Term 3 A.Math Class Test (Revision Paper 1) Solution

Answer all the questions.

Question 1:

1 **(a)** Given that $\log_b (x^3 y) = p$ and $\log_b \left(\dfrac{y}{x^2}\right) = q$, express $\log_b \sqrt{xy}$ in terms of p and q. [6]

(b) Solve the equation $\log_x 8 + 2 \log_2 x = 7$. [4]

Solution:

1 (a)	$\log_b (x^3 y) = p$ $\log_b x^3 + \log_b y = p$ $3 \log_b x + \log_b y = p$ ---------- (1) $\log_b \left(\dfrac{y}{x^2}\right) = q$ $\log_b y - \log_b x^2 = q$ $\log_b y - 2 \log_b x = q$ ---------- (2) (1) – (2), $5 \log_b x = p - q$ $\log_b x = \dfrac{p-q}{5}$ ---------- (3) Substitute (3) into (2). $\log_b y - 2\left(\dfrac{p-q}{5}\right) = q$ $\log_b y = q + 2\left(\dfrac{p-q}{5}\right)$ $\log_b y = q + \dfrac{2}{5}p - \dfrac{2}{5}q$ $\log_b y = \dfrac{2}{5}p + \dfrac{3}{5}q$ $\log_b \sqrt{xy}$ $= \log_b (xy)^{\frac{1}{2}}$ $= \dfrac{1}{2}(\log_b x + \log_b y)$

	$= \frac{1}{2}\left(\frac{p - q}{5} + \frac{2}{5}p + \frac{3}{5}q\right)$ $= \frac{1}{2}\left(\frac{p - q}{5} + \frac{2}{5}p + \frac{3}{5}q\right)$ $= \frac{1}{2}\left(\frac{3p + 2q}{5}\right)$ $= \frac{1}{10}(3p + 2q)$
1(b)	$\log_x 8 + 2\log_2 x = 7$ $\log_x 2^3 + 2\log_2 x = 7$ $\frac{3}{\log_2 x} + 2\log_2 x = 7$ Let $y = \log_2 x$. $\frac{3}{y} + 2y = 7$ $3 + 2y^2 = 7y$ $2y^2 - 7y + 3 = 0$ $(2y - 1)(y - 3) = 0$ $2y - 1 = 0$ or $y - 3 = 0$ $y = \frac{1}{2}$ $y = 3$ When $y = \frac{1}{2}$, $\log_2 x = \frac{1}{2}$ $x = 2^{\frac{1}{2}}$ $x = \sqrt{2}$ $x = 1.414$ $x \approx 1.41$ When $y = 3$, $\log_2 x = 3$ $x = 2^3$ $x = 8$

	$\therefore\ x = \sqrt{2}$ [Accept $x = 1.41$] or $x = 8$

Question 2:

2 **(i)** Use the substitution $y = 2^x$ to express the equation $2^{3x+1} + 7\left(2^x\right) = 4 - 4^x$ as a cubic equation in y. [1]

(ii) Show that $y = \frac{1}{2}$ is the only real solution of this equation. [4]

(iii) Hence solve the equation $2^{3x+1} + 7\left(2^x\right) = 4 - 4^x$. [1]

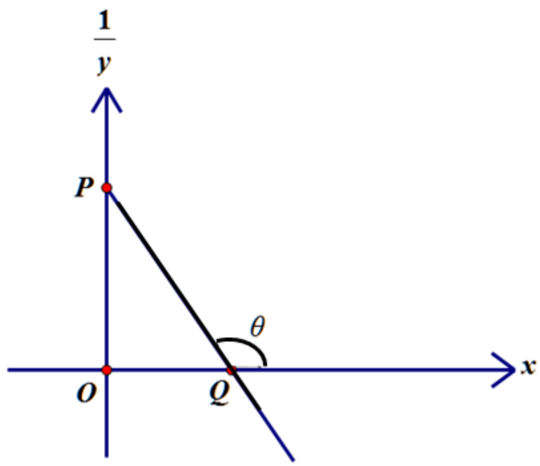
Solution:

2(i)	$2^{3x+1} + 7\left(2^x\right) = 4 - 4^x$ $2\left(2^{3x}\right) + 7\left(2^x\right) = 4 - 2^{2x}$ $2\left(2^{3x}\right) + 2^{2x} + 7\left(2^x\right) - 4 = 0$ $2\left(2^x\right)^3 + \left(2^x\right)^2 + 7\left(2^x\right) - 4 = 0$ $2y^3 + y^2 + 7y - 4 = 0$
2(ii)	Let $f(y) = 2y^3 + y^2 + 7y - 4$. $f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 + 7\left(\frac{1}{2}\right) - 4$ $= 0$ $\therefore\ (2y - 1)$ is a factor of $f(y)$.

	$\begin{array}{r} y^2 + y + 4 \\ 2y - 1 \overline{\smash{\big)} 2y^3 + y^2 + 7y - 4} \\ \underline{2y^3 - y^2 } \\ 2y^2 + 7y - 4 \\ \underline{2y^2 - y } \\ 8y - 4 \\ \underline{8y - 4} \\ \underline{0} \end{array}$ $\mathrm{f}(y) = 0$ $2y^3 + y^2 + 7y - 4 = 0$ $(2y - 1)(y^2 + y + 4) = 0$ $2y - 1 = 0$ or $y^2 + y + 4 = 0$ $2y = 1 \qquad y = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(4)}}{2(1)}$ $y = \dfrac{1}{2} \qquad = \dfrac{-1 \pm \sqrt{-15}}{2}$ (no real solution) Hence $y = \dfrac{1}{2}$ is the only real solution of this equation.
2(iii)	$y = 2^x$ $2^x = \dfrac{1}{2}$ $2^x = 2^{-1}$ $\therefore x = -1$

Question 3:

3 The two variables x and y are related by the equation $2y-5xy=1$. The diagram shows part of a straight line graph obtained by plotting $\frac{1}{y}$ against x.



