



QUANTITIES AND MEASUREMENT TUTORIAL

Self-Check Questions

- S1** List the SI base quantities and their units.
- S2** What is meant by *base* units and *derived* units?
- S3** State two ways in which an equation that is dimensionally consistent be physically incorrect.
- S4** What is meant by *systematic error* and *random error*?
- S5** Give examples of systematic errors and random errors, and how can they be eliminated/reduced.
- S6** What is meant by *accuracy* and *precision*?
- S7** What is meant by a *scalar* and *vector* quantity?
- S8** Show, with a diagram, how a vector is resolved into its perpendicular components?

Self-Practice Questions

- 1** What does **not** cause a systematic error?
- A** a bent needle on an analogue voltmeter
B a metre rule that is worn down at both ends
C the zero error on a micrometer
D varying reaction time of a person using a stop-watch
- 2** The diameter of a wire, known to be 0.28 mm at room temperature, is measured with an instrument that gives readings to 0.001 mm.

Readings are taken, at room temperature, at three different points along the wire. Two perpendicular values are taken at each point.

The six readings obtained, in mm are 0.258, 0.258, 0.258, 0.259, 0.259 and 0.258.

What best describes this set of readings?

	accurate	Precise
A	No	Yes
B	No	No
C	Yes	Yes
D	Yes	No

- 3 A wire of uniform circular cross-section has a diameter d and length L . A potential difference V between the ends of the wire give rise to a current I in the wire.

The resistivity of the material of the wire is given by the expression

$$\rho = \frac{\pi d^2 V}{4LI}.$$

In one experiment, the following measurements are made.

$$d = 1.20 \pm 0.01 \text{ cm}$$

$$I = 1.50 \pm 0.05 \text{ A}$$

$$L = 100 \pm 1 \text{ cm}$$

$$V = 5.0 \pm 0.1 \text{ V}$$

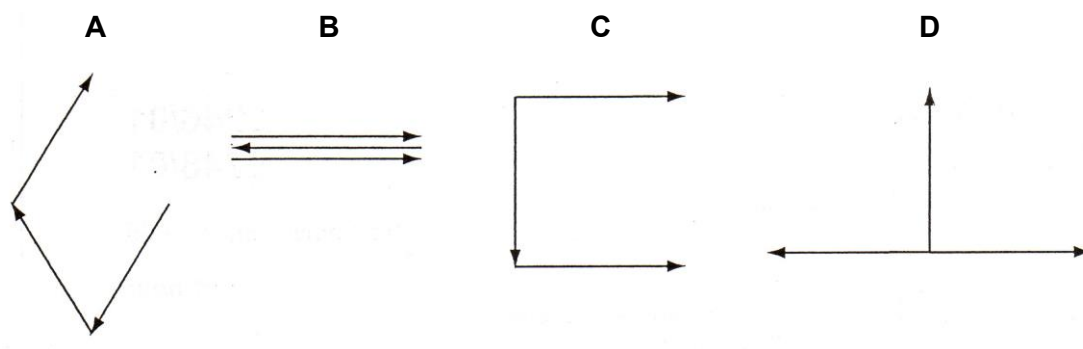
Which measurement gives rise to the most uncertainty in the value for the resistivity?

- A d
 - B I
 - C L
 - D V
- 4 A boat changes its velocity from 8 m s^{-1} due North to 6 m s^{-1} due East.

What is the change in velocity?

- A 2 m s^{-1} at a direction of 37° East of North
 - B 10 m s^{-1} at a direction of 37° East of South
 - C 2 m s^{-1} at a direction of 53° East of North
 - D 10 m s^{-1} at a direction of 53° West of South
- 5 Each diagram shows three vectors of equal magnitude.

In which diagram is the magnitude of the resultant vector different from the other three?



Answer Key

1 D 2 A 3 B 4 B 5 C

Discussion Questions

SI units

- 1 The Stefan constant σ is measured in $\text{W m}^{-2} \text{K}^{-4}$ and is given by

$$\sigma = \frac{ak^4}{h^3c^2}$$

where a is a dimensionless constant, k is the Boltzman constant and has units J K^{-1} , and c is the speed of light.

Determine the SI base unit of h . Show your working clearly.

- 2 The theory of gas flow through small diameter tubes at low pressures is given as

$$Q = \frac{kr^3(p_1 - p_2)}{l} \sqrt{\frac{M}{RT}}$$

where k is a number without units, r is the radius of the tube, p_1 and p_2 are the pressures at each end of the tube of length l , M is the molar mass of the gas (units: kg mol^{-1}), R is the molar gas constant (units: $\text{J K}^{-1} \text{mol}^{-1}$) and T is the temperature.

Use the equation to determine the base units of Q . Show your working clearly.

- 3 A body moving through air at high speed v experiences a retarding force $F = kA\rho v^x$ where A is the area of the body, ρ is the density of air and k is a dimensionless constant.

Using unit analysis, determine the value of x . Show your working clearly.

Errors and Uncertainties

- 4 Carry out the following arithmetic operations and express your answers to the correct number of significant figures and/or decimal places.

(a) The sum of the numbers 756, 37.2, 0.83 and 2.5.

(b) The value of $1.6523 - 0.015$.

(c) The product of 3.2 and 3.563.

(d) The quotient of 5.6 and π .

- 5 A student makes measurements from which he calculates the speed of sound as 326.15 m s^{-1} . He estimates that his result is accurate only to $\pm 3\%$.

Express the speed of sound, together with its associated uncertainty.

- 6 The density of a steel ball was determined by measuring its mass and diameter. The mass was measured within 1% and the diameter within 3%.

Calculate the percentage error in the calculated density.

