ANDERSON SERANGOON JUNIOR COLLEGE

H2 MATHEMATICS **9758**

JC2 ASP Paper 1 (100 marks)

27 July 2026

3 hours

Additional Material(s): List of Formulae (MF 27)

CANDIDATE NAME

CLASS /

READ THESE INSTRUCTIONS FIRST

Write your name and class in the boxes above.
Please write clearly and use capital letters.
Write in dark blue or black pen. HB pencil may be used for graphs and diagrams only.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions and write your answers in this booklet.
Do not tear out any part of this booklet.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

All work must be handed in at the end of the examination. If you have used any additional paper, please insert them inside this booklet.
The number of marks is given in brackets [ ] at the end of each question or part question.

Question number	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total	

This document consists of 13 printed pages and 3 blank pages.

[Turn Over

1	Solve the simultaneous equations	
	$$\frac{z^*}{z}+1=-\mathrm{i}\left(w^*-3\right) \text{ and } w=-2z+\mathrm{i},$$	
	where z and w are non-zero complex numbers. Leave your answer in the form $a+\mathrm{i}b$ where a and b are real numbers.	[5]
	Solution	
	$\frac{z^*}{z}+1=-\mathrm{i}\left(w^*-3\right)\cdots\cdots(1)$	
	From $w=-2z+\mathrm{i}\cdots\cdots(2)$	
	$w^*=-2z^*-\mathrm{i}$	
	Subst into (1)	
	$$z^*+z=-\mathrm{i}z\left(-2z^*-\mathrm{i}-3\right)$$	
	$z^*+z=2\mathrm{i}\|z\|^2-z+3\mathrm{i}z\cdots\cdots(3)$	
	Let $z=x+\mathrm{i}y$	
	From (3),	
	$2x=2\mathrm{i}\left(x^2+y^2\right)-\left(x+\mathrm{i}y\right)+3\mathrm{i}\left(x+\mathrm{i}y\right)$	
	Comparing the real and imaginary parts	
	$2x=-x-3y$ and $0=2\left(x^2+y^2\right)-y+3x$	
	$\Rightarrow x=-y$ and so $0=2\left(y^2+y^2\right)-y-3y$	
	$$0=4y\left(y-1\right)$$	
	$y=0$ (rejected since z is non-zero complex number or $y=1$	
	$\therefore x=-1$	
	So $z=-1+\mathrm{i}$	
	And from (2) $w=-2\left(-1+\mathrm{i}\right)+\mathrm{i}=2-\mathrm{i}$	
2	A curve is defined by the parametric equations $$x=\frac{1}{t}, \qquad y=t^2, \qquad \text{for } 0<t<1.$$ Show that the equation of the normal to the curve at the point $P\left(\frac{1}{p},p^2\right)$ is $$2p^4y-px=2p^6-1$$ Hence show that the normal at P cuts the curve exactly once.	[6]
	Solution	
	(a) $x=\frac{1}{t}$ $\qquad y=t^2$ $\frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{1}{t^2}$ $\qquad \frac{\mathrm{d}y}{\mathrm{d}t}=2t$	

	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2t}{-\frac{1}{t^2}}=-2t^3$	
	Equation of normal at point P: $y-p^2=\frac{1}{2p^3}\left(x-\frac{1}{p}\right)$	
	$2p^4y-2p^6=px-1$ $2p^4y-px=2p^6-1$	
	To check if the normal cuts the curve again, $2p^4\left(t^2\right)-p\left(\frac{1}{t}\right)=2p^6-1$	
	$2p^4t^3-p+(1-2p^6)t=0$	
	$(t-p)\left(2p^4t^2+At+1\right)=0$	
	Comparing coefficient of $t:1-Ap=1-2p^6$ $A=2p^5$ $\therefore(t-p)\left(2p^4t^2+2p^5t+1\right)=0$ $t=p$ or $2p^4t^2+2p^5t+1=0\cdots\cdots(1)$	
	For $2p^4t^2+2p^5t+1=0$, Discriminant $=\left(2p^5\right)^2-4\left(2p^4\right)(1)$	
	$=4p^{10}-8p^4$ $=4p^4\left(p^6-2\right)<0$ since $0<p<1$ (Need to state the reason) Hence (1) has no real solution. Since there is only 1 real solution for t, so the normal at P cuts the curve only once.	
3	Andy and his fiancée signed up for a new 4-room flat in Boon Keng. They take up a housing loan of $450,000 provided by BEST bank for the purchase. The couple pay a fixed monthly instalment of $\$A$ on the first day of each month. Interest is charged on the last day of each year at a fixed rate of 1.6% of the remaining loan amount at the beginning of that year. If the first instalment is paid in January 2026,	
	(i) Show that the amount the couple owe the bank at the end of 2027 is $\$[464515.2-24.192A]$.	[1]
	(ii) Given that A is 1500, find the date and amount of the final repayment to the nearest cent.	[5]
	Solution	
	(i) Amt owe at the end of 2017 = (1.016)(1.016)(450000) – 1.016(12A) – 12A = $\$[464515.2-24.192A]$	
	(ii)	
	<table><tr><td>year</td><td>Amt owed at the beginning</td><td>Amt owed at the end of the year after paying 18000</td></tr></table>	

