



Secondary 3 Mathematics
WA3 Revision Time Trial — Bedok South (2023)

Duration: 47 minutes

Score: _____ / 31

Topics tested: Graphs of Functions; Graphs in Practical Situations; Further Trigonometry; Arc Length and Sector Area.

Section A: Graphs of Functions [11 marks]

1. The variables x and y are connected by the equation $y = 1 - x^3 + 3x^2$.

The table below shows some corresponding values of x and y , correct to 1 decimal place where appropriate.

x	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y	5	1.9	1	p	3	4.4	5	4.1	1

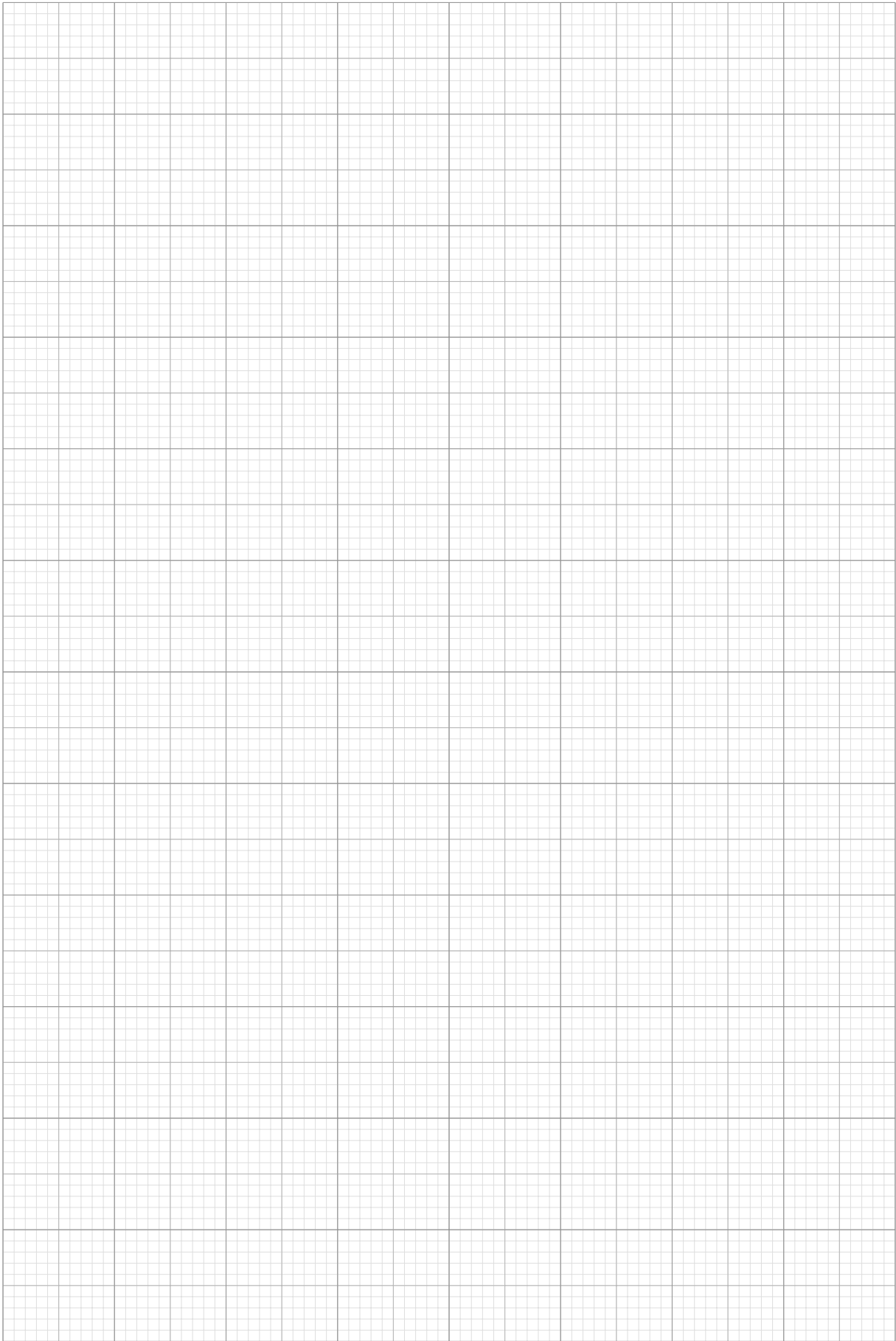
(a) Calculate the value of p , giving your answer correct to 1 decimal place.

[1]

Answer $p =$

(b) Using a scale of 2 cm to represent 0.5 units on a horizontal x -axis for $-1 \leq x \leq 3$ and 2 cm to represent 1 unit on a vertical y -axis for $-1 \leq y \leq 7$, on the graph paper provided on the next page, plot the points given in the table and join them with a smooth curve.

