Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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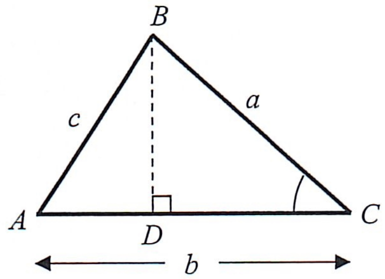
MA304.3.3 – Cosine Rule

At the end of this unit, you should be able to
• **use the cosine rule to solve triangles**

E1. Given the triangle below,



(a) find CD in terms of a and (angle) C

$\cos(C) = \frac{CD}{a}$

$CD = a\cos(C)$

(b) hence express AD in terms of a, b and C

$AD = b - a\cos(C)$

(c) find also BD in terms of a and C

$\sin(C) = \frac{BD}{a}$

$BD = a\sin(C)$

(d) using ΔABD, show by Pythagoras theorem on AD, AB and BD,

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$c^2 = (a\sin(C))^2 + (b - a\cos(C))^2$

$= b^2 - 2ab\cos(C) + (a\cos(C))^2 + (a\sin(C))^2$

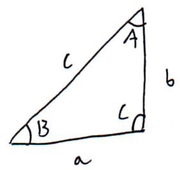
$= b^2 - 2ab\cos(C) + a^2(\cos^2(C) + \sin^2(C))$

$= a^2 + b^2 - 2ab\cos(C)$ (shown)

In a triangle with sides *a*, *b* and *c*, and with the angles opposite the sides as *A*, *B* and *C* respectively, the cosine rule states that

$$c^2 = a^2 + b^2 - 2ab\cos C$$

E2. Express $\cos C$ in terms of a, b and c. By interchanging the variables, or otherwise, write down similar expressions for $\cos A$ and $\cos B$.



$c^2 = a^2 + b^2 - 2ab \cos C$

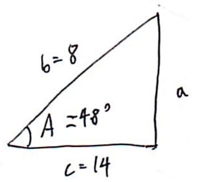
$2ab \cos C = a^2 + b^2 - c^2$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

E3. A triangle ABC is such that $\measuredangle A = 48°$, $b = 8$ cm and $c = 14$ cm. Find the length BC.



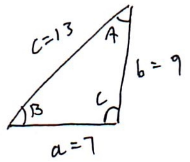
$a^2 = b^2 + c^2 - 2bc \cos(48°)$

$a^2 = 260 - 224 \cos(48°)$

$a = 10.5$ cm (3 s.f.)

$\therefore BC = 10.5$ cm

P1. Solve the triangle with $a = 7$ cm, $b = 9$ cm and $c = 13$ cm. What do you observe about the relationship between the lengths of each side and the corresponding angle (i.e. a and A) across all three pairs?



$7^2 = 13^2 + 9^2 - 234 \cos A$

$234 \cos A = 201$

$\cos A = \frac{201}{234}$

$\measuredangle A = \cos^{-1}\left(\frac{201}{234}\right)$

$= 30.8°$ (1d.p.)

$9^2 = 13^2 + 7^2 - 182 \cos B$

$182 \cos B = 137$

$\cos B = \frac{137}{182}$

$\measuredangle B = \cos^{-1}\left(\frac{137}{182}\right)$

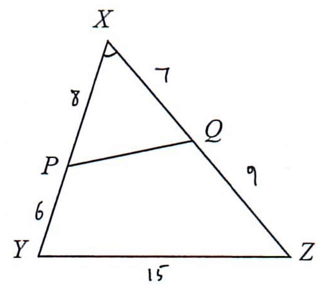
$= 41.2°$ (1d.p.)

$\measuredangle C = 180° - 30.8° - 41.2°$

$= 108°$

The longer the side, the larger the corresponding angle.

P2. In the figure $XP = 8$ cm, $XQ = 7$ cm, $XY = 14$ cm, $XZ = 16$ cm and $YZ = 15$ cm. Find the length of PQ.



$15^2 = 14^2 + 16^2 - 448 \cos(YXZ)$

$448 \cos(\angle YXZ) = 227$

$\cos(\angle YXZ) = \frac{227}{448}$

$\measuredangle YXZ = \cos^{-1}\left(\frac{227}{448}\right)$

$PQ^2 = 8^2 + 7^2 - 112 \cos\left(\cos^{-1}\left(\frac{227}{448}\right)\right)$

$PQ^2 = 56.25$

$PQ = 7.5$ cm