1^{st}	450000	1.016(450000) -18000
2^{nd}	1.016(450000) -12A	(1.016)(1.016)(450000) – 1.016(18000) – 18000
3^{rd}	(1.016^2)(450000) – 1.016(12A) – 12A	(1.016)(1.016^2)(450000) – (1.016)(1.016)(18000) – (1.016)(18000) – 18000
	…	…
n^{th}		(1.016^n)(450000) – (1.016^{n-1})(18000) – (1.016^{n-2})(18000) – …... – 18000

Amount of money owe at the end of nth year

$$= 450000(1.016)^n - 18000(1 + 1.016 + 1.016^2 + ... + 1.016^{n-1})$$

Consider

$$450000(1.016)^n - 18000\left(\frac{1\left(1.016^n - 1\right)}{1.016 - 1}\right) \le 0$$

$$450000(1.016)^n - 1125000\left(1.016^n - 1\right) \le 0$$

Using G.C, $n \ge 32.2$

When n = 32,

Amount owe at the end of 32 years

$$= \$450000(1.016)^{32} - \$1125000\left(1.016^{32} - 1\right) = \$3233.601$$

Since they will be paying \$1500 each month, they will finished the payment on 1st March 2058. The last payment is \$233.60.

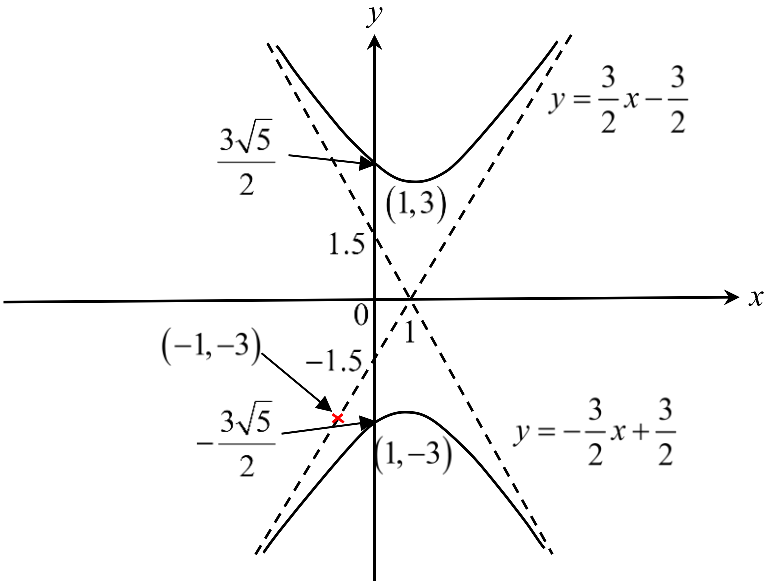
4	Without using a calculator, solve the inequality $\dfrac{2x+4}{\left(x-2\right)^2} \ge \dfrac{1}{x+1}$.	[4]
	Hence, solve $\dfrac{2e^{-x}+4}{\left(e^{-x}-2\right)^2} \ge \dfrac{e^x}{1+e^x}$.	[3]
	Solution	

	$\dfrac{2x+4}{(x-2)^2} \geq \dfrac{1}{x+1}$	
	$\dfrac{2x+4}{(x-2)^2} - \dfrac{1}{x+1} \geq 0$	
	$\dfrac{(2x+4)(x+1)-(x-2)^2}{(x-2)^2(x+1)} \geq 0$	
	$\dfrac{2x^2+6x+4-(x^2-4x+4)}{(x-2)^2(x+1)} \geq 0$	
	$\dfrac{x(x+10)}{(x-2)^2(x+1)} \geq 0$	
		
	$\therefore -10 \leq x < -1$ or $0 \leq x < 2$ or $x > 2$	
	OR	
	$\therefore -10 \leq x < -1$ or $x \geq 0,\ x \neq 2$	
	$\dfrac{2e^{-x}+4}{(e^{-x}-2)^2} \geq \dfrac{e^x}{1+e^x}$	
	$\dfrac{2e^{-x}+4}{(e^{-x}-2)^2} \geq \dfrac{1}{1+e^{-x}}$	
	Replace x with e^{-x}	
	$\therefore -10 \leq e^{-x} < -1$ (no solution as $e^{-x} > 0$ for all real values of x)	
	or $0 \leq e^{-x} < 2$ $\Rightarrow e^{-x} > 0$ and $e^{-x} < 2$ $x \in \mathbb{R}$ and $-x < \ln 2$ $\therefore x > -\ln 2$ or $e^{-x} > 2$ or $x < -\ln 2$	
	$\therefore x \in \mathbb{R} \setminus \{-\ln 2\}$	
5	The rate at which the number of people infected with Middle East Respiratory Syndrome (MERS), x, is varying at any time t, is proportional to the difference between the number of infected people and the number of deaths due to MERS. It is also known that the number of deaths due to MERS is proportional to the square of the number of people infected with MERS. Initially there were 10 people infected and the number infected remains constant when it reaches 100. Show that $\dfrac{dx}{dt} = \dfrac{k}{100}(100x - x^2)$, where k is a constant. Hence find x in terms of k and t in the form $x = \dfrac{p}{1+qe^{-kt}}$ where p and q are constants to be determined.	[7]