[3]



(c) Use your graph to find all the solutions of $1 - x^3 + 3x^2 = 2$.

[2]

Answer

(d) By drawing a suitable tangent, find the gradient of the curve $y = 1 - x^3 + 3x^2$ at the point where $x = 2.5$.

[2]

Answer

(e) (i) On the same axes, draw the line $y = -2x + 5$ for $-1 \leq x \leq 3$.

[2]

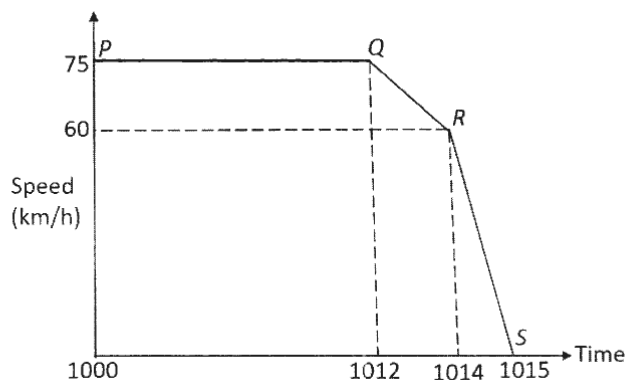
(ii) Hence, use your graph to find the range of values of x such that $1 - x^3 + 3x^2 \geq -2x + 5$ in the range $-1 \leq x \leq 3$.

[1]

Answer

Section B: Graphs in Practical Situations [4 marks]

2. The diagram shows the speed-time graph of a car which travelled from P to S .



(a) How long was the car travelling at constant speed?

[1]

Answer

(b) What is the distance travelled by the car from P to S ?

[2]

Answer

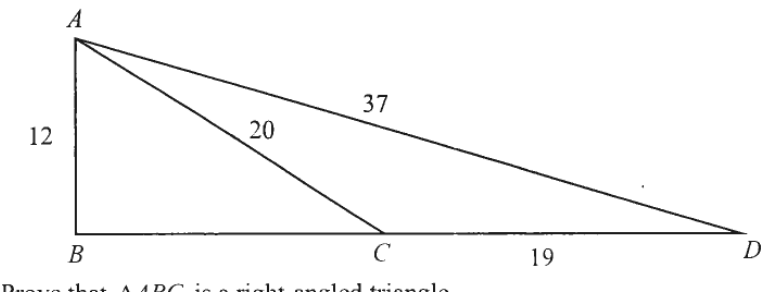
(c) What is the deceleration from Q to R in km/h^2 ?

[1]

Answer

Section C: Further Trigonometry [5 marks]

3. ABD is a triangle with $AB = 12$ cm, $AC = 20$ cm, $CD = 19$ cm, $AD = 37$ cm and $BD = 35$ cm.



(a) Prove that $\triangle ABC$ is a right-angled triangle. [2]

Answer
.....
.....

(b) Express the following as a fraction in its lowest term.

(i) $\sin \angle ACD$ [1]

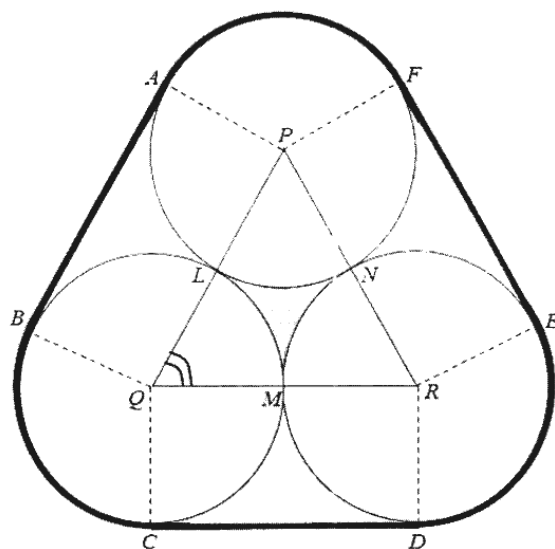
Answer

(ii) $\cos \angle CAD$ [2]

Answer

Section D: Arc Length and Sector Area [11 marks]

4. The diagram shows the cross-section of 3 circular tins of the same size with centres P , Q and R . The tins touch one another externally at L , M and N .



(a) (i) Show that $\angle PQR = \frac{\pi}{3}$ radians. [2]

Answer

.....

.....

(ii) Given that the area of $\triangle PQR = 6.93 \text{ cm}^2$, show that the length QL is approximately 2.00 cm. [2]

(b) Calculate the shaded area.

