



CANDIDATE
NAME

CG

MATHEMATICS

Paper 1

9758/01

20 August 2026

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

3 hours

READ THESE INSTRUCTIONS FIRST

Write your CG, index number and name on the work you hand in.

Answer **all** questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

- 1** A function f is defined by $f(x) = ax^3 + bx^2 + cx + d$. The graph of $y = f(x)$ has turning points at $(-1, 4)$ and $\left(\frac{1}{2}, -\frac{11}{4}\right)$. Find the values of a , b , c , and d . [4]

- 2** A geometric series has first term a and common ratio r . The sum to infinity of the series is 16 and its fourth term is $\frac{3}{16}$.

(a) Show that r satisfies the equation $256r^4 - 256r^3 + 3 = 0$. Given that $|r| < 0.5$, solve this equation and hence show that $a = 12$. [5]

(b) Find the exact value of the sum to infinity of the even-numbered terms of the geometric series. [2]

- 3** It is given that

$$f(x) = \begin{cases} \frac{10}{9-4x^2} & \text{for } -1 \leq x < 1, \\ 2 & \text{for } 1 \leq x < 3, \end{cases}$$

and that $f(x) = f(x+4)$ for all real values of x .

(a) Evaluate $f(-49)$ [1]

(b) Sketch the graph of $y = f(x)$ for $-2 \leq x \leq 7$. [3]

(c) Find the exact value of $\int_4^6 f(x) \, dx$. [3]

- 4** **(a)** Find the exact solutions of the equation $|x^2 - 6| = 2x + 7$. [4]

(b) Sketch, on the same axes, the graphs of $y = |x^2 - 6|$ and $y = 2x + 7$.

Hence solve exactly the inequality $|x^2 - 6| < 2x + 7$. [3]

- 5 The sum of the first n terms of a sequence $u_1, u_2, u_3, \dots$ is given by

$$\sum_{r=1}^n u_r = a - \frac{2a}{(n+2)!}, \text{ where } a \text{ is a positive constant.}$$

- (a) Find u_n in terms of a , expressing your answer as a single algebraic fraction. [2]
- (b) Explain why the series $\sum_{r=1}^{\infty} u_r$ converges. [1]
- (c) Find $\sum_{r=7}^{\infty} u_r$ in terms of a . [2]
- (d) Find the least value of n for which the sum of the first n terms of the sequence is within $a \times 10^{-8}$ of its sum to infinity. [3]
- 6 (a) Describe a sequence of transformations that transform the graph of $y = \ln x$ onto the graph of $y = \ln(ax + b)$, where a and b are positive constants. [2]
- (b) The curve $y = f(x)$ cuts the axes at $(0, -3)$ and $(-1, 0)$. It is given that the equation of the asymptote is $x = 2$. State, if it is possible to do so, the equations of any asymptotes for the following curves and the coordinates of any points where the curves cut the axes.
- (i) $y = f(|x|)$, [2]
- (ii) $y = \frac{1}{f(x)}$. [2]
- 7 A curve has parametric equations $x = \cos 2\theta$, $y = \ln(\sin 2\theta)$, $0 < \theta < \frac{\pi}{2}$.
- (a) Sketch the curve, stating the equations of the two vertical asymptotes. [2]
- (b) State the equation of the line of symmetry of the curve. [1]
- (c) Given that the gradient of the curve at a point P is between -1.5 and 1.5 , find the range of possible values of the x -coordinate of P . [4]
- 8 A curve has equation $\frac{1}{x} - \frac{1}{y} = 3$, where $x < \frac{1}{3}$.
- (a) Show that $\frac{dy}{dx} = \frac{y^2}{x^2}$. [2]
- (b) Using calculus, find the equation of the tangent to the curve at the point where $x = -1$, giving your answer in the form $y = mx + c$. [3]
- (c) Given that y increases at a constant rate of 5 units per second, find the rate of change of x when $y = 1$. [3]

- 9** (a) Without using a calculator, solve the equation $z^2 = 9 - 40i$, leaving your answers in the form $a + bi$, where $a, b \in \mathbb{R}$. [5]
- (b) It is given that the Maclaurin series for e^x can be generalised to expressions of the form e^{wx} for all complex numbers w and real numbers x .
- (i) Find the first six non-zero terms of the Maclaurin series for e^{ix} . [2]
- (ii) By considering the Maclaurin series for $\cos x$ and $\sin x$, verify that $e^{ix} = \cos x + i \sin x$. [1]
- 10** It is given that $y = e^{\sin^{-1}(3x)}$, where $-\frac{1}{3} < x < \frac{1}{3}$.
- (a) Show that $(1 - 9x^2) \frac{d^2y}{dx^2} - 9x \frac{dy}{dx} - 9y = 0$. [3]
- (b) By differentiating the above result, find the Maclaurin series for y , up to and including the term in x^3 . [4]
- (c) Using your result in part (b) and the standard series in the List of Formulae (MF27), obtain the Maclaurin series for $\sin^{-1}(3x)$, up to and including the term in x^3 . [3]
- 11** The plane π_1 has equation $x + 2y + 2z = 14$. The line l_1 passes through the point A with coordinates $(1, 0, 2)$ and is parallel to $\mathbf{i} + \mathbf{j}$.
- (a) Find the acute angle between π_1 and l_1 . [2]
- (b) Find the coordinates of the point of intersection of l_1 and π_1 . [3]
- (c) Find the position vector of the foot of perpendicular from A to π_1 . [3]
- The line l_2 is the reflection of l_1 in π_1 .
- (d) Show that l_2 is parallel to $-\mathbf{i} + \mathbf{j} + 4\mathbf{k}$. [2]
- (e) The plane π_2 contains l_1 and l_2 . Find a cartesian equation of π_2 . [3]
- (f) Explain whether π_1 and π_2 are perpendicular. [1]

- 12** Scientists are investigating the toxin levels in marine fish living in a particular habitat. The rate of change of the toxin level in a marine fish, u milligrams (mg), with respect to time, t days, is proportional to the difference between the toxin level in the habitat and the toxin level in the fish. It may be assumed that the toxin level in the habitat, a mg, is constant. Two models describe the fluctuation of the toxin level over time.

Model A is a continuous model using differential equations.

- (a) Write down a differential equation for this model. [1]
- (b) By solving the differential equation in part (a), show that $u = a - Ae^{-kt}$, where A and k are constants. [3]

The scientists narrow their investigation to a particular species of marine fish for which $a = 23$. Based on published data, when a fish of this species has an initial toxin level of 13 mg, its toxin level increases to 19.3 mg after 15 days.

- (c) Find the toxin level in the fish after 30 days. [4]
- (d) Sketch the graph of u against t . [2]

Model B is a discrete model using the recurrence relation $u_{n+1} = b + 0.96u_n$, where u_n is the toxin level in the fish on the n th day and b is a constant. The starting term is u_0 . This recurrence relation has a general solution $u_n = C + (0.96)^n D$ for some constants C and D .

- (e) Use the earlier published data on the toxin levels to find the values of C and D , giving your answers correct to 3 decimal places. [2]
- (f) Describe how the toxin level changes over time based on this model. [1]

The fish will die if the toxin level in its body reaches 25 mg. A study reveals that many of the fish have died from the toxin in recent years.

- (g) Explain which of the models, A or B, should be used in view of this study. [1]

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