	Solution	
	Let D represent the number of deaths due to MERS.	
	$\frac{\mathrm{d}x}{\mathrm{d}t} \propto (x-D)$	
	$\frac{\mathrm{d}x}{\mathrm{d}t} = k(x-D)$	
	$\frac{\mathrm{d}x}{\mathrm{d}t} = k(x-Ax^2)$	
	When $x=100, \frac{\mathrm{d}x}{\mathrm{d}t}=0$	
	$0=k\left(100-A(100)^2\right)$	
	$A=\frac{1}{100}$	
	$\frac{\mathrm{d}x}{\mathrm{d}t} = k\left(x-\frac{1}{100}x^2\right) = \frac{k}{100}\left(100x-x^2\right)$, shown	
	Integrating $\Rightarrow \int \frac{1}{100x-x^2}\,\mathrm{d}x = \int \frac{k}{100}\,\mathrm{d}t$	
	$\int \frac{1}{(100-x)x}\,\mathrm{d}x = \int \frac{k}{100}\,\mathrm{d}t$	
	$\frac{1}{100}\int \frac{1}{x}+\frac{1}{100-x}\,\mathrm{d}x = \int \frac{k}{100}\,\mathrm{d}t$	
	$\int \frac{1}{x}+\frac{1}{100-x}\,\mathrm{d}x = \int k\,\mathrm{d}t$	
	$\ln\lvert x\rvert - \ln\lvert 100-x\rvert = kt + \mathrm{C}$	
	$\ln\left\lvert \frac{x}{100-x}\right\rvert = kt+\mathrm{C}$	
	$\frac{x}{100-x} = B\mathrm{e}^{kt}$ where $B=\pm\mathrm{e}^{C}$	
	$x = 100B\mathrm{e}^{kt} - Bx\mathrm{e}^{kt}$	
	$x + Bx\mathrm{e}^{kt} = 100B\mathrm{e}^{kt}$	
	$x\left(1+B\mathrm{e}^{kt}\right) = 100B\mathrm{e}^{kt}$	
	$x = \frac{100B\mathrm{e}^{kt}}{1+B\mathrm{e}^{kt}}$	
	When $t=0, x=10$	
	$10=\frac{100B}{1+B}$	
	$10+10B=100B$	
	$90B=10$	
	$B=\frac{1}{9}$	

