Anglo-Chinese School

(Independent)

FINAL EXAMINATION 2016

YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS

HIGHER LEVEL

PAPER 1

5 October 2016

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- No calculators are allowed for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet, and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics HL and Further Mathematics HL information booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**

Section A (60 Marks)		Section B (60 Marks)	
Question	**Marks**	**Question**	**Marks**
1			
2		**10**	
3			
4			
5		**11**	
6			
7			
8		**12**	
9			
Subtotal		**Subtotal**	
TOTAL	**/ 120**		

This question paper consists of 12 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

Find the term independent of x in the expansion of $\left(\frac{1}{x}-1\right)\left(1-\sqrt{x}\right)^{10}$.

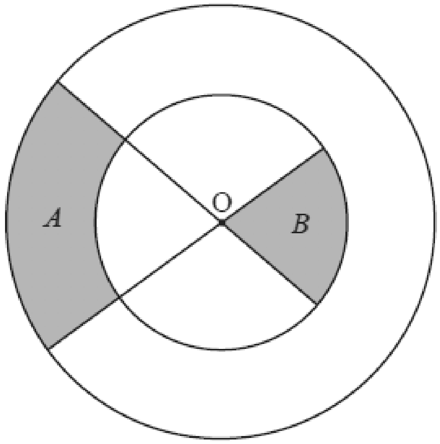
2. [Maximum mark: 7]

(i) Find the sum of all the 2-digit natural numbers that are exactly divisible by 9. [3]

(ii) Express the sum of all the 2-digit natural numbers that are NOT divisible by 9 in sigma notations of the form $\sum_{r=1}^{m}(r+9)-\sum_{r=1}^{n}(kr+k)$, where k, m and n are to be determined. [4]

3. [Maximum mark: 4]

The diagram below shows two straight lines intersecting at O and two concentric circles, each with center O. The outer circle has radius R and the inner circle has radius r.



Consider the shaded regions with areas A and B. Given that the area of shaded region A is 3 times that of the area of shaded region B, prove that $R = 2r$.

4. [Maximum mark: 7]

The first 3 terms of a geometric sequence are $\frac{1}{2^x}, \frac{1}{2^{2x}}, \frac{1}{2^{3x}}, \ldots$. Given that $x = 0.5$,

(i) find the sum to infinity in the form $\sqrt{a} + b$. [3]

(ii) find the ***product*** of the first 11 terms in the form $\frac{1}{2^k}$. [4]

5. [Maximum mark: 7]

Prove by the method of mathematical induction that $n! > 2^{n-1}$ for all $n > 2,\ n \in \mathbb{Z}^+$.

6. [Maximum mark: 6]

The zeros of the cubic function $f(x) = 8x^3 - 16x^2 - 8x + 16$ are α, β and γ. By considering the product of roots or otherwise, form a cubic equation of the form $Ay^3 + By^2 + Cy + D = 0$, whose roots are $\alpha^2\beta\gamma,\ \alpha\beta^2\gamma$ and $\alpha\beta\gamma^2$.

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7. [Maximum mark: 5]

Given that $\arcsin(k) = \frac{\pi}{4} - \arctan\left(\frac{1}{\sqrt{3}}\right)$, show that $k = \frac{\sqrt{3}-1}{2\sqrt{2}}$.

8. [Maximum mark: 7]

(i) Solve the inequality $\frac{1}{a+x} \geq \frac{1}{a-x}$, where $a > 0$. [4]

(ii) Hence or otherwise, solve $\frac{1}{2+e^y} \geq \frac{1}{2-e^y}$. [3]

9. [Maximum mark: 11]

(i) Given that $f(x)=\frac{1}{\sqrt{x}}$, find $f'(x)$ using first principles. [6]

(ii) Find $\frac{dy}{dx}$ given that $2^{xy}=e^{x}\cos x$. [5]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

10. [Maximum mark: 22]

Consider the functions given below.

