RAFFLES INSTITUTION
2025 YEAR 5 TIMED PRACTICE
Higher 2

MATHEMATICS **9758**

2 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

[Turn over

1 Given that the curve $y = ax + b|x|$ passes through $(-1, -4)$ and intersects another curve $y = cx - 3$ at $x = -3$ and $x = 3$, find the values of a, b and c. [4]

2 Referred to an origin O, the position vectors of three points A, B and C are $3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$, $\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ and $\alpha\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ respectively.

(a) Determine the value of α for which points O, A, B and C are coplanar. [2]

(b) Given that $\overrightarrow{OD} = \overrightarrow{OA} + \lambda\overrightarrow{OB}$, find the value of λ for which $\overrightarrow{OD}$ is perpendicular to the y – axis. [2]

(c) Find the exact area of triangle OAD. [2]

3 The function f is defined by

$$\mathrm{f} : x \mapsto (3 - x)(x + 1), \text{ where } x \in \mathbb{R}, \ x \leq 1.$$

(a) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

(b) Sketch on the same diagram the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, giving the coordinates of any points where the curves cross the axes. [3]

The function g is defined by

$$\mathrm{g} : x \mapsto \mathrm{e}^{-x}, \quad \text{where } x \in \mathbb{R}, \quad x \geq 0.$$

(c) Show that the composite function fg exists and state the range of fg. [2]

4 It is given that $2 + \mathrm{i}$ is a root of the equation

$$2z^4 + az^3 + bz^2 + az - 15 = 0.$$

(a) Find the values of the real numbers a and b, and without the use of a calculator, find the other roots of the equation. [6]

(b) Deduce the roots of the equation $2w^4 + a\mathrm{i}w^3 - bw^2 - a\mathrm{i}w - 15 = 0$. [2]

5 **Do not use a calculator in answering this question.**

The curve C has equation $x^2 + 3xy - y^2 + 4 = 0$.

(a) Find the exact x-coordinates of the stationary points of C. [4]

(b) For the stationary point with $x < 0$, determine, by differentiation, whether it is a maximum or minimum. [4]

6 **(a)** Express $x^3 - 8$ in the form $(x-2)(ax^2 + bx + c)$, where a, b and c are constants to be determined. [1]

(b) Without using a calculator, solve the inequality $\dfrac{x^2}{x+1} \geq \dfrac{8}{x(x+1)}$. [4]

(c) Hence, solve $\dfrac{(q+2)^2}{|q+2|+1} \geq \dfrac{8}{|q+2|(|q+2|+1)}$. [3]

7 A particle moves on the cartesian plane in such a way that at time t, its position is given by the parametric equations

$$x = t\cos t,\ y = t\sin t.$$

The particle starts moving from the origin.

(a) Find the equation of the normal to the path of the particle at $t = \dfrac{\pi}{2}$, leaving your answer in exact form. [5]

(b) It is given that the instantaneous speed of the particle at time t is defined as

$$\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}.$$

(i) Find an expression for $\sqrt{\left(\dfrac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)^2}$ in terms of t, giving your answer in its simplest form. [2]

(ii) Find the exact instantaneous speed of the particle when it crosses the positive x-axis for the first time. [2]

(iii) Explain if the particle will ever come to rest. [1]

8 A home interior designer is looking to design a pull-down wardrobe lift. To test his concept, the designer built a small-scale model of it, in which points (x, y, z) are defined relative to a corner of the model at $O(0, 0, 0)$. The rods used in the model can be modelled by equations of straight lines. The main rod can be modelled by the line l_1 with equation

$$\mathbf{r} = \begin{pmatrix} 5 \\ -5 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \lambda \in \mathbb{R}.$$

Another rod, line l_2, passes through the points $A(5, -5, 8)$ and $B(8, -4, 3)$.

(a) A supporting wire connects B to a point F on l_1 such that $\overrightarrow{BF}$ is perpendicular to l_1. Find the position vector of F. [3]

(b) A retractable rod, line l_R, is placed such that it can be modelled by reflecting l_2 about l_1. Find a vector equation of l_R. [3]

The panels of the small-scale model can be modelled by equations of planes. The main panel, plane p_1, contains the main rod l_1 and an attachment point $C(0, 0, 6)$.

(c) Show that p_1 can be represented by the cartesian equation $-x + y + 5z = 30$. [3]

(d) A decorative plank, plane p_2, has equation

$$p_2 : -3x + \alpha y + 15z = \beta,$$

where $\alpha, \beta \in \mathbb{R}$. If the distance between p_1 and p_2 is exactly $3\sqrt{3}$ units, find the possible values of α and β. [3]