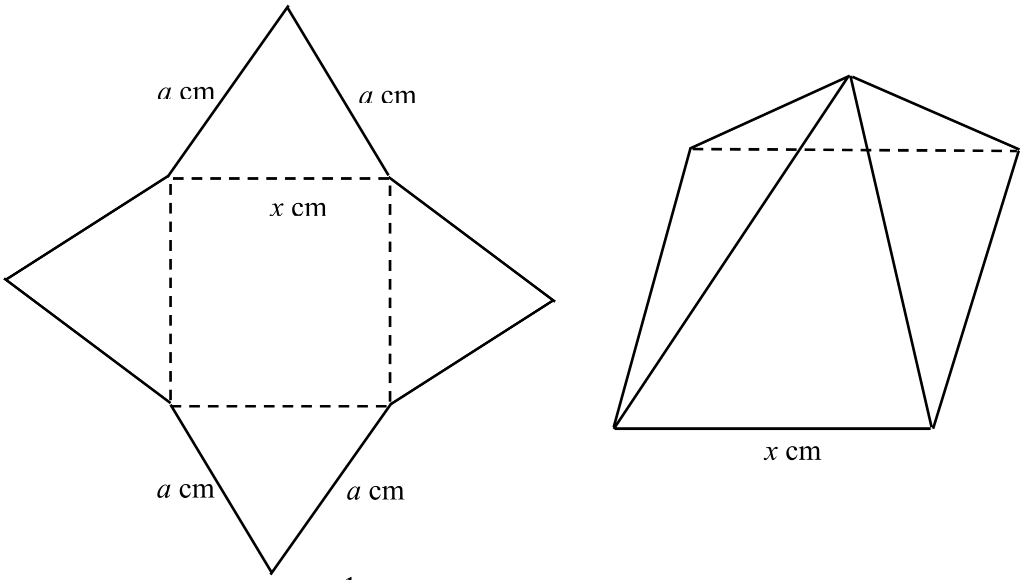
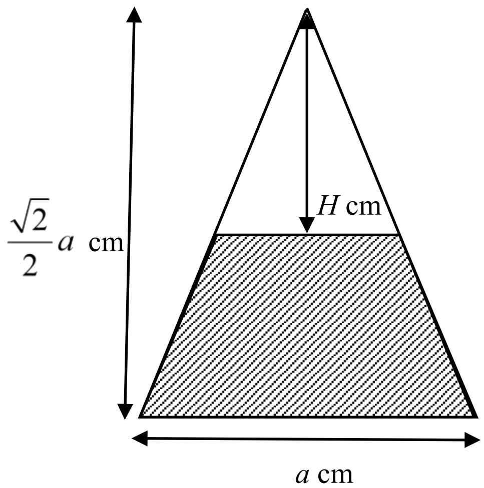
	$\therefore x=\dfrac{100\left(\frac{1}{9}\right)e^{kt}}{1+\left(\frac{1}{9}\right)e^{kt}}$	
	$=\dfrac{100e^{kt}}{9+e^{kt}}$	
	$=\dfrac{100}{\left(\frac{9+e^{kt}}{e^{kt}}\right)}=\dfrac{100}{1+9e^{-kt}}$, where p = 100 and q = 9	
6	The functions f and g are defined as follows: $\mathrm{f}: x \mapsto -\left\lvert x^2+2x\right\rvert, \quad a<x\le 0,$ $\mathrm{g}: x \mapsto -\sqrt{x+1}, \quad x>-1.$	
	(i) State the least value of a for the inverse function of f to exist. Hence, find f^{-1} in similar form.	[4]
	For the following parts, use the value of a found in part **(i)**.	
	(ii) Write down ff^{-1} in similar form.	[1]
	(iii) Find the rule for gf in the form $bx + c$, where $b, c \in \mathbb{R}$. State its range.	[3]
	Solution	
	(i) Least $a=-1$	
	$\mathrm{f}(x)=-\left(-\left(x^2+2x\right)\right)=x^2+2x$	
	Let $y=\mathrm{f}(x)=x^2+2x$, $-1<x\le 0$	
	$y=(x+1)^2-1$	
	$y+1=(x+1)^2$	
	$x=-1\pm\sqrt{y+1}$	
	$x=-1+\sqrt{y+1}\ (\because -1<x\le 0)$	
	$\mathrm{f}^{-1}: x \mapsto -1+\sqrt{x+1}, \quad -1<x\le 0$	
	(ii) $\mathrm{ff}^{-1}: x \mapsto x, \quad -1<x\le 0$	
	(iii) $\mathrm{gf}(x)=-\sqrt{x^2+2x+1}$	
	$=-\sqrt{(x+1)^2}$	
	$=-\lvert x+1\rvert$	
	$=-x-1 \quad (\because D_{\mathrm{gf}}=(-1,0])$	
	$R_{\mathrm{gf}}=[-1,0)$	
7	**(i)** The curve with equation $\dfrac{y^2}{9}-x^2=1$ undergoes a two-step transformations to become curve C with equation $\dfrac{y^2}{9}-\dfrac{(x-1)^2}{4}=1$.	[2]

	State the two transformations involved for the curve $\frac{y^2}{9} - x^2 = 1$.	
	(ii) Draw a sketch of the curve C, labelling clearly the equation(s) of its asymptote(s), intersection with the axes and the coordinates of any turning points.	[3]
	(iii) Show that the point $(-1,-3)$ lies on the line $y = mx + m - 3$ for all real values of m.	[1]
	(iv) Hence using the diagram drawn in **(ii)**, find the range of values of k such that the equation $\frac{(kx+k-3)^2}{9} - \frac{(x-1)^2}{4} = 1$ has 2 negative real roots.	[2]
	Solution	
	(i) Method 1 $\frac{y^2}{9} - x^2 = 1 \to \frac{y^2}{9} - \frac{x^2}{4} = 1 \to \frac{y^2}{9} - \frac{(x-1)^2}{4} = 1$	
	Scaling parallel to the x axis by a scale factor of 2.	
	Translation of 1 unit in the positive x direction.	
	Method 2 $\frac{y^2}{9} - x^2 = 1 \to \frac{y^2}{9} - \left(x - \frac{1}{2}\right)^2 = 1 \to \frac{y^2}{9} - \frac{(x-1)^2}{4} = 1$	
	Translation of 0.5 units in the positive x direction	
	Scaling parallel to the x axis by a scale factor of 2.	



	(iii) When $x = -1$, $\text{RHS} = m(-1) + m - 3 = -3 = \text{LHS}$	
	Hence $(-1,-3)$ lies on the line $y = mx + m - 3$ for all m.	
	(iv) From the sketch, the range of values of k are $k > \frac{3\sqrt{5}}{2} + 3$ or $k < -\frac{3}{2}$	
8	**(i)** Given that $y = \sqrt{4 + \sin 2x}$, show that $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 2y^2 = 8$.	[3]