$$f(x) = 2x - x^2, \quad x < 1$$

$$g(x) = \frac{1}{x}, \; x \neq 0$$

$$h(x) = x(x-2), \quad x < 0$$

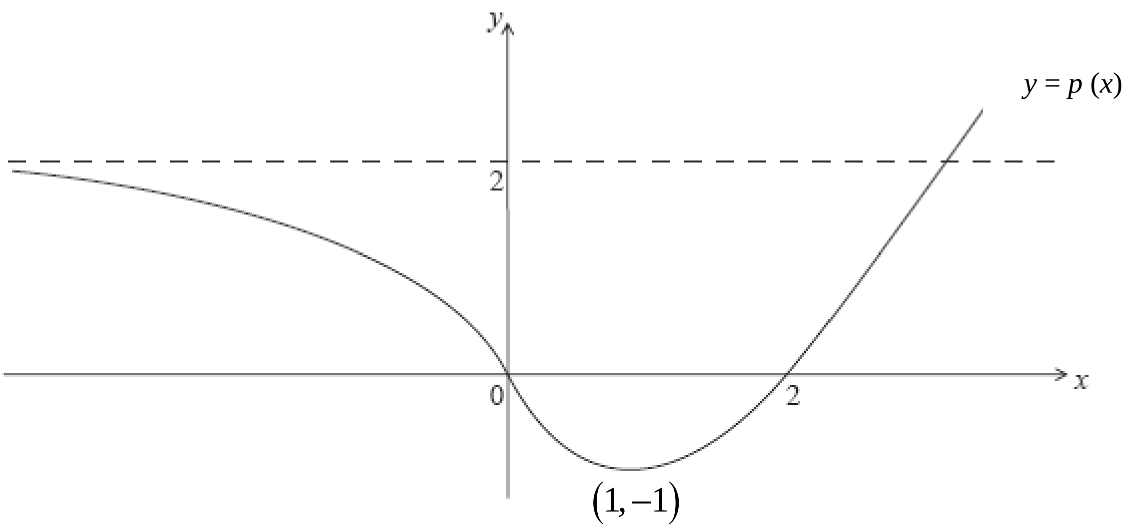
(a) Find $f^{-1}(x)$ if it exists, stating clearly the domain of the function. [4]

(b) (i) Show that gh exist.

(ii) Find gh in similar form, stating clearly the domain of the function.

(iii) By considering $g^2(x)$, express $g^{2017}h(x)$ in terms of $h(x)$ only. [7]

The diagram shows the graph of $p(x)$. The graph has a horizontal asymptote at $y = 2$ and minimum point at $(1, -1)$.



(c) Sketch the graph of $y = 2p(1-x)$. [4]

(d) The graph of $y = p(x)$ was translated by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, followed by the transformation $p(|x|)$. Sketch the graph after these transformations. [3]

(e) Sketch the graph of $y = \dfrac{1}{p(x)}$. [4]

Do **not** write solutions on this page.

11. [Maximum mark: 21]

Consider the points A (1, −1, 4) and B (2, − 2, 5)).

(a) Calculate the cosine of the angle between $\overrightarrow{OA}$ and $\overrightarrow{AB}$ [3]

(b) Find a vector equation of the line L_1 which passes through A and B. [2]

The line L_2 has equation $\dfrac{x-2}{2}=\dfrac{y-4}{1}=\dfrac{z-7}{3}$.

(c) Show that the lines L_1 and L_2 intersect and find the coordinates of their point of intersection. [5]

(d) Find the foot of the perpendicular from point C (2, −10, 7) to L_2. [5]

(e) Find C' , the mirror image of point C reflected in the line L_2. [2]

Given that θ is the angle between the unit vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, where $0 \le \theta \le \pi$.

(f) (i) Show that the area of the parallelogram formed by the vectors $(\boldsymbol{u}-\boldsymbol{v})$ and $\boldsymbol{v}$. is the same as the area of the parallelogram formed by $(\boldsymbol{u}+\boldsymbol{v})$ and $\boldsymbol{v}$.

(ii) Hence or otherwise find the area of the parallelogram in terms of θ. [4]

12. [Maximum mark: 17]

(a) Given that $\arg(x+iy)=\alpha,$ where $x, y \in \mathbb{R},$ find in terms of α and/or $\pi,$ the value of $\arg(y+ix)$. [2]

(b) Use De Moivre's theorem to find all the roots of the equation $\left(\dfrac{w}{2i}\right)^3=1$, where w is a complex number in the form of $re^{i\theta},$ where r > 0 and $-\pi < \theta \le \pi$. [5]

(c) (i) Given that $z=\cos\theta+i\sin\theta,$ show that $z^n+z^{-n}=2\cos n\theta$.

(ii) Show that $z^2-5z-5z^{-1}+z^{-2}+6=2(\cos 2\theta-5\cos\theta+3)$.

(iii) Hence, solve $z^2-5z-5z^{-1}+z^{-2}+6=0$ giving the root(s) in the Cartesian form. [10]

END OF PAPER