

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 (a) If $64m^{3x} = 27$, find the exact value of m^x . [2]

(b) State the condition for m , in order for $64m^{3x} = 27$ to be valid. [1]

2 **(a)** Given that $y = \frac{2x}{9 - x^2}$, find $\frac{dy}{dx}$. [2]

(b) Hence, find $\int \frac{9 + x^2}{(9 - x^2)^2} dx$. [2]

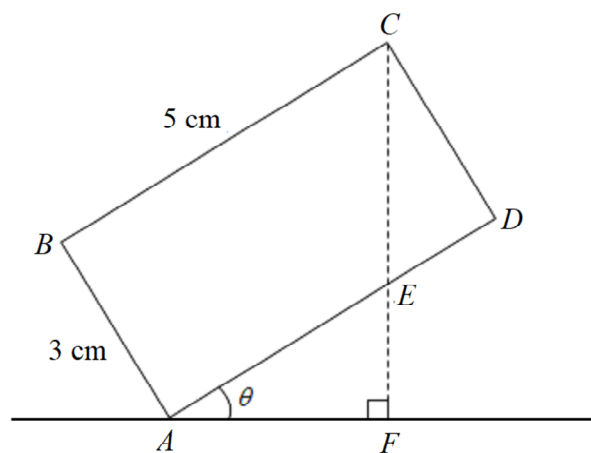
- 3** Show that the equation $e^z(2e^z - 1) = 10$ has only 1 solution and find its value correct to 2 decimal places. [5]

4 The equation of a polynomial is given by $f(x) = 3x^3 - 11x^2 + 7x + 1$.

(a) Find the remainder when $f(x)$ is divided by $x + 2$. [1]

(b) Solve the equation $3x^3 - 11x^2 + 7x + 1 = 0$. [7]

5



The diagram above shows a rectangle $ABCD$ with $AB = 3\text{ cm}$ and $BC = 5\text{ cm}$.

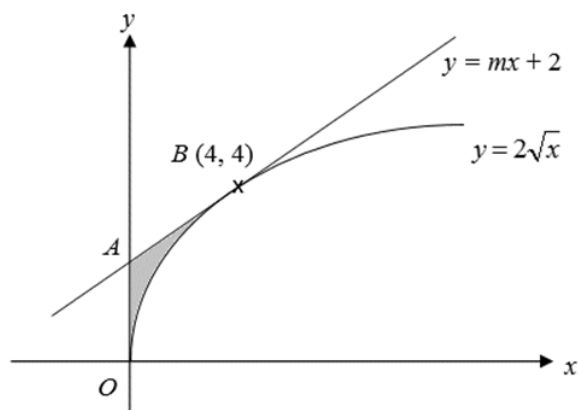
The side AD of the rectangle makes an acute angle θ with the horizontal ground.

- (a) Show that $CF = 3\cos\theta + 5\sin\theta$, where CF is the height of C above the ground. [3]

(b) Express CF in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(c) Find the maximum value of CF and the corresponding value of θ . [3]

6



In the diagram, the line $y = mx + 2$ is the tangent to the curve $y = 2\sqrt{x}$ at the point $B(4, 4)$.

(a) Find the coordinates of A . [1]

(b) Find the value of m . [2]

(c) Find the area of the shaded region. [4]