	(ii) By repeated differentiation of this result, find the Maclaurin's series for y, in ascending powers of x, up to and including the term in x^3.	[3]
	(iii) Deduce the Maclaurin's series expansion for $\sqrt{\dfrac{4+\sin 2x}{1-\sin 2x}}$ up to and including the term in x^2, given that x is small.	[3]
	Solution	
	(i) $y=\sqrt{4+\sin 2x}$ or $y^2=4+\sin 2x$	
	$\dfrac{dy}{dx}=\dfrac{\cos 2x}{\sqrt{4+\sin 2x}}$ or $2y\dfrac{dy}{dx}=2\cos 2x$	
	$\left(\dfrac{dy}{dx}\right)^2+y\dfrac{d^2y}{dx^2}=-2\sin 2x$	
	$\left(\dfrac{dy}{dx}\right)^2+y\dfrac{d^2y}{dx^2}=-2\left(y^2-4\right)$	
	$y\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2+2y^2=8$ (Shown)	
	(ii) $y\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2+2y^2=8$	
	$y\dfrac{d^3y}{dx^3}+\left(\dfrac{dy}{dx}\right)\left(\dfrac{d^2y}{dx^2}\right)+2\left(\dfrac{dy}{dx}\right)\left(\dfrac{d^2y}{dx^2}\right)+4y\left(\dfrac{dy}{dx}\right)=0$	
	$y\dfrac{d^3y}{dx^3}+3\left(\dfrac{dy}{dx}\right)\left(\dfrac{d^2y}{dx^2}\right)+4y\left(\dfrac{dy}{dx}\right)=0$	
	When $x=0$,	
	$y=2,\ \dfrac{dy}{dx}=\dfrac{1}{2},\ \dfrac{d^2y}{dx^2}=-\dfrac{1}{8},\ \dfrac{d^3y}{dx^3}=-\dfrac{61}{32}$	
	A1 - At least 2 terms correctly evaluated.	
	$\therefore y=2+\dfrac{1}{2}x-\dfrac{1}{16}x^2-\dfrac{61}{192}x^3+\ldots$	
	(iii) $\sqrt{\dfrac{4+\sin 2x}{1-\sin 2x}}=\sqrt{4+\sin 2x}\left(1-\sin 2x\right)^{-1/2}$	
	$\approx\left(2+\dfrac{1}{2}x-\dfrac{1}{16}x^2\right)\left(1-2x\right)^{-1/2}$	
	$\approx\left(2+\dfrac{1}{2}x-\dfrac{1}{16}x^2\right)\left(1+(-2x)\left(-\dfrac{1}{2}\right)+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(-2x)^2\right)$	
	$=\left(2+\dfrac{1}{2}x-\dfrac{1}{16}x^2\right)\left(1+x+\dfrac{3}{2}x^2\right)$	
	$\approx 2+\left(\dfrac{1}{2}+2\right)x+\left(-\dfrac{1}{16}+3+\dfrac{1}{2}\right)x^2$	

	$= 2 + \frac{5}{2}x + \frac{55}{16}x^2$	
	Give only 2 marks if the student used small angle approx for $\sqrt{4+\sin 2x}$.	
9	A piece of metal with negligible thickness has been cut into a shape that is made up of four isosceles triangles each with base x cm and fixed sides a cm. Their bases frame to a form a square with sides of length x cm. A right pyramid is formed by folding along the dotted lines as shown in the diagram below.  [Volume of a pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$]	
	(i) Show that the volume of the pyramid is $\frac{x^2}{3}\sqrt{a^2 - \frac{x^2}{2}}$ cm^3.	[2]
	(ii) Find the value of x, in terms of a, that will give maximum volume for the pyramid.	[4]
	(iii)  To make the pyramid into a paperweight with negligible thickness, a viscous fluid is pumped into the interior at a rate of 1 cm^3/s. Given that H cm is the perpendicular distance from the apex of the pyramid to the viscous fluid surface, $x = a$ and the	

	height of the pyramid is $\frac{\sqrt{2}}{2}a$ cm, find the rate at which H is changing when $H=\frac{a}{2}$, giving your answer in terms of a. [The diagram above shows the cross sectional area of the pyramid.]	[3]
	Solution	
	(i) Let the height of the isosceles triangle be k cm. $k^2+\frac{x^2}{4}=a^2$ $k^2=a^2-\frac{x^2}{4}$	
	Therefore, height of the pyramid = $\sqrt{a^2-\frac{x^2}{4}-\frac{x^2}{4}}=\sqrt{a^2-\frac{x^2}{2}}$	
	Volume of pyramid, $V=\frac{x^2}{3}\sqrt{a^2-\frac{x^2}{2}}$	
	(ii) $\frac{\mathrm{d}V}{\mathrm{d}x}=\frac{4x}{3}\sqrt{a^2-\frac{x^2}{2}}+\frac{x^2}{3}\left(\frac{1}{2}\right)\frac{-x}{\sqrt{a^2-\frac{x^2}{2}}}$	
	$=\frac{4x\left(a^2-\frac{x^2}{2}\right)-x^3}{6\sqrt{a^2-\frac{x^2}{2}}}$ $=\frac{4a^2x-3x^3}{6\sqrt{a^2-\frac{x^2}{2}}}$ $=\frac{2x\left(a-\frac{\sqrt{3}}{2}x\right)\left(a+\frac{\sqrt{3}}{2}x\right)}{3\sqrt{a^2-\frac{x^2}{2}}}$	
	When $\frac{\mathrm{d}V}{\mathrm{d}x}=0$, $\frac{2x\left(a-\frac{\sqrt{3}}{2}x\right)\left(a+\frac{\sqrt{3}}{2}x\right)}{3\sqrt{a^2-\frac{x^2}{2}}}=0$	
	$\frac{2x}{3}\left(a^2-\frac{x^2}{2}\right)=\frac{x^3}{6}$	

