Topical Revision Paper 1

Term 1 Unit 1 Advanced Algebraic Manipulation

Question 1:

Given that $x^2 - 4x = y^2 - 4y$ and that $x \neq y$, find the value of $3\sqrt{x+y}$. [3]

Solution:

$x^2 - 4x = y^2 - 4y \Rightarrow (x-y)(x+y) = 4(x-y)$

Since $x \neq y$, $x - y \neq 0 \Rightarrow x + y = 4$

Hence $3\sqrt{x+y} = 3\sqrt{4}$

$= 6$

Question 2:

Simplify into a single fraction $\frac{x^3+8y^3}{x^2-4y^2} - \frac{16xy^2-4x^3}{(4y-2x)^2}$. [4]

Solution:

$$\frac{x^3+8y^3}{x^2-4y^2} - \frac{16xy^2-4x^3}{(4y-2x)^2}$$

$$= \frac{(x+2y)(x^2-2xy+4y^2)}{(x+2y)(x-2y)} - \frac{4x(2y-x)(2y+x)}{4(2y-x)^2}$$

$$= \frac{x^2-2xy+4y^2}{x-2y} - \frac{2xy+x^2}{2y-x}$$

$$= \frac{2x^2+4y^2}{x-2y}$$

Question 3:

(a) Factorise completely $2a^3 + 6a^2b - 8ab^2 - 24b^3$. [3]

(b) Make q the subject of the formula $pq = \sqrt{4q^2 - 9p^2}$. [3]

Solution:

a) $2a^3 + 6a^2b - 8ab^2 - 24b^3$

$= 2a^2(a+3b) - 8b^2(a+3b)$

$= (2a^2 - 8b^2)(a+3b)$

$= 2(a-2b)(a+2b)(a+3b)$

(b) $pq = \sqrt{4q^2 - 9p^2}$

$\Rightarrow p^2q^2 = 4q^2 - 9p^2$

$\Rightarrow q^2(4-p^2) = 9p^2$

$\Rightarrow q = \pm\sqrt{\dfrac{9p^2}{4-p^2}}$ or $\pm\dfrac{3p}{\sqrt{4-p^2}}$

Question 4:

(a) It is given that $R = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$.

(i) Find the value of R when $v = \sqrt{5}$ and $c = 3$. [1]

(ii) Express v in terms of R and c. [3]

(b) Factorise completely $x^4 - x^2 + 14xy - 49y^2$. [3]

Solution:

(a)

$R = \dfrac{1}{\sqrt{1-\dfrac{5}{9}}}$

$= \dfrac{1}{\left(\dfrac{2}{3}\right)}$

$= \dfrac{3}{2}$

(b)

$$R^2 = \frac{1}{1-\frac{v^2}{c^2}}$$

$$= \frac{c^2}{c^2 - v^2}$$

$$c^2 - v^2 = \frac{c^2}{R^2}$$

$$v = \pm\sqrt{c^2 - \frac{c^2}{R^2}} \quad \text{or} \quad v = \pm\sqrt{\frac{R^2c^2 - c^2}{R^2}}$$

$$\text{or } v = \pm\frac{c}{R}\sqrt{R^2 - 1}$$

(c)

$$x^4 - x^2 + 14xy - 49y^2$$

$$= x^4 - \left(x^2 - 14xy + 49y^2\right)$$

$$= x^4 - \left(x - 7y\right)^2$$

$$= \left(x^2 - x + 7y\right)\left(x^2 + x - 7y\right)$$

Question 5:

(a) Given that $4a - 3b = a + 2b$, find the value of $\frac{2ab}{a^2 + 2b^2}$. [3]

(b) Solve for x and y if $\sqrt{x} - 3y^3 - 1 = 2\sqrt{x} - 5y^3 - 3 = 0$. [4]

Solution:

(a)

$$4a - 3b = a + 2b$$

$$3a = 5b$$

$$b = \frac{3}{5}\text{a}$$

$$\frac{2ab}{a^2 + 2b^2} = \frac{2a\left(\frac{3a}{5}\right)}{a^2 + 2\left(\frac{3a}{5}\right)^2} = \frac{30}{43}$$

(b)

$\sqrt{x}-3y^3=1$

$\Rightarrow 2\sqrt{x}-6y^3=2-----(1)$

$2\sqrt{x}-5y^3=3-------(2)$

$(1)-(2):-y^3=-1$

$\Rightarrow y=1$ sub into (1)

$\sqrt{x}-3=1$

$\sqrt{x}=4$

$\Rightarrow x=16$