**RAFFLES INSTITUTION**
**H2 Mathematics (9758)**
**2025 Year 5**

## Additional Practice Questions for Chapter 5: Complex Numbers

**1** Given that $(-2+3\text{i})^2+\lambda(-2+3\text{i})+\mu=0$, find the real numbers $\lambda$ and $\mu$. [6]

[ $\lambda=4,\ \mu=13$ ]

**2** The complex number $x+\text{i}y$ is such that $(x+\text{i}y)^2=\text{i}$. Find the possible values of $x,\ y\in\mathbb{R}$, giving your answers in exact form. [4]

Hence find the possible values of the complex number $w$ such that $w^2=-\text{i}$. [2]

[ $x=\frac{1}{\sqrt{2}},\ y=\frac{1}{\sqrt{2}},\ \ x=-\frac{1}{\sqrt{2}},\ y=-\frac{1}{\sqrt{2}};\ \frac{1}{\sqrt{2}}-\text{i}\frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}+\text{i}\frac{1}{\sqrt{2}}$ ]

**3** The complex number $z$ is given by $z=1+2a\text{i}$, where $a$ is a non-zero real number. Given that $z^3$ is real, find the possible values of $z$. [3]

[ $z=1+\sqrt{3}\,\text{i}$ or $z=1-\sqrt{3}\,\text{i}$ ]

**4** The complex number $w$ is such that $w=a+\text{i}b$, where $a$ and $b$ are non-zero real numbers. The complex conjugate of $w$ is denoted by $w^*$. Given that $\frac{w^2}{w^*}$ is purely imaginary, find the possible values of $w$ in terms of $a$. [5]

[ $a+\text{i}\frac{a}{\sqrt{3}}$ or $a-\text{i}\frac{a}{\sqrt{3}}$ ]

**5** **(a)** Solve the simultaneous equations

$$\text{i}w+z=-1-\text{i},\ 2z-(1+\text{i})w=\frac{40}{6-2\text{i}}.$$ [3]

**(b)** Two complex numbers $w$ and $z$ are such that

$$w^*=z-2\text{i},\qquad |w|^2=z+6.$$

Find $w$ in the form $a+\text{i}b$, where $a$ and $b$ are real and positive. [4]

[**(a)** $w=-2+2\text{i},\ z=1+\text{i}$ **(b)** $w=2+2\text{i}$ ]

**6** The complex numbers $s$ and $w$ satisfy the equations

$$s - w = 6\text{i} \qquad \text{and} \qquad sw = 10.$$

Given that $\text{Re}(s) > 0$, solve the equations for $s$ and $w$, giving all answers in the form $x + \text{i}y$, where $x$ and $y$ are real. [4]

Hence find a solution to the following equations

$$u - v = -6 \qquad \text{and} \qquad uv = -10.$$

Give your answers for $u$ and $v$ in the form $x + \text{i}y$, where $x$ and $y$ are real. [2]

[ $s = 1 + 3\text{i}$, $w = 1 - 3\text{i}$ ; $u = -3 + \text{i}$ and $v = 3 + \text{i}$ ]

**7** **Do not use a calculator in answering this question.**

Given that $2 - 3\text{i}$ is a root of the equation

$$z^4 - 10z^3 + 48z^2 - 122z + 143 = 0,$$

solve the equation, giving your answers in exact form. [4]

[ $2 + 3\text{i}$ and $3 \pm \sqrt{2}\,\text{i}$ ]

**8** It is given that $-1 + 2\text{i}$ satisfies the equation $z^2 + (a - \text{i})z + 16 + b\text{i} = 0$, where $a, b \in \mathbb{R}$.
Find the value of $a$ and $b$ and the other root. [5]

[ $a = 15$ and $b = -27$, $-14 - \text{i}$ ]

**9** **Do not use a calculator in answering this question.**

**(a)** Verify that $-1 + 5\text{i}$ is a root of the equation $w^2 + (-1 - 8\text{i})w + (-17 + 7\text{i}) = 0$. Hence, or otherwise, find the second root of the equation in cartesian form, $p + \text{i}q$, showing your working. [5]

**(b)** The equation $z^3 - 5z^2 + 16z + k = 0$, where $k$ is a real constant, has a root $z = 1 + a\text{i}$, where $a$ is a positive real constant. Find the values of $a$ and $k$, showing your working. [5]

[**(a)** $2 + 3\text{i}$ **(b)** $a = 3$, $k = -30$ ]

**10** **Do not use a calculator in answering this question.**

**(a)** Find the roots of the equation $z^2(1-\mathrm{i})-2z+(5+5\mathrm{i})=0$, giving your answers in cartesian form $a+\mathrm{i}b$. [3]

**(b)** **(i)** Given that $\omega=1-\mathrm{i}$, find $\omega^2$, $\omega^3$ and $\omega^4$ in cartesian form. Given also that

$$\omega^4+p\omega^3+39\omega^2+q\omega+58=0,$$

where $p$ and $q$ are real, find $p$ and $q$. [4]

