Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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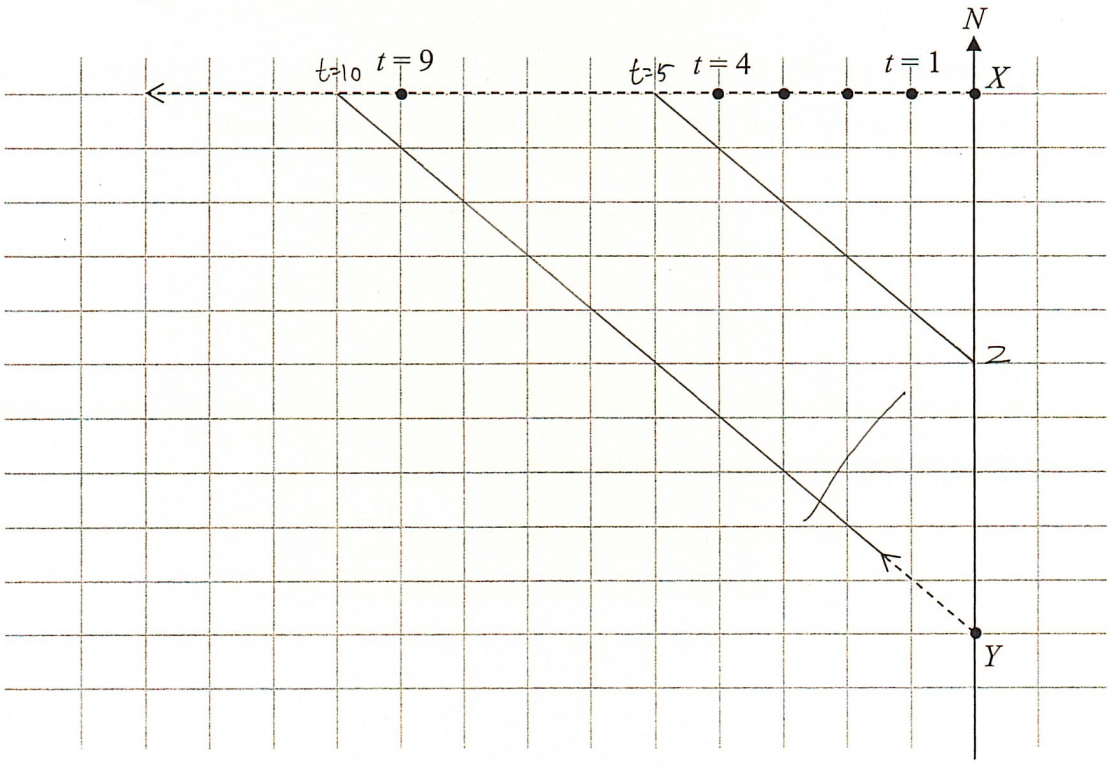
MA304.4.X2 – Angle of Interception

Ever wonder how a missile is fired to intercept another which approaching or flying towards another target? Some questions to ponder:

- Does the angle of interception (angle of firing) depend on the speed of both missiles?
- Does the angle of interception depend on the distance between the missiles?

Consider a car X which is 100 metres due North of another Y. Let t be the time measured in seconds and initially ($t = 0$) car X starts to move to the West at a speed of 10 ms^{-1}.

(a) The 'map' below shows the location of car X at different time t.



Each grid line represents a distance of 10 metres from the next grid line.

If car Y is to move at a bearing of 315°, at a speed of $10\sqrt{2}$ ms^{-1}, plot its location at different times. Will car Y meet car X?

(b) Another car Z is initially 50 metres due South of car X (i.e. halfway between cars X and Y). It starts to move also at a speed of $10\sqrt{2}$ ms^{-1} to meet car X. Should the bearing of travel of car Z be smaller, larger or equal to 315° ?

Equal to 315°

(c) As cars Y and Z move to meet car X, both move off at the same time as car X. Which car will meet X first? How much faster would the first car meet car X before the second car?

Z will meet X first.
Time difference = $10-5$ s
$= 5$ s

(d) A fourth car, car W, is only able to travel at a speed of 10 ms^{-1}. If it is initially also 50 metres due South of car X, will it be able to meet car X eventually?

No. They are travelling at the same speed.

(e) Car Y decides to increase its speed to $20\sqrt{2}$ ms^{-1}, twice its initial speed. Should its bearing of travel be greater, smaller or still equal to 315° ?

Bearing of travel $= 360^\circ - \sin^{-1}\left(\frac{1}{2\sqrt{2}}\right)^\circ$
$= 337.3^\circ$ (1dp.)

In interception problems, if the speeds of the two objects are fixed, the angle of interception does not depend on the distance between the two points.

S1. Ship A which was originally due north of Ship B at a distance of 80 km travels at a speed of 100 km/h at a bearing of 049°. Ship B sets out at the same time in an attempt to intercept Ship A. If Ship B travels at 150 km/h, determine the bearing at which Ship B must set sail. Determine also the time taken for the interception to take place.

Let θ be the bearing.

Bearing $< 049°$.

$100 \sin(44°) = 150 \sin\theta$

$\theta = \sin^{-1}\left(\frac{100\sin(44°)}{150}\right)$

$= 030.2°$ (1 d.p.)

Time taken $= 80 \div (150\cos\theta - 100\cos 44°)$ h

$= 1.25$ h (3 s.f.)

$= 1$ h 15 min