Find

(i) the coordinates of P and Q, [3]

(ii) the value of θ in degrees. [2]

Solution:

3(i)	$2y-5xy=1$ $y(2-5x)=1$ $\frac{1}{y}=2-5x$ When $x=0, \frac{1}{y}=2$ $\therefore P(0,2)$ When $y=0, x=\frac{2}{5}$ $\therefore Q(\frac{2}{5},0)$

3(ii)	$\tan\theta = -5$ $\alpha = 78.7°$ $\theta = 180° - 78.7°$ $= 101.3°$ (to 1 d.p.)

Question 4:

The expression $ax^3 + x^2 + bx$ has remainder 3 when it is divided by $x+1$. When the expression is divided by $x-2$, the remainder is 6. Find the remainder when the expression is divided by $x-3$.

[4]

4	Let $ax^3 + x^2 + bx$ be f(x) f(x) = $ax^3 + x^2 + bx$ $\mathrm{f}(-1) = 3$ $-a - b = 2 ---(1)$ $\mathrm{f}(2) = 6$ $8a + 4 + 2b = 6$ $8a + 2b = 2$ $4a + b = 1 ---(2)$ (1) + (2) : $3a = 3$ $a = 1$ $b = -3$ $\therefore \mathrm{f}(x) = x(x^2 + x - 3)$ $\mathrm{f}(3) = 27$ Therefore, remainder = 27

Question 5:

5 **(i)** Factorise the expression $3x^3 - 8x^2 - 5x + 6$ completely. [3]

(ii) Hence, sketch the graph of $y = 3x^3 - 8x^2 - 5x + 6$, indicate clearly the coordinates of the x-intercepts and y- intercepts. [3]

Solution:

Let $f(x) = 3x^3 - 8x^2 - 5x + 6$.

$$\begin{aligned} f(-1) &= 3(-1)^3 - 8(-1)^2 - 5(-1) + 6 \\ &= -3 - 8 + 5 + 6 \\ &= 0 \end{aligned}$$

$(x + 1)$ is a factor of $f(x)$

$3x^3 - 8x^2 - 5x + 6 = (x + 1)(ax^2 + bx + c)$

Equating coefficients of x^3: $a = 3$

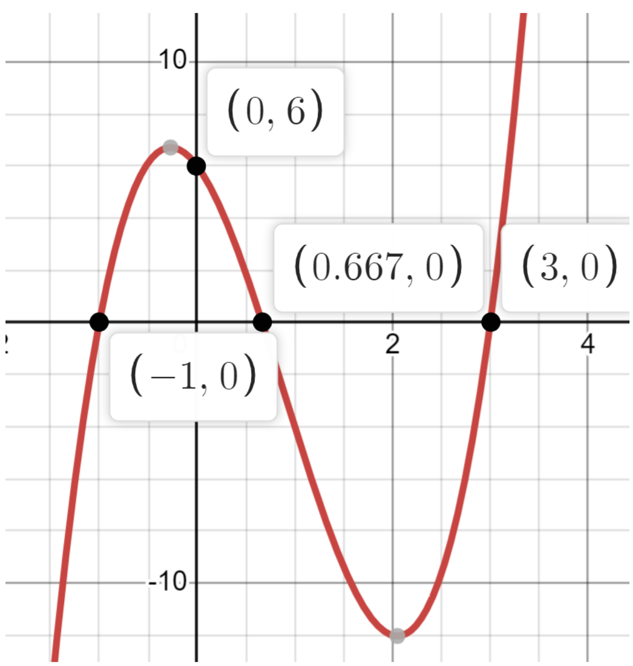
Equating coefficients of x^2: $-8 = a + b$

$b = -11$

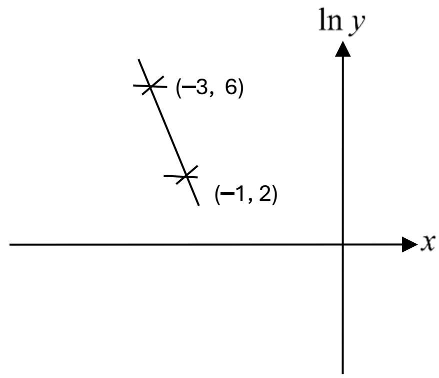
Equating constant term: $6 = c$

$$\begin{aligned} 3x^3 - 8x^2 - 5x + 6 &= (x + 1)(3x^2 - 11x + 6) \\ &= (x + 1)(3x - 2)(x - 3) \end{aligned}$$

(ii)



Question 6:
The figure shows part of a straight line graph obtained by plotting values of the variables indicated. Express y as a function of x. [3]



Solution:

$$gradient = \frac{6-2}{-3+1}$$

$$gradient = -2$$

$$\ln y = -2x + C$$

$$Point : (-1, 2)$$

$$2 = -2(-1) + C$$

$$C = 0$$

$$\ln y = -2x$$

$$y = e^{-2x}$$

Question 7:
Using long division, find the quotient and remainder when $x^5 - 3x^4 + 2x^3 - 2x^2 + 3x + 1$ is divided by $2x^2 + 3$. [3]

Solution: