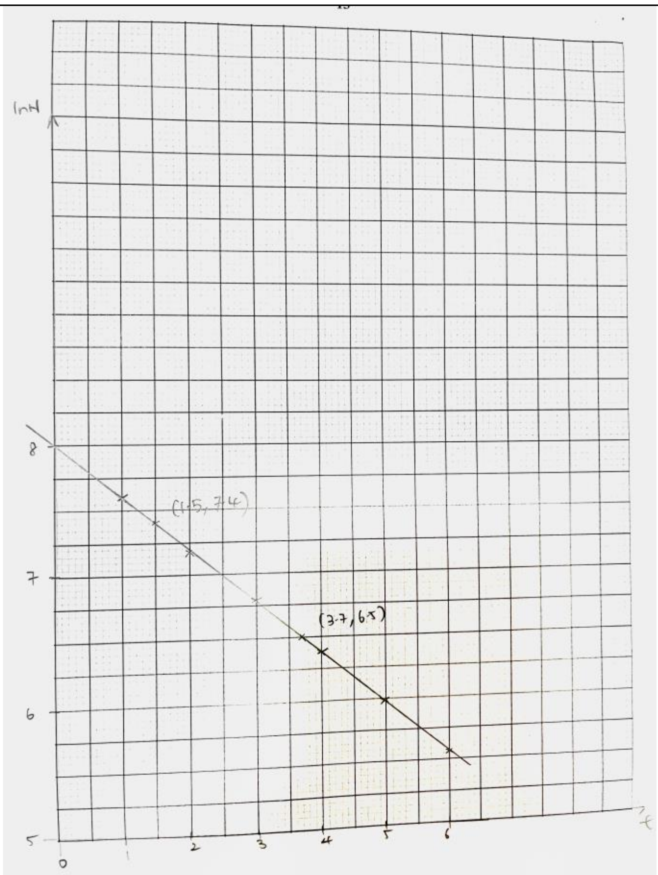
2022 AMath Prelim Paper 2 Answer Scheme		
1(a)	$\sec\theta - \dfrac{\cos\theta}{1+\sin\theta}$	
	$= \dfrac{1}{\cos\theta} - \dfrac{\cos\theta}{1+\sin\theta}$	M1
	$= \dfrac{1+\sin\theta - \cos^2\theta}{\cos\theta(1+\sin\theta)}$	
	$= \dfrac{1+\sin\theta-(1-\sin^2\theta)}{\cos\theta(1+\sin\theta)}$	M1
	$= \dfrac{\sin^2\theta+\sin\theta}{\cos\theta(1+\sin\theta)}$	
	$= \dfrac{\sin\theta(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$	M1
	$= \dfrac{\sin\theta}{\cos\theta}$	M1
	$= \tan\theta$	
1(b)	$\tan\theta = 2\sin\theta$	
	$\tan\theta - 2\sin\theta = 0$	
	$\dfrac{\sin\theta}{\cos\theta} - 2\sin\theta = 0$	M1
	$\sin\theta - 2\sin\theta\cos\theta = 0$	
	$\sin\theta(1-2\cos\theta)=0$	
	$\sin\theta = 0$ *or* $1-2\cos\theta = 0$	M1
	$\theta = 0(\text{NA}), \pi, 2\pi(\text{NA})$ $\quad \cos\theta = \dfrac{1}{2}$	A1
	$\text{Acute}\angle = \dfrac{\pi}{3}$	
	$\theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$	A1
2(i)	$r = 5$	B1
	$(x-5)^2+(y-7)^2 = 25$	B1

2(ii)	$y=-\frac{4}{3}x+\frac{16}{3}$ Gradient $CQ=\frac{3}{4}$ $y=\frac{3}{4}x+c$ At$(5,7), c=\frac{13}{4}$ $y=\frac{3}{4}x+\frac{13}{4}$	M1 M1 A1
2(iii)	$-\frac{4}{3}x+\frac{16}{3}=\frac{3}{4}x+\frac{13}{4}$ $x=1$ $y=4$ $Q(1,4)$	M1 A1
2(iv)	Let $R(x,y)$ $\left(\frac{1+x}{2},\frac{4+y}{2}\right)=(5,7)$ $\frac{1+x}{2}=5$ or $\frac{4+y}{2}=7$ $x=9$ or $y=10$ $R(9,10)$ Subst $(9,10)$into $4x+3y=k$ $4(9)+3(10)=k$ $k=66$	M1 M1 A1
3(i)	$f\left(\frac{3}{2}\right)=2\left(\frac{3}{2}\right)^3-3\left(\frac{3}{2}\right)^2+2\left(\frac{3}{2}\right)-3$ $=0$ Since $R=0,$ by factor theorem, $2x-3$ is a factor	B1
3(ii)	By Long division, $f(x)=(2x-3)(x^2+1)$	M1 A1

