quantities from Raffles and Gryphon respectively.

	Total amount spent in Raffles ($)	Total amount spent in Gryphon ($)
Kayden	255	249.50
Leo	274	265

(ii) Using your matrix found in **(i)**, find the value of a and of b. [2]

(iii) The total amount spent by Kayden and Leo buying the quantities from Raffles and Gryphon respectively can be represented by matrix, $\mathbf{D} = \begin{pmatrix} 255 & 249.50 \\ 274 & 265 \end{pmatrix}$.

To entice customers, Raffles and Gryphon each runs a promotion offering a 15% discount and a 12% discount on the orders they receive respectively.

(a) Using a 2×2 matrix **E**, find the product **DE** which will give the discounted prices that Kayden and Leo will pay to Raffles and to Gryphon. [2]

(b) Hence, explain which company should Leo order from if he wants to keep his costs as low as possible. [1]

13 It is given that $\mathrm{f}(x)=3ax^3+(a-2b)x^2-x+2$ is exactly divisible by $x-1$ but leaves a remainder of -10 when it is divided by $x+1$.

(i) Calculate the values of a and b. [3]

(ii) Factorise $\mathrm{f}(x)$ completely. [2]

13 **(iii)** Hence, solve the equation $3a\left(4^{y}\right)+2\left(2^{-y}\right)=1-\left(a-2b\right)\left(2^{y}\right)$. [2]

(iv) Show that one of the solutions from part **(iii)** may be written in the form $1-\log_{p} q$, where p, q are constants and $p \neq q$. [2]

END OF PAPER