	$x=\frac{2\sqrt{3}}{3}a$ or $-\frac{2\sqrt{3}}{3}a$ (rejected $\because x>0$)	
	When $x<\frac{2\sqrt{3}}{3}a$, $\frac{1}{\sqrt{a^2-\frac{x^2}{2}}}>0$, $\frac{2}{3}x>0$, $a-\frac{\sqrt{3}}{2}x>0$ and $a+\frac{\sqrt{3}}{2}x>0$, thus $\frac{\mathrm{d}V}{\mathrm{d}x}>0$. When $x>\frac{2\sqrt{3}}{3}a$, $\frac{1}{\sqrt{a^2-\frac{x^2}{2}}}>0$, $\frac{2}{3}x>0$, $a-\frac{\sqrt{3}}{2}x<0$ and $a+\frac{\sqrt{3}}{2}x>0$, thus $\frac{\mathrm{d}V}{\mathrm{d}x}<0$. Therefore $x=\frac{2\sqrt{3}}{3}a$ gives maximum volume.	
	(iii) Volume of pyramidal empty space, W cm^3, in the pyramid as it is being filled up $=\frac{1}{3}b^2H$, where b is the length of the square base $\frac{b}{H}=\sqrt{2}$ Therefore $W=\frac{2}{3}H^3 \Rightarrow \frac{\mathrm{d}W}{\mathrm{d}H}=2H^2$	
	When $H=\frac{a}{2}$	
	$\frac{\mathrm{d}W}{\mathrm{d}H}\times\frac{\mathrm{d}H}{\mathrm{d}t}=\frac{\mathrm{d}W}{\mathrm{d}t}$ $2\left(\frac{a}{2}\right)^2\times\frac{\mathrm{d}H}{\mathrm{d}t}=-1$	
	$\frac{\mathrm{d}H}{\mathrm{d}t}=-\frac{2}{a^2}$	
	H is decreasing at a rate of $\frac{2}{a^2}$ cm/s.	
10	**(a)** The sequence of positive real numbers x_1, x_2, x_3,… satisfies the recurrence relation $x_{n+1}=\frac{6}{x_n+1}$ for $n\geq 1$. **(i)** If this sequence converges to l, determine the exact value of l.	[2]
	(ii) Determine the behaviour of the sequence when $x_1=3$.	[1]
	(iii) Show that $\frac{1}{x_{n+1}}-\frac{1}{l}=\frac{1}{6}(x_n-l)$. Hence, show that if $x_n>l$, then $x_{n+1}<l<x_n$.	[3]

(b) Given that $\displaystyle\sum_{n=2}^{N}\frac{n+1}{n^2(n+2)^2}=\frac{13}{144}-\frac{1}{4(N+1)^2}-\frac{1}{4(N+2)^2}$,

(i) find $\displaystyle\sum_{n=6}^{N}\frac{n}{(n^2-1)^2}$. [3]

(ii) Show that $\displaystyle\sum_{n=2}^{N}\frac{n}{(n+2)^4}<\frac{13}{144}$. [2]

Solution

(ai) When $n\to\infty$, $x_n\to l$, $x_{n+1}\to l$.

$$l=\frac{6}{l+1}$$

$$l^2+l-6=0$$

$$(l+3)(l-2)=0$$

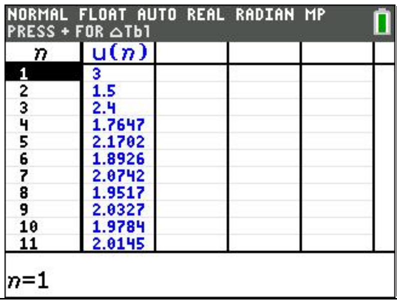
$$l=-3 \text{ or } l=2$$

Since $l>0$, $l=2$

(ii) The odd numbered terms form a decreasing sequence that converges to 2 and the even numbered terms form an increasing sequence that converges to 2. The whole sequence converges to 2.

OR

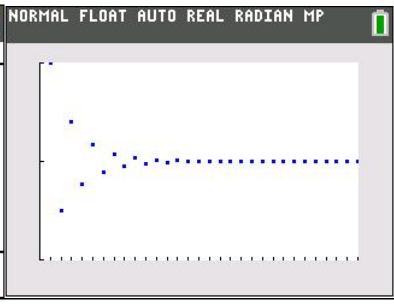
The sequence alternates about the value 2 and converges to 2 eventually.



NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl

n	u(n)
1	3
2	1.5
3	2.4
4	1.7647
5	2.1702
6	1.8926
7	2.0742
8	1.9517
9	2.0327
10	1.9784
11	2.0145

n=1



(iii) $\displaystyle\frac{1}{x_{n+1}}-\frac{1}{l}=\frac{x_n+1}{6}-\frac{l+1}{6}=\frac{1}{6}(x_n-l)$ (shown)

Given $x_n>l$,

$$\frac{1}{x_{n+1}}-\frac{1}{l}=\frac{1}{6}(x_n-l)\quad>0$$

$$\frac{1}{x_{n+1}}>\frac{1}{l}$$

Since both sides are positive, $x_{n+1}<l$

$x_{n+1}<l<x_n$ (shown)

(bi) $\displaystyle\sum_{n=6}^{N}\frac{n}{(n^2-1)^2}=\sum_{n=6}^{N}\frac{n}{(n-1)^2(n+1)^2}$

	Replacing n by $(n+1)$, we get $\sum_{n=5}^{N-1}\frac{n+1}{n^2(n+2)^2}$	
	$=\sum_{n=2}^{N-1}\frac{n}{n^2(n+2)^2}-\sum_{n=2}^{4}\frac{n}{n^2(n+2)^2}$	
	$=\frac{13}{144}-\frac{1}{4N^2}-\frac{1}{4(N+1)^2}-\left(\frac{13}{144}-\frac{1}{100}-\frac{1}{144}\right)$ $=\frac{61}{3600}-\frac{1}{4N^2}-\frac{1}{4(N+1)^2}$	
	(ii) Since $(n+2)^4>n^2(n+2)^2$ and $n<n+1$, $\sum_{n=2}^{N}\frac{n}{(n+2)^4}<\sum_{n=2}^{N}\frac{n+1}{n^2(n+2)^2}$	
	$\sum_{n=2}^{N}\frac{n}{(n+2)^4}<\frac{13}{144}-\frac{1}{4(N+1)^2}-\frac{1}{4(N+2)^2}$	
	Since $-\frac{1}{4(N+1)^2}<0$ and $-\frac{1}{4(N+2)^2}<0$ $\sum_{n=2}^{N}\frac{n}{(n+2)^4}<\frac{13}{144}$	
	A1 – For the reason	
11	**(a)** If $0<a<b$, solve $\int_0^b x\lvert a-x\rvert\ \mathrm{d}x$, leaving your answers in terms of a and b.	[2]
	(b)(i) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{3-x}{\sqrt{1-x}}\right)$.	[1]
	(ii) Find $\int\frac{3-x}{x^2-3x+2}\mathrm{d}x$.	[2]
	(iii) Hence find $\int\frac{1+x}{(1-x)^{\frac{3}{2}}}\tan^{-1}\sqrt{1-x}\ \mathrm{d}x$.	[3]
	(c) The region R is the finite region enclosed by the curve $(y-1)^2=1-x$ and the y-axis. The region S is the region in the 2nd quadrant enclosed by the curve $y=2\tan\left(x+\frac{\pi}{4}\right)$ and the axes.	
	Find the total volume generated when region R and S is rotated through 2π radians about the x-axis, leaving your answers in exact form.	[4]
	Solution	
	(a) $\int_0^b x\lvert a-x\rvert\ \mathrm{d}x=\int_0^a x(a-x)\ \mathrm{d}x+\int_a^b x(x-a)\ \mathrm{d}x$	
	$=\left[\frac{ax^2}{2}-\frac{x^3}{3}\right]_0^a+\left[\frac{x^3}{3}-\frac{ax^2}{2}\right]_a^b$	

	$=\left(\frac{a^3}{2}-\frac{a^3}{3}\right)+\left(\frac{b^3}{3}-\frac{ab^2}{2}\right)-\left(\frac{a^3}{3}-\frac{a^3}{2}\right)$	
	$=\frac{a^3}{3}+\frac{b^3}{3}-\frac{ab^2}{2}$	
	(bi) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{3-x}{\sqrt{1-x}}\right)=\frac{-\sqrt{1-x}-(3-x)\left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{1-x}}\right)}{1-x}$	
	$=\frac{-2+2x+3-x}{2(1-x)^{3/2}}=\frac{x+1}{2(1-x)^{3/2}}$	
	(ii) $\int\frac{3-x}{x^2-3x+2}\mathrm{d}x=\int\frac{3-x}{(x-2)(x-1)}\mathrm{d}x$	
	$=\int\left[\frac{1}{x-2}-\frac{2}{x-1}\right]\mathrm{d}x$	
	$=\ln\|x-2\|-2\ln\|x-1\|+c$	
	(iii) $\int\frac{1+x}{(1-x)^{\frac{3}{2}}}\tan^{-1}\sqrt{1-x}\ \mathrm{d}x$ $=2\left(\frac{3-x}{\sqrt{1-x}}\right)\tan^{-1}\sqrt{1-x}-\int 2\left(\frac{3-x}{\sqrt{1-x}}\right)\left(\frac{1}{1+1-x}\right)\left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{1-x}}\right)\ \mathrm{d}x$	
	$=2\left(\frac{3-x}{\sqrt{1-x}}\right)\tan^{-1}\sqrt{1-x}+\int\frac{3-x}{(2-x)(1-x)}\ \mathrm{d}x$	
	$=2\left(\frac{3-x}{\sqrt{1-x}}\right)\tan^{-1}\sqrt{1-x}+\ln\|x-2\|-2\ln\|x-1\|+c$	
	(c) Vol $=\pi\int_{-\frac{\pi}{4}}^{0}4\tan^2\left(x+\frac{\pi}{4}\right)\mathrm{d}x+\pi\int_0^1\left[\left(1+\sqrt{1-x}\right)^2-\left(1-\sqrt{1-x}\right)^2\right]\mathrm{d}x$	
	$=4\pi\int_{-\frac{\pi}{4}}^{0}\left(\sec^2\left(x+\frac{\pi}{4}\right)-1\right)\mathrm{d}x+\pi\int_0^1 4\sqrt{1-x}\ \mathrm{d}x$	
	$=4\pi\left[\tan\left(x+\frac{\pi}{4}\right)-x\right]_{-\frac{\pi}{4}}^{0}+4\pi\left[\frac{(1-x)^{\frac{3}{2}}}{-\frac{3}{2}}\right]_0^1$	
	$=4\pi\left(1-\frac{\pi}{4}\right)+\frac{8}{3}\pi=\frac{20}{3}\pi-\pi^2$	
12	A drone carrying a bomb departs from a point $A(-1,1,-3)$. It flies in a direction of $-a\mathbf{i}+\mathbf{j}+2a\mathbf{k}$, where $a>0$, across a lake to a target location. It is given that the surface of the lake is part of a plane with equation $x-z=2$.	
	(i) Determine the value of a if the path of the drone makes an angle of $\frac{\pi}{3}$ radians with the surface of the lake.	[3]