3(iii)	$\dfrac{-x^2+7x+8}{2x^3-3x^2+2x-3}=\dfrac{A}{2x-3}+\dfrac{Bx+C}{x^2+1}$ $-x^2+7x+8=A(x^2+1)+Bx+C(2x-3)$ When $x=1.5$; $16.25=3.25A$ $A=5$ When $x=0$; $8=5-3C$ $C=-1$ When $x=1$; $14=5(2)+(B-1)(-1)$ $B=-3$ $\dfrac{5}{2x-3}-\dfrac{3x+1}{x^2+1}$	M1 M1 M1 – substitution of values/comparing coefficients M1 – 2 values correct A1
4(a)	$\log_2(3x+2)-\log_{\frac{1}{2}}(x-1)=1$ $\log_2(3x+2)-\dfrac{\log_2(x-1)}{\log_2\frac{1}{2}}=1$ $\log_2(3x+2)(x-1)=1$ $3x^2-x-4=0$ $(3x-4)(x+1)=0$ $x=\dfrac{4}{3}$ or $x=-1$(NA)	M1 M1 M1 M1 A1
4(b)	$2\lg\dfrac{1}{3}(a+b)$ $=\lg\dfrac{1}{9}(a+b)^2$ $=\lg\dfrac{1}{9}(a^2+2ab+b^2)$ $=\lg\dfrac{1}{9}(9ab)$ $=\lg ab$ $=\lg a+\lg b$	M1 M1 M1 M1
5(i)	$9\tan^2 x=24\sec x-25$ $9(\sec^2 x-1)-24\sec x-25=0$ $9\sec^2 x-24\sec x+16=0$ $(3\sec x-4)^2=0$ $\sec x=\dfrac{4}{3}$ $\cos x=\dfrac{3}{4}$ $x=41.4°, 318.6°$	M1 M1 M1 A1

5(ii)	$3\operatorname{cosec}^2\left(2x+\frac{\pi}{4}\right)=4$ $\frac{3}{\sin^2\left(2x+\frac{\pi}{4}\right)}=4$ $\sin^2\left(2x+\frac{\pi}{4}\right)=\frac{3}{4}$ $\sin\left(2x+\frac{\pi}{4}\right)=\pm\frac{\sqrt{3}}{2}$ Acute$\angle=\frac{\pi}{3}$ $2x+\frac{\pi}{4}=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3}$ $x=\frac{\pi}{24},\frac{5\pi}{24},\frac{13\pi}{24}$	M1 M1 M1 A1
6(i)	$\ln N=\ln(Ab^{-t})$ $\ln N=\ln A+\ln b^{-t}$ $\ln N=\ln A-t\ln b$ Plot $\ln N$ against t $\ln A=8$ $A=2980$(Accept $2700-3000$) $-\ln b=\frac{7.4-6.5}{1.5-3.7}$ $b=1.51$(Accept $1.45-1.55$)	M1 M1 M1 A1 M1 A1

		
6(ii)	$N = 2980.957987(1.05445)^{-15}$ $N = 6.48$ $N \approx 6$ trees(Accept $5-7$ trees)	M1 A1

7(i)	$\dfrac{dy}{dx}=-\dfrac{1}{2}e^{-\frac{x}{2}}$ At $x=-6;\dfrac{dy}{dx}=-\dfrac{1}{2}e^{3}$ $y=-\dfrac{1}{2}e^{3}x+c$ At$(-6,e^{3});e^{3}=-\dfrac{1}{2}e^{3}(-6)+c$ $c=-2e^{3}$ $y=-\dfrac{1}{2}e^{3}x-2e^{3}$ At $y=0;-\dfrac{1}{2}e^{3}x-2e^{3}=0$ $x=-4$ $B(-4,0)$	M1 M1 M1 A1
7(ii)	Area of triangle $=\dfrac{1}{2}\times 2\times e^{3}$ $=e^{3}$ $\int_{-6}^{2} e^{-\frac{x}{2}}dx$ $=\left[-2e^{-\frac{x}{2}}\right]_{-6}^{2}$ $=-2e^{-1}-(-2e^{3})$ $=2e^{3}-\dfrac{2}{e^{1}}$ Area of shaded region $=2e^{3}-\dfrac{2}{e^{1}}-e^{3}$ $=19.3\text{ units}^{2}$	M1 M1 M1 M1 M1 A1
8(i)	By long division	M1