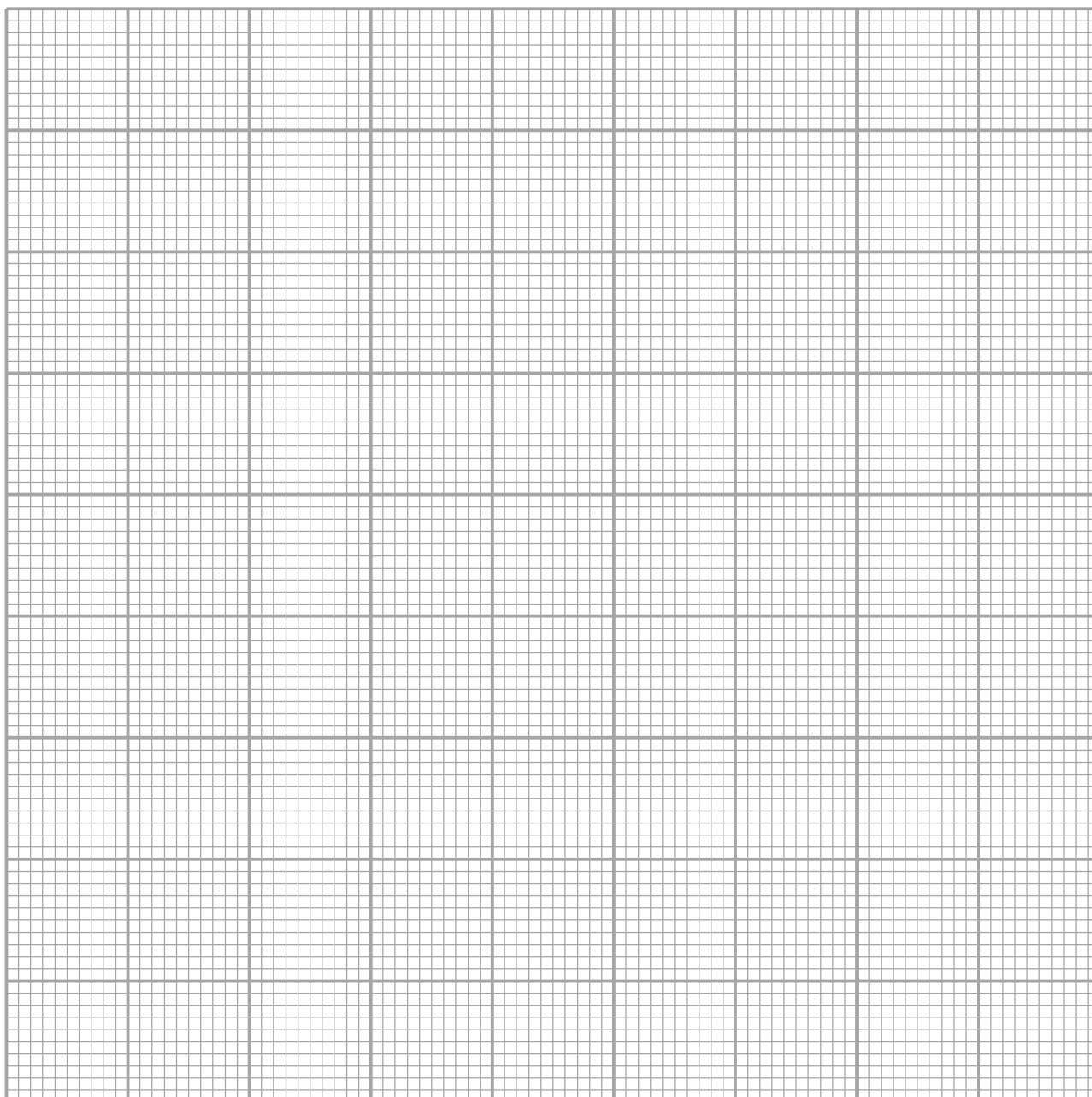
- 7 The table shows experimental values of two variables, x and y .

x	10	20	30	40
y	0.06	0.38	2.37	14.70

It is known that x and y are related by the equation $y = \frac{b^x}{10^A}$, where A and b are constants.

- (a) Draw a straight line graph of $\lg y$ against x , using a scale of 2 cm to represent 5 units on the x -axis and 2 cm to represent 0.5 units on the $\lg y$ -axis.

[3]



(b) Use your graph to estimate the value of A and of b . [4]

(c) On the same diagram, draw the line representing $y^4 = 10^{-x}$ and **hence**, find the value of x for which $10^{\frac{A-x}{4}} = b^x$. [3]

- 8** A particle moves in a straight line so that at time t seconds after leaving a fixed point O , its displacement, s m, is given by $s = 5 - 5e^{-t} - 0.4t$.

(a) Find the initial velocity of the particle. [3]

(b) Find the value of t when the particle is instantaneously at rest. [3]

(c) Find the acceleration of the particle at this instant, correct to 1 decimal place. [2]

(d) Find the total distance travelled from $t = 0$ to $t = 5$. [3]

(e) Show that the particle is again at O at some instant during the thirteenth second after first passing through O . [2]

9 The equation of the curve is $y = -x^2 + 5x - 4$.

(a) Find the set of values of x for which the curve lies below the line $y = -10$. [4]

(b) The line $y = -x + c$ is a tangent to the curve $y = -x^2 + 5x - 4$. Find the value of c . [3]

- 10 (a)** Expand and simplify the first three terms in the expansion of $\left(3 + \frac{x}{2}\right)^{10}$, in increasing powers of x .

[2]

- (b)** Using your answer from **part (a)**, find the value of $(3.04)^{10}$, correct to 3 significant figures.

[2]

- (c) In the expansion of $\left(x^2 - \frac{k}{x}\right)^9$, where k is a positive constant, the term independent of x is 5376. Find the value of k .

[4]

11 A circle passes through the origin O and cuts the x and y axes at 3 and 4 respectively.

(a) Explain why the coordinates of the centre are $(1.5, 2)$. [1]

(b) Hence, find the equation of the circle, leaving your answer in general form. [3]

(c) Given that OP is the diameter of the circle, find the equation of the tangent at P . [5]

- (d) Two tangents to the circle, which are parallel to the y -axis, meet the tangent found in **part (c)**. Find the coordinates of the points of intersections. [4]

- (e) Determine if the point $R(1, 4.3)$ lies inside or outside the circle. [3]

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