**(ii)** Using the values of $p$ and $q$ in part **(b)(i)**, express $\omega^4+p\omega^3+39\omega^2+q\omega+58$ as the product of two quadratic factors. [3]

[**(a)** $-1+2\mathrm{i}$ or $2-\mathrm{i}$ **(bi)** $p=-6,\ q=-66$ **(bii)** $(\omega^2-2\omega+2)(\omega^2-4\omega+29)$ ]

**11** A graphic calculator is **not** to be used in answering this question.

**(i)** It is given that $z_1=1+\sqrt{3}\mathrm{i}$. Find the value of $z_1{}^3$, showing clearly how you obtain your answer. [3]

**(ii)** Given that $1+\sqrt{3}\mathrm{i}$ is a root of the equation

$$2z^3+az^2+bz+4=0,$$

Find the values of the real numbers $a$ and $b$. [4]

**(iii)** For these values of $a$ and $b$, solve the equation in part **(ii)**, and show all the roots on an Argand diagram. [4]

[**(i)** $-8$ **(ii)** $a=-3$, $b=6$ **(iii)** $z_1=1+\sqrt{3}\mathrm{i}$, $z_2=1-\sqrt{3}\mathrm{i}$ and $z_3=-\dfrac{1}{2}$ ]

**12** Given that $z_1=1-2\mathrm{i}$ is a root of the quadratic equation $z^2+az+b=0$ where $a,\ b\in\mathbb{R}$, find the values of $a$ and $b$. Mark on an Argand diagram the points $P_1$ and $P_2$, which represent the roots $z_1$ and $z_2$ of the quadratic equation $z^2+az+b=0$. A third point $P_3$ is on the real axis such that $P_1$, $P_2$ and $P_3$ form a triangle which encloses the origin with an area of 8 square units. Find the complex number represented by the point $P_3$. [5]

[ $a=-2,\ b=5$ ; $z_3=-3$ ]

**13** The complex number $z$ and $w$ are given by and $z=-3+2\mathrm{i}$ and $w=5+4\mathrm{i}$. In an Argand diagram, the points $Z$ and $W$ represent the complex numbers $z$ and $w$ respectively, and $Z'$ represents $z*$, the complex conjugate of $z$. The point $P$ is such that $ZWZ'P$ is a parallelogram. Find the complex number $p$ represented by $P$.

[ $-11-4\mathrm{i}$ ]

**14** The complex number $7+5\mathrm{i}$ is represented by the point $A$ in an Argand diagram with origin $O$. Given that $OABC$ is a rectangle described in a clockwise sense with $OC = 2OA$, find the complex numbers represented by the points $B$ and $C$ in the form $x+\mathrm{i}y$, where $x$ and $y$ are real.

[$b = 17-9\mathrm{i}$, $c = 10-14\mathrm{i}$]

**15** The complex numbers $a$, $b$, $c$ and $d$ are represented by the points $A$, $B$, $C$ and $D$ respectively on the Argand diagram such that $ABCD$ is a square described in the anti-clockwise sense. It is given that $a = 2\mathrm{i}$ and $d = \left(\sqrt{3}+1\right)+\left(\sqrt{3}+1\right)\mathrm{i}$.

**(i)** Find the complex numbers $b$ and $c$, leaving your answers in the form $x + \mathrm{i}y$, where $x$ and $y$ are real numbers to be determined exactly.

**(ii)** The points $A$, $B$, $C$ and $D$ lie on the circle of radius $k$, centred at the point $M$ representing the complex number $m$. State the values of $m$ and $k$.

[**(i)** $b = \left(\sqrt{3}-1\right)+\left(1-\sqrt{3}\right)\mathrm{i}$, $c = 2\sqrt{3}$ **(ii)** $m = \sqrt{3}+\mathrm{i}$ and $k = 2$ units]

**16** The complex numbers $z_1$ and $z_2$ are given by $z_1 = -1+\mathrm{i}a$ and $z_2 = \sqrt{2}-\sqrt{2}\mathrm{i}$ where $a$ is a real number. Given that $\operatorname{Im}\left(\dfrac{z_1}{z_2}\right) = \dfrac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$, show that $a = \sqrt{3}$. [2]

State the modulus and argument of each of $z_1$ and $z_2$. [2]

Sketch an Argand diagram with origin $O$, showing the points $P$, $Q$ and $R$ representing the complex numbers $z_1$, $z_2$ and $(z_1+z_2)$ respectively. Give a geometrical description of $OPRQ$. [2]

Deduce from your diagram that $\tan\dfrac{5\pi}{24} = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{2}-1}$. [3]

[$|z_1| = 2$, $\arg(z_1) = \dfrac{2\pi}{3}$; $|z_2| = 2$, $\arg(z_2) = -\dfrac{\pi}{4}$; rhombus]

**THE END**