	$-2+\dfrac{6}{3-5x}$	A1
8(ii)	$x\left[\dfrac{2(-5)}{3-5x}\right]+2\ln(3-5x)$ $=\dfrac{-10x}{3-5x}+2\ln(3-5x)$	M1, M1 for each term A1
8(iii)	$\int \dfrac{-10x}{3-5x}+2\ln(3-5x)dx=x\ln(3-5x)^2+c$ $\int -\left(-2+\dfrac{6}{3-5x}\right)+\int 2\ln(3-5x)dx=x\ln(3-5x)^2+c$ $\left[2x+\dfrac{6}{5}\ln(3-5x)+c_1\right]+\int 2\ln(3-5x)dx=x\ln(3-5x)^2+c$ $\int 2\ln(3-5x)dx=x\ln(3-5x)^2-2x-\dfrac{6}{5}\ln(3-5x)+c_2$ $\int \ln(3-5x)dx=\dfrac{x\ln(3-5x)^2}{2}-x-\dfrac{3}{5}\ln(3-5x)+c_3$ $\int \ln(3-5x)dx=x\ln(3-5x)-x-\dfrac{3}{5}\ln(3-5x)+c_3$	M1 M1 M1 M1 A1
9(i)	$\cos\theta=\dfrac{AP}{28}$ $AP=28\cos\theta$ $\sin\theta=\dfrac{CQ}{4}$ $CQ=4\sin\theta$ $L=AD+DC+CB+AB$ $=4\sin\theta+28\cos\theta+28\sin\theta-4\cos\theta+4+28$ $=32+32\sin\theta+24\cos\theta$	 M1 M1 M1
9(ii)	$\tan\alpha=0.75$ $\alpha=36.9°$ $R=\sqrt{32^2+24^2}$ $R=40$ $32+40\sin(\theta+36.9°)$	M1 M1 A1
9(iii)	$32+40\sin(\theta+36.9°)=70$ $\sin(\theta+36.9°)=0.95$ $\theta+36.9°=71.8051°,108.1948°$ $\theta=34.9°,71.3°$	M1 A1

9(iv)	Since $\sin(\theta+36.9°)\le 1$ $40\le\sin(\theta+36.9°)\le 40$ $32+40\sin(\theta+36.9°)\le 32+40$ $L\le 72$ $\therefore L$ should not exceed 72.	M1 A1
10	$\mathrm{f}'(x)=\int(3\cos 3x-4\sin 2x)dx$ $=\dfrac{3\sin 3x}{3}-\dfrac{4(-\cos 2x)}{2}+c$ $=\sin 3x+2\cos 2x+c_1$ $\mathrm{f}(x)=\int(\sin 3x+2\cos 2x+c_1)dx$ $=\dfrac{-\cos 3x}{3}+\dfrac{2\sin 2x}{2}+c_1x+c_2$ $=\dfrac{-\cos 3x}{3}+\sin 2x+c_1x+c_2$ $\mathrm{f}(0)=0$ $0-\dfrac{1}{3}+c_2=0$ $c_2=\dfrac{1}{3}$ $\mathrm{f}(x)=\dfrac{-\cos 3x}{3}+\sin 2x+c_1x+\dfrac{1}{3}$ $0-0+\dfrac{\pi}{2}c_1+\dfrac{1}{3}=\dfrac{5}{6}$ $c_1=\dfrac{1}{\pi}$ $f(x)=\dfrac{-\cos 3x}{3}+\sin 2x+\dfrac{x}{\pi}+\dfrac{1}{3}$ $f(\dfrac{\pi}{3})=\dfrac{-\cos\pi}{3}+\sin 2\left(\dfrac{\pi}{3}\right)+\dfrac{\pi}{3}\times\dfrac{1}{\pi}+\dfrac{1}{3}$ $=1+\dfrac{\sqrt{3}}{2}$	A1, A1 A1, A1 M1 M1 M1 M1, M1