[3]

Answer cm^2

(c) The circular tins are bounded together by a piece of rope $ABCDEF$.
Calculate the total length of the rope used.

[4]

Answer cm

— END OF PAPER —

Section A: Graphs of Functions [11 marks]

1(a) At $x = 0.5$: $y = 1 - 0.125 + 0.75 = 1.625$. $\therefore p = \mathbf{1.6}$ (1 d.p.). [1]

1(b) Axes ruled with the given scale (x : 2 cm per 0.5 unit from -1 to 3 ; y : 2 cm per unit from -1 to 7), all nine points plotted and joined with a smooth curve: local minimum at $(0, 1)$, rising to a local maximum at $(2, 5)$, then falling to $(3, 1)$. [3]

1(c) $1 - x^3 + 3x^2 = 2$ is where the curve meets the horizontal line $y = 2$:
 $x \approx \mathbf{-0.53, 0.65}$ and $\mathbf{2.88}$ (accept ± 0.05 each). [2]

1(d) Tangent drawn at $x = 2.5$.

Exact gradient: $\frac{dy}{dx} = -3x^2 + 6x = -18.75 + 15 = \mathbf{-3.75}$ (accept ± 0.5). [2]

1(e)(i) Line $y = -2x + 5$ drawn through e.g. $(0, 5)$, $(2, 1)$ and $(3, -1)$. [2]

1(e)(ii) The curve is on or above the line for $\mathbf{1 \leq x \leq 3}$. [1]

Section B: Graphs in Practical Situations [4 marks]

2(a) Constant speed from 1000 to 1012, i.e. **12 minutes**. [1]

2(b) Distance = area under graph

$$= \frac{1}{2} \left(\frac{15}{60} + \frac{14}{60} \right) (60) + \frac{1}{2} \left(\frac{14}{60} + \frac{12}{60} \right) (15) = 14.5 + 3.25 = \mathbf{17.75 \text{ km.}}$$
 [2]

2(c) From Q to R the speed falls from 75 to 60 km/h in 2 minutes $= \frac{1}{30}$ h.

$$\text{Deceleration} = \frac{15}{1/30} = \mathbf{450 \text{ km/h}^2}.$$
 [1]

Section C: Further Trigonometry [5 marks]

3(a) $BC = BD - CD = 35 - 19 = 16 \text{ cm.}$

$$AB^2 + BC^2 = 12^2 + 16^2 = 400 \text{ and } AC^2 = 20^2 = 400.$$

Since $AB^2 + BC^2 = AC^2$, by the **Converse of Pythagoras' Theorem**, $\triangle ABC$ is a right-angled triangle. [2]

$$\mathbf{3(b)(i)} \quad \sin \angle ACD = \sin(180^\circ - \angle ACB) = \sin \angle ACB = \frac{12}{20} = \frac{\mathbf{3}}{\mathbf{5}}.$$
 [1]

3(b)(ii) By the Cosine Rule in triangle ACD :

$$\cos \angle CAD = \frac{20^2 + 37^2 - 19^2}{2(20)(37)} = \frac{1408}{1480} = \frac{\mathbf{176}}{\mathbf{185}}.$$
 [2]

Section D: Arc Length and Sector Area [11 marks]

4(a)(i) $PQ = QR = RP$ (each is the sum of two equal radii),
so $\triangle PQR$ is equilateral and $\angle PQR = 60^\circ = \frac{180^\circ}{3} = \frac{\pi}{3}$ radians (shown). [2]

4(a)(ii) Let the radius be r , so $PQ = QR = 2r$.
 $\frac{1}{2}(2r)^2 \sin \frac{\pi}{3} = 6.93 \Rightarrow 4r^2 = \frac{2(6.93)}{\sin(\pi/3)} = 16.004 \dots$
 $r = 2.0002 \dots \approx \mathbf{2.00}$ cm (3 s.f.), and $QL = r$ (shown). [2]

4(b) Area of one sector (e.g. QLM) $= \frac{1}{2}(2)^2 \left(\frac{\pi}{3}\right) = 2.0944 \text{ cm}^2$.
Shaded area = area of $\triangle PQR - 3$ sectors $= 6.93 - 3(2.0944) = \mathbf{0.647 \text{ cm}^2}$ (3 s.f.). [3]

4(c) At each tin the rope wraps an angle of $2\pi - 2\left(\frac{\pi}{2}\right) - \frac{\pi}{3} = \frac{2\pi}{3}$.

Each arc $= 2\left(\frac{2\pi}{3}\right) = \frac{4\pi}{3}$ cm; each straight section $= PQ = 4$ cm.

Total length $= 3\left(\frac{4\pi}{3}\right) + 3(4) = 4\pi + 12 = \mathbf{24.6}$ cm (3 s.f.). [4]