	It is given that $a = 2$.	
	(ii) A launcher is launched to intercept the drone. It moves in a path with equation $\frac{10-x}{2} = -z,\ y = m$, where m is a real constant. Given that the launcher is successful in intercepting the drone, find the point of interception.	[3]
	(iii) At the point of interception, the drone falls perpendicular to the lake and meets the surface of the lake at *N*. Find the position vector of *N*.	[3]
	(iv) Denoting the line that contains the drone's path as L_1 and L_1' being the reflection of L_1 in the surface of the lake, find a vector equation of L_1'.	[3]
	Solution	
	(i) $\sin\frac{\pi}{3} = \left\lvert \frac{\begin{pmatrix} -a \\ 1 \\ 2a \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}{\sqrt{5a^2+1}\sqrt{2}} \right\rvert$	
	$(6a)^2 = 6\left(5a^2+1\right)$	
	$a = 1$ or $a = -1$ (rejected since $a > 0$)	
	(ii) Given that $a = 2$. $l_{\textbf{drone}}:\ \mathbf{r} = \begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix},\ \lambda \in \mathbb{R}$ $l_{\textbf{launcher}}:\ \mathbf{r} = \begin{pmatrix} 10 \\ m \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix},\ \mu \in \mathbb{R}$ $\begin{pmatrix} -1-2\lambda \\ 1+\lambda \\ -3+4\lambda \end{pmatrix} = \begin{pmatrix} 10+2\mu \\ m \\ \mu \end{pmatrix}$	
	$-2\mu - 2\lambda = 11$ $-\mu + 4\lambda = 3$	
	$\lambda = -\frac{1}{2},\ \mu = -5$	
	$m = \frac{1}{2}$	
	Coordinates of point of intersection are $\left(0, \frac{1}{2}, -5\right)$.	
	(iii) Let the point of interception be *P*.	

	$\ell_{NP}: \mathbf{r} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ -5 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix},\ \alpha \in \mathbb{R}$ $\begin{pmatrix} \alpha \\ \frac{1}{2} \\ -5-\alpha \end{pmatrix} . \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = 2$	
	$2\alpha = -3$	
	$\alpha = -\frac{3}{2}$	
	$\overrightarrow{ON} = \frac{1}{2}\begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix}$	
	OR $\overrightarrow{NP} = \left[\begin{pmatrix} 1 \\ -\frac{1}{2} \\ -2 \end{pmatrix} \bullet \frac{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}{\sqrt{2}} \right] \frac{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}{\sqrt{2}}$	
	$= \frac{1}{2}\begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$	
	$\overrightarrow{ON} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ -5 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix}$	
	(iv) Let P’ be the reflected point of P in the lake. Using ratio theorem, $\overrightarrow{ON} = \frac{\overrightarrow{OP} + \overrightarrow{OP'}}{2}$ $\Rightarrow \overrightarrow{OP'} = 2\overrightarrow{ON} - \overrightarrow{OP} = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 0 \\ 1 \\ -10 \end{pmatrix} = \begin{pmatrix} -3 \\ \frac{1}{2} \\ -2 \end{pmatrix}$	
	The direction vector of the reflected line is $\begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix} - \begin{pmatrix} -3 \\ \frac{1}{2} \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ \frac{1}{2} \\ -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$	

	$\therefore L_1': \mathbf{r} = \begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix} + \beta \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}, \beta \in \mathbb{R}$	

End of Paper