- 7 A student uses the equation $R = \frac{\rho l}{\pi r^2}$ to determine the resistivity ρ of an alloy called nichrome, where r is the radius and l is the length of a nichrome wire and R is its resistance.

Numerical values from one experiment were

$$r = (3.15 \pm 0.05) \times 10^{-5} \text{ m}$$

$$l = (2.71 \pm 0.01) \text{ m}$$

$$R = 939 \pm 3 \, \Omega$$

Calculate the resistivity of nichrome together with its uncertainty. Show the method you used to determine the uncertainty.

- 8 A student attempts to measure the diameter of a steel ball by using a metre rule to measure four similar balls in a row, as shown in Fig. 8.1.

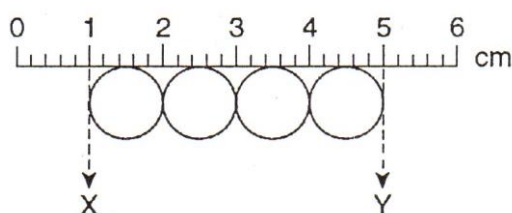


Fig. 8.1

The student estimates the positions on the scale to be as follows:

$$X = (1.0 \pm 0.1) \text{ cm}$$

$$Y = (5.0 \pm 0.1) \text{ cm}$$

Calculate the radius of a steel ball, with its associated uncertainty.

- 9 Give **reasoned** estimates of the following quantities. In each case, give your answer in an SI unit.

(a) The area of the main island of Singapore [2]

(b) The acceleration of a train on the Singapore rapid transit system [2]

Scalars and vectors

- 10 A tennis ball approaches a tennis racquet at 25 m s^{-1} and leaves the racquet in the opposite direction at 37 m s^{-1} .

Determine the change in velocity of the tennis ball.

- 11 A car turns into a road so that its velocity changes from 120 m s^{-1} due West to 160 m s^{-1} due North-East.

With the use of a vector diagram or otherwise, calculate the change in velocity.

- 12 At two instants, $t = 0 \text{ s}$ and $t = 2.0 \times 10^{-7} \text{ s}$, the velocity of an electron moving in a horizontal plane in vacuum is as shown in Fig. 12.1

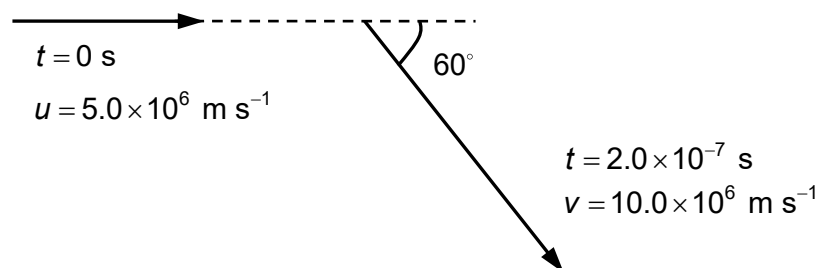
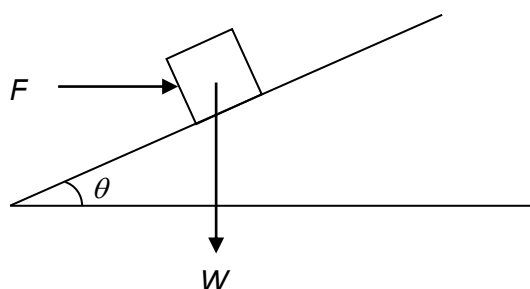


Fig. 12.1

- (a) Calculate the magnitude of the change in velocity which takes place over this time interval.
- (b) Hence, determine the magnitude and direction of the average acceleration of the electron over this time interval.
- 13 A boy pulls a rope attached to a sled with a force of 100 N. The rope makes an angle of 30° with the horizontal ground.
- Calculate the effective value of the force tending to move the sled along the ground, and the effective value tending to lift the sled vertically.
- 14 A horizontal force F is applied to a body of weight W on a smooth plane which is inclined at an angle θ to the horizontal.



Resolve vectors F and W into components parallel to, and perpendicular to the slope. Show your workings clearly, using a vector diagram.

Challenging Question:

- 15** The bore of a capillary tube was measured by introducing a thread of mercury and determining its length and mass. To measure its length, two readings at both ends of the mercury thread were taken using a travelling microscope with a vernier scale. The observations were:

Vernier microscope readings: one end, $R_1 = 10.926 \pm 0.002$ cm
 other end, $R_2 = 4.758 \pm 0.002$ cm

Mass of empty crucible : $m_1 = 6.610 \pm 0.002 \text{ g}$

Mass of crucible with mercury : $m_2 = 6.884 \pm 0.002$ g

- (a) Given that the density of the mercury is 13.6 g cm^{-3} accurate within $\pm 0.1\%$, calculate the diameter of the bore with its associated uncertainty.
- (b) Explain the advantage of this method as compared to the method of direct measurement of the diameter.

Answers to selected discussion questions:

- | | | | |
|----------|--|--------------|--|
| 1 | $\text{kg m}^2 \text{s}^{-1}$ | 10 | 62 m s^{-1} |
| 2 | kg s^{-1} | 11 | 260 m s^{-1} at an angle of 26° North of East |
| 3 | 2 | 12(a) | $8.66 \times 10^6 \text{ m s}^{-1}$ |
| 5 | $(330 \pm 10) \text{ m s}^{-1}$ | 12(b) | $4.33 \times 10^{13} \text{ m s}^{-2}$ |
| 6 | 10% | 13 | 87 N, 50 N |
| 7 | $\rho = (1.08 \pm 0.04) \times 10^{-6} \Omega \text{ m}$ | 15 | $d = (0.0645 \pm 0.0005) \text{ cm}$ |
| 8 | $(0.50 \pm 0.03) \text